|

Notes for Paper 55 Part 6

The General Dynamical Equations of Physics :

Newtonian Dynamics

The equations of classical dynamics are always given by ECE theory in any frame of reference moving arbitrarily with respect to another frame. They are given by Cartan geometry :

$$T = d \wedge q + \omega \wedge q \quad -(1)$$
$$R = d \wedge \omega + \omega \wedge \omega \quad -(2)$$
$$D \wedge T = R \wedge q \quad -(3)$$
$$D \wedge R = 0 . \quad -(4)$$

The notation is fully explained elsewhere. Einstein Hilbert equation is pure translation :

$$T = 0 \quad -(5)$$
$$R \wedge q = 0 \quad -(6)$$
$$D \wedge R = 0 \quad -(7)$$

Pure rotation is

$$T = D \wedge q \quad -(8)$$
$$d \wedge T = 0 . \quad -(9)$$

Interaction of rotation and translation is given in any frame by eqs. (1) - (4).

The Newton equations are given by eq. (4)

2)

and the equation of pure rotational dynamics by eq. (8). This is the Euler equation. Both the Newton and Euler equations are approximations in the classical non-relativistic limit.

<u>Newton Equations</u>

Eqn. (7) is:

$$d \wedge R^a{}_b = R^a{}_c \wedge \omega^c{}_b - \omega^a{}_c \wedge R^c{}_b \quad - (10)$$

and eq. (2) is:

$$d \wedge \omega^a{}_b = R^a{}_b - \omega^a{}_c \wedge \omega^c{}_b \quad - (11)$$

Write these as:

$$d \wedge \omega^a{}_b = j^a{}_b \quad - (12)$$

$$d \wedge R^a{}_b = j_R{}^a{}_b \quad - (13)$$

The vector form of these equations is known from previous work. Also from eq. (1):

$$d \wedge q^a = T^a - \omega^a{}_b \wedge q^b \quad - (14)$$

i.e.

$$d \wedge q^a = j^a. \quad - (15)$$

So eqs. (12) to (15) put the equations of dynamics in the same form as those of electrodynamics.

3) The Riemann form is:

$$R^a{}_{b\mu\nu} = - R^a{}_{b\nu\mu} \quad -(16)$$

and so has orbital and spin components as for the torsion form. From eq. (13) we can write:

$$\boxed{\underline{\nabla} \cdot \underline{R}^a{}_b(\text{orbital}) = j^a{}_b{}^{(0)}} \quad -(17)$$

Now recognize ~~that~~:

$$\underline{R}^a{}_b(\text{orbital}) = \frac{kR}{c^2} \underline{F}^a{}_b \quad -(18)$$

where k is Einstein's constant and R is the scalar curvature of ECE theory. Here $\underline{F}^a{}_b$ has the units of force. If:

$$\underline{F}^a{}_b = m_1 \underline{g}^a{}_b \quad -(19)$$

eq. (17) is the Newton inverse square law:

$$\underline{F}^a{}_b = - \frac{G m_1 m_2}{r^2} \underline{k}^a{}_b = m_1 \underline{g}^a{}_b \quad -(20)$$

where

$$k = \frac{8\pi G}{c^2}. \quad -(21)$$

4)

The classical Newton inverse square law is the special case:

$$a = b. \quad -(22)$$

More generally the acceleration due to gravity is a rank two tensor, and so is the Newtonian force. They both come from the second Bianchi identity of Cartan geometry, and from the orbital part of the Riemann form.

The spin part of the Riemann form is entirely neglected in Newtonian dynamics. The interaction of the orbital and spin parts of the Riemann form are governed by equations which have the same structure as the Faraday law and Ampère Maxwell law of ECE theory.