1)

NOTES FOR PAPER 53, Part 2.

Convention for Field Tensor

The basic structure of the resonance equation is:

$$d \wedge F + \omega \wedge F = \mu_0 j_{int} = R_{int} \wedge A \quad -(1)$$

$$F = d \wedge A + \omega \wedge A. \quad -(2)$$

We adopt the convention:

$$\partial_\mu = \left(\frac{1}{c} \frac{\partial}{\partial t}, -\underline{\nabla} \right) \quad -(3)$$

$$\partial^\mu = g^{\mu\sigma} \partial_\sigma \quad -(4)$$

$$A^{\nu a} = (A^{0a}, \underline{A}^a), \quad B^{\nu a} = (B^{0a}, \underline{B}^a) \quad -(5)$$

and

$$\boxed{F^{\mu\nu} = \partial^\mu A^{\nu a} - \partial^\nu A^{\mu a} + \omega^{\mu a}{}_b A^{\nu b} - \omega^{\nu a}{}_b A^{\mu b}} \quad -(6)$$

where:

$$\partial^\mu A^{\nu a} := g^{\mu\sigma} \partial_\sigma B^{\nu a} \text{ etc.} \quad -(7)$$

Then this convention allows us to define:

$$\underline{E}^a = -\frac{\partial \underline{A}^a}{\partial t} - c \underline{\nabla} A^{0a} - c\, \omega^{0a}{}_b \underline{A}^b + c\, \underline{\omega}^a{}_b A^{0b}$$

$$-(8)$$

$$\underline{B}^a = \underline{\nabla} \times \underline{A}^a - \underline{\omega}^a{}_b \times \underline{A}^b. \quad -(9)$$

Then the field equation (1) is:

$$\partial_\mu \tilde{F}^{\mu\nu a} = \mu_0 \tilde{j}^{\nu a}_{int} - \omega^a_{\mu b} \tilde{F}^{\mu\nu b} \quad -(10)$$

or

$$\boxed{\partial_\mu \tilde{F}^{\mu\nu a} + \omega^a_{\mu b} \tilde{F}^{\mu\nu b} = \mu_0 \tilde{j}^{\nu a}_{int}} \quad -(11)$$

2) In vector notation, eqn. (11) is:

$$\underline{\nabla} \cdot \underline{B}^a + \underline{\omega}^a{}_b \cdot \underline{B}^b = \mu_0 \tilde{J}^{0a}_{int} \quad -(12)$$

$$\underline{\nabla} \times \underline{E}^a + \frac{\partial \underline{B}^a}{\partial t} + \underline{\omega}^a{}_b \times \underline{E}^b + \omega^a_{0b} \underline{B}^b = \mu_0 \tilde{\underline{J}}^a_{int} \quad -(13)$$

where we have used:

$$\omega^a_{\mu b} = \left(\omega^a_{0b}, -\underline{\omega}^a{}_b \right) \quad -(14)$$

The Hodge duals of eqns (12) and (13) are:

$$\underline{\nabla} \cdot \underline{E}^a + \underline{\omega}^a{}_b \cdot \underline{E}^b = \mu_0 \tilde{J}^{0a}_{int} \quad -(15)$$

$$\underline{\nabla} \times \underline{B}^a - \frac{1}{c^2} \frac{\partial \underline{E}^a}{\partial t} + \underline{\omega}^a{}_b \times \underline{B}^a - \frac{1}{c^2} \omega^a_{0b} \underline{E}^a = \frac{\mu_0}{c} \tilde{\underline{J}}^a_{int} \quad -(16)$$

By substituting eqns (8) and (9) into eqns. (12) - (16) four linear inhomogeneous equations are obtained.