3. ANALYTICAL SOLUTION.

Eq. (100) is a development of the linear inhomogeneous {37} class of equations:

$$\frac{d^2y}{dx^2} + a\frac{dy}{dx} + by = f(x). \quad -(101)$$

In the special case:

$$f(x) = 0 \quad -(102)$$

Eq. (101) reduces to the linear homogeneous class

$$\frac{d^2y}{dx^2} + a\frac{dy}{dx} + by = 0 \quad -(103)$$

whose general solution is:

$$y = c_1 e^{r_1 x} + c_2 e^{r_2 x}, \quad r_1 \neq r_2, \quad -(104)$$

with the auxiliary equation

$$r^2 + ar + b = 0. \quad -(105)$$

Eq. (104) holds when the roots of Eq. (103) are real and unequal, i.e. $r_1 \neq r_2$. If the roots of Eq. (103) are imaginary ($\alpha \pm i\beta$), then:

$$\begin{aligned} y &= e^{\alpha x}\left(c_1 \cos\beta x + c_2 \sin\beta x\right) \\ &= \mu e^{\alpha x} \sin(\beta x + \delta). \end{aligned} \quad -(106)$$

Now let:

$$y = u \quad -(107)$$

be the general solution of

$$y'' + ay' + by = 0 \qquad -(108)$$

and let

$$y = v \qquad -(109)$$

be any solution of

$$y'' + ay' + by = f(x) \qquad -(110)$$

then

$$y = u + v \qquad -(111)$$

is a solution of Eq. (101). The function u is the complementary function and v is the particular integral. One must find by inspection a function v that satisfies:

$$v'' + av' + bv = f(x). \qquad -(112)$$

Eq. (100) of Section 2 is a special case of the linear inhomogeneous class (101) and Eq. (100) can be rewritten as

$$m\ddot{x} + b\dot{x} + kx = F_0 \cos\omega t. \qquad -(113)$$

This is the equation of driven oscillation {37}. In Eq. (113) the external driving force varies harmonically with time, and is applied to the oscillator. The total force on the particle is:

$$F = -kx - b\dot{x} + F_0 \cos\omega t \qquad -(114)$$

and consists of a linear restoring force, $-kx$, (Hooke's Law), and a viscous damping force

$-b\dot{x}$. Therefore the master equation (35) of ECE theory has all these features and is also more richly structured. In this Section an analytical solution of Eq. (99) is found in a well defined approximation using the properties of the linear inhomogeneous class of equations (101).

Resonance solutions of Eq. (113) are found from the complementary function $x_c(t)$ and the particular integral $x_p(t)$. The former is:

$$x_c(t) = e^{-\beta t}\left(A_1 \exp\left(\left(\beta^2-\omega_0^2\right)^{1/2} t\right) + A_2 \exp\left(-\left(\beta^2-\omega_0^2\right)^{1/2} t\right)\right) \quad -(115)$$

and the latter is {37}:

$$x_p(t) = D\cos(\omega t - \delta). \quad -(116)$$

It follows that

$$x_p(t) = A\left(\left(\omega_0^2-\omega^2\right)^2 + 4\omega^2\beta^2\right)^{-1/2}\cos(\omega t - \delta) \quad -(117)$$

where

$$\delta = \tan^{-1}\left(\frac{2\omega\beta}{\omega_0^2-\omega^2}\right). \quad -(118)$$

The general solution is:

$$x(t) = x_c(t) + x_p(t). \quad -(119)$$

The term $x_c(t)$ represents transient effects that depend on the initial conditions. These damp out with time because of the factor $e^{-\beta t}$. The term $x_p(t)$ represents steady state effects which dominate for $t \gg 1/\beta$. The quantity δ is the phase difference between the driving force and the resultant motion, i.e. a delay between the application of force and the response of the system. For a fixed ω_0, as ω increases from 0, the phase increases from $\delta = 0$ at $\omega = 0$ to δ at $\pi/2$ and to π as $\omega \to \infty$.

The amplitude resonance frequency ω_R is that at which the amplitude D is a maximum. It is defined by:

$$\left.\frac{dD}{d\omega}\right|_{\omega=\omega_R} = 0 \quad -(120)$$

i.e.

$$\omega_R = \left(\omega_0^2 - 2\beta^2\right)^{1/2}. \quad -(121)$$

We see that for an equation such as (92) in which ω_0 and β are both zero, there is no resonance. In an equation in which ω_0 is zero but β is non-zero the resonance frequency ω_R is pure imaginary and unphysical. Therefore the requirement for resonance is that ω_0 and D be non-zero. If the amplitude D is initially zero it cannot be maximized from Eq. (120). These conditions are very important for the resonant acquisition of energy and for resonant counter-gravitation.

The degree of damping in an oscillatory system is described by the quality factor:

$$Q = \frac{\omega_R}{2\beta}. \quad -(122)$$

In loudspeakers for example {37} the values of Q may be a few hundred, in quartz crystal oscillators or tuning forks up to 10,000. Highly tuned electric circuits (of interest to extracting resonance energy from ECE space-time) may have Q up to 100,000 {37}. This is the order of magnitude of the amplification observed by the Mexican Group. The oscillation of electrons in atoms leads to optical radiation. The sharpness of the spectral lines is limited {37} by the damping due to loss of energy by radiation (radiation damping). The minimum width of a line is, classically, about:

$$\Delta\omega = 2 \times 10^{-8}\,\omega. \quad -(123)$$

The Q of such an oscillation is therefore of the order 10^{7}. The largest known Q occurs from radiation from a gas laser, about 10^{14}. Therefore resonant energy form ECE space-time and resonant counter-gravitation are also governed by such features. A current j (barebones notation) is set up by Eq. (21) and can set electrons in a circuit or within a material into resonant motion, producing a resonance current from space-time as observed experimentally {1-35}. Eq. (21) shows that the current is generated by the geometry of space-time itself.

