1)

Notes for Paper 52, Part 7

THIRD LINEAR INHOMOGENEOUS STRUCTURE AND CROSS - CHECKS.

The third linear inhomogeneous structure is given by

$$\underline{\nabla} \times \underline{E}^a + \frac{\partial \underline{B}^a}{\partial t} = \mu_0 \underline{\tilde{j}}^a \quad -(1)$$

$$\underline{E}^a = -\frac{\partial \underline{A}^a}{\partial t} - c \underline{\nabla} A^{a0} + c \omega^a{}_b \underline{A}^b - c A^{0b} \underline{\omega}^a{}_b \quad -(2)$$

$$\underline{B}^a = \underline{\nabla} \times \underline{A}^a - \underline{\omega}^a{}_b \times \underline{A}^b \quad -(3)$$

Using:

$$\frac{\partial}{\partial t} \underline{\nabla} \times \underline{A}^a = \underline{\nabla} \times \frac{\partial \underline{A}^a}{\partial t} \quad -(4)$$

and:

$$\underline{\nabla} \times \underline{\nabla} A^{a0} = \underline{0} \quad -(5)$$

we find:

$$\boxed{\frac{\partial}{\partial t}\left(\underline{\omega}^a{}_b \times \underline{A}^b\right) + c \underline{\nabla} \times \left(A^{0b} \underline{\omega}^a{}_b\right) - c \underline{\nabla} \times \left(\omega^a{}_b \underline{A}^b\right) = \mu_0 \underline{\tilde{j}}^a} \quad -(6)$$

This is a first order differential in the potential. It can be seen that if the spin connection is zero (standard model), the current $\underline{\tilde{j}}^a$ is zero, meaning no current from spacetime.

2) <u>Cross-Check</u>

In the limit of the free electromagnetic field:

$$\omega_{\mu b}^{a} = -\kappa \epsilon^{a}{}_{bc} q V_{\mu}^{c} \quad -(7)$$

$$j^{a} = 0 \quad -(8)$$

From eqn. (7):

$$(\omega^{a}{}_{0b}, -\underline{\omega}^{a}{}_{b}) = -\kappa \epsilon^{a}{}_{bc} (q V_{0}^{c}, -\underline{q V}^{c})$$

i.e.

$$\omega^{a}{}_{0b} = -\kappa \epsilon^{a}{}_{bc} q V_{0}^{c} \quad -(9)$$

$$\underline{\omega}^{a}{}_{b} = -\kappa \epsilon^{a}{}_{bc} \underline{q V}^{c} \quad -(10)$$

If $a = 1$:

$$\underline{\omega}^{a}{}_{b} \times \underline{A}^{b} = \underline{\omega}^{1}{}_{2} \times \underline{A}^{2} + \underline{\omega}^{1}{}_{3} \times \underline{A}^{3}$$

$$= -\frac{\kappa}{A^{(0)}} (\underline{A}^{3} \times \underline{A}^{2} + \underline{A}^{2} \times \underline{A}^{3}) \quad -(11)$$

$$= 0$$

$$c A^{0b} \underline{\nabla} \times \underline{\omega}^{1}{}_{b} = \frac{c}{A^{(0)}} A^{02} \underline{\nabla} \times \underline{A}^{3} + \frac{c}{A^{(0)}} A^{03} \underline{\nabla} \times \underline{A}^{2} \quad -(12)$$

$$-c\omega^{10}{}_{c} \underline{\nabla} \times \underline{A}^{b} = -\frac{cA^{03}}{A^{(0)}} \underline{\nabla} \times \underline{A}^{2} - \frac{cA^{02}}{A^{(0)}} \underline{\nabla} \times \underline{A}^{3} \quad -(13)$$

and

$$(12) + (13) = 0 \quad -(14)$$

We have checked self-consistency with eqns. (7) and (8), Q.E.D.