1)

Notes for Paper 54, Part 4

Derivation of the Cartan Torsion and Bianchi Identity from the ECE Lagrangian Density.

It was shown in part 3 that the ECE lagrangian density is:

$$\mathcal{L} = c^2 T + D_\mu q^a_{\ \nu} D^\mu q^\nu_{\ a} \quad -(1)$$

and that the Euler Lagrange equation of ECE theory is:

$$\frac{\partial \mathcal{L}}{\partial q^\nu_{\ a}} = D^\mu \left(\frac{\partial \mathcal{L}}{\partial (D^\mu q^\nu_{\ a})} \right) \quad -(2)$$

Using eq (1) in eq. (2) gives the ECE Lemma:

$$D^\mu \left(D_\mu q^a_{\ \nu} \right) = 0, \quad -(3)$$

which is the same as:

$$\square q^a_{\ \nu} = R q^a_{\ \nu} \quad -(4)$$

where

$$R := q^\lambda_{\ a} \left(\partial^\mu \left(\Gamma^\nu_{\mu\lambda} q^a_{\ \nu} \right) - \partial^\mu \left(\omega^a_{\ \mu b} q^b_{\ \lambda} \right) \right)$$

$$-(5)$$

II) Eq. (3) or (4) is the fundamental wave equation of generally covariant quantum mechanics. The ECE wave equation is obtained using the Einstein Ansatz:

$$R = -kT \quad -(6)$$

in eq. (4). Therefore eqs (1) to (6) unify relativity and wave mechanics. This is not possible in the standard model.

By reference to volume 2 of M. W. Evans, Generally Covariant Unified Field Theory (Abramis, 2006, in press) the Cartan torsion is the difference of two tetrad postulates. Eq. (3) shows that the ECE wave equation is the covariant derivative of the tetrad postulate. The Lagrangian density is ∂T plus the product of two tetrad postulates. The latter is the fundamental necessity for a vector field to be independent of its components and basis elements.

The Cartan torsion is therefore an inevitable consequence of the tetrad postulate. This was shown in volume 2 as follows.

III) Consider two tetrad postulates:

$$\partial_\mu q^a_\lambda + \omega^a_{\mu b} q^b_\lambda = \Gamma^\nu_{\mu\lambda} q^a_\nu \quad -(7)$$

$$\partial_\lambda q^a_\mu + \omega^a_{\lambda b} q^b_\mu = \Gamma^\nu_{\lambda\mu} q^a_\nu \quad -(8)$$

All that has been done is to change the index labelling, so we have written out the tetrad postulate twice. Now subtract eq. (8) from eq. (7):

$$\partial_\mu q^a_\lambda - \partial_\lambda q^a_\mu + \omega^a_{\mu b} q^b_\lambda - \omega^a_{\lambda b} q^b_\mu$$
$$= \Gamma^\nu_{\mu\lambda} q^a_\nu - \Gamma^\nu_{\lambda\mu} q^a_\nu, \quad -(9)$$

to obtain the Cartan torsion:

$$T^a_{\mu\lambda} = \Gamma^\nu_{\mu\lambda} q^a_\nu - \Gamma^\nu_{\lambda\mu} q^a_\nu \quad -(10)$$
$$= T^\nu_{\mu\lambda} q^a_\nu .$$

In differential form notation eq. (9) is:

$$T^a_{\mu\nu} = (d \wedge q^a)_{\mu\nu} + \omega^a_{\mu b} \wedge q^b_\nu \quad -(11)$$

which is the first Cartan structure equation.

In standard notation in differential geometry eq. (10) is written as:

$$T^a = d \wedge q^a + \omega^a{}_b \wedge q^b. \quad -(11)$$

The ECE Ansatz:

$$F^a = A^{(0)} T^a \quad -(12)$$

defines the electromagnetic tensor.

So we have obtained the electromagnetic tensor from the lagrangian density using the tetrad postulate.

Finally, the field equations of ECE theory have been obtained from the first Bianchi identity:

$$d \wedge T^a + \omega^a{}_b \wedge T^b := R^a{}_b \wedge q^b \quad -(13)$$

which states that the covariant derivative of the Cartan torsion is a cyclic sum over Riemann tensor elements.

v) The identity (134) again follows from the tetrad postulate, as proven in all detail in chapter 17 of vol. 1 of M. W. Evans, Generally Covariant Unified Field Theory (Abramis 2005). Using the Ansatz (13) and the Ansatz:

$$A^a_\mu = A^{(0)} q^a_\mu \quad - (14)$$

in eq. (134) produces the homogeneous and inhomogeneous ECE field equations:

$$d \wedge F^a = \mu_0 j^a = A^{(0)} \left(R^a{}_b \wedge q^b - \omega^a{}_b \wedge T^b \right) \quad - (15)$$

and its Hodge dual:

$$d \wedge \tilde{F}^a = \mu_0 J^a = A^{(0)} \left(\tilde{R}^a{}_b \wedge q^b - \omega^a{}_b \wedge \tilde{T}^b \right). \quad - (16)$$

So the wave and field equations of ECE theory can be obtained from the Lagrangian density (1).