Resonance in kinetic energy (T) is defined by the value of ω for which T is a maximum, where {37}:

$$T = \frac{1}{2} m \dot{x}^{2}. \quad -(124)$$

It is found from:

$$\left. \frac{d\langle T \rangle}{d\omega} \right|_{\omega = \omega_E} = 0 \quad -(125)$$

and is

$$\omega_E = \omega_0 \quad -(126)$$

where

$$\langle T \rangle = \frac{m A^2}{4} \omega^2 \left(\left(\omega_0^2 - \omega^2 \right)^2 + 4\omega^2 \beta^2 \right)^{-1/2}. \quad -(127)$$

The potential energy is proportional to the square of the amplitude, and occurs at the same frequency as amplitude resonance. The kinetic and potential energies resonate at different frequencies because the damped oscillator is not a conservative system {37} of dynamics. Energy is continuously exchanged with the driving system. In energy from ECE space-time energy is therefore continuously exchanged between space-time and the circuit or material, total energy being conserved by Neother's Theorem.

Atomic systems within a material taking resonant energy from ECE space-time can be represented classically as linear oscillators. When light falls on matter it causes the atoms and molecules to vibrate. Similarly ECE space-time causes the atoms and molecules to vibrate, light being ECE space-time within the factor $A^{(0)}$ of Eq. (12). A resonant frequency occurs at one of the spectral frequencies of the system. When light (i.e. ECE space-time) having one of the resonant frequencies of the atomic or molecular system falls on the material, electromagnetic energy (i.e. energy from ECE space-time) is absorbed, causing the atom or molecule to oscillate with large amplitude. This is what happens in a circuit or material such as that of the Mexican Group {1-35}. A large amount of energy is resonantly absorbed from ECE space-time. This can be released as electric current or power, the governing equation is equation (35). Large electromagnetic fields (ECE space-time dynamics) are produced by the oscillating electric charges. Electric circuits are non-mechanical oscillations. Therefore resonance theory and electric circuit theory can be used to explain energy from space-time. The mechanism is clear from Eq. (35), i.e.:

$$j = \frac{A^{(0)}}{\mu_0} \left(d \wedge (d \wedge q) + d \wedge (\omega \wedge q) \right). \quad -(128)$$

The current j is picked up from ECE space-time and is represented by q and ω of Eq. (128), a driven damped oscillator equation. Amplitude, kinetic energy and potential energy resonances occur. The electrons in a well designed circuit or material oscillate in constructive interference, producing a surge of current and electric power. This is observed experimentally in the reproducible and repeatable work of the Mexican group of AIAS {1-35}.

These qualitative remarks are underlined as follows with an analytical solution of Eq. (99) with well defined approximations. First use

$$\omega^a_{\mu b} = -\kappa \epsilon^a{}_{bc} q^c_\mu \qquad -(129)$$

so

$$A^{b0}\,\underline{\omega}^a{}_b = \omega^{a0}{}_b\,\underline{A}^b, \qquad -(130)$$

$$\underline{\omega}^a{}_b \times \underline{A}^b = \underline{0}. \qquad -(131)$$

Then use

$$\nabla^2 \underline{A}^a = -\frac{\omega_0^2}{c^2}\underline{A}^a, \qquad -(132)$$

$$\underline{\nabla}\cdot\underline{A}^a = 0, \qquad -(133)$$

$$\partial A^{0a}/\partial t = 0, \qquad -(134)$$

with:

$$\underline{\nabla}\times(\underline{\nabla}\times\underline{A}) = -\nabla^2\underline{A} + \underline{\nabla}(\underline{\nabla}\cdot\underline{A}). \qquad -(135)$$

Eq. (99) then simplifies to

$$\frac{1}{c^2}\frac{\partial^2 \underline{A}^a}{\partial t^2} + \frac{\omega_0^2}{c^2}\underline{A}^a = \frac{\mu_0}{c}\underline{\tilde{J}}^a. \qquad -(136)$$

This is an undamped driven oscillator, it has the structure of Eq. (100) with

$$\beta = 0. \qquad -(137)$$

From Eqs. (132) and (133)

$$\underline{A}^a = \frac{A^{(0)}}{\sqrt{2}}(\underline{i} - i\underline{j})e^{-i\omega_0 Z/c} \qquad -(137)$$

is a possible solution. From the analytical solution of Eq. (100) already discussed in this

Section:

$$A^{(0)} = A_c^{(0)} + A_p^{(0)} \qquad -(138)$$

where

$$A_c^{(0)} = A_1 e^{i\omega_0 t} + A_2 e^{-i\omega_0 t}, \quad -(139)$$

$$A_\rho^{(0)} = D, \quad -(140)$$

assuming:

$$\frac{\mu_0}{c} \underline{\tilde{J}}^a = A^a (\underline{i} - i\underline{j}) \cos\omega t. \quad -(141)$$

Resonance occurs at

$$\omega_R = \omega_0 \quad -(142)$$

with:

$$\delta = 0, \quad Q \to \infty, \quad D \to \infty. \quad -(143)$$

In this case there is a surge of current of infinite amplitude:

$$\underline{\tilde{J}}^a \longrightarrow \infty \quad -(144)$$

because there is no damping. This simple illustration, using well defined approximations, shows how resonant energy from space-time occurs mathematically within ECE theory. More realistic results with finite damping can be produced numerically from Eq. (99), and under certian conditions will reproduce the factor of 100,000 amplification observed by the Mexican Group {1-35} and found independently to be reproducible and repeatable.