Notes for Paper 52, Part 4

The d'Alembert wave equation is generalized in ECE theory to:

$$\boxed{d \wedge (d \wedge A) + d \wedge (\omega \wedge A) = \mu_0 j} \quad -(1)$$

In the standard model the same wave equation is:

$$d \wedge (d \wedge A) = 0 \quad -(2)$$

which is the Poincaré Lemma. In these notes it is proven that resonance solutions may occur of eqn. (1) but not of eqn (2).

Proof

Eqn (1) is a development of the linear inhomogeneous class of equations:

$$\frac{d^2 y}{dx^2} + a\frac{dy}{dx} + by = f(x) \quad -(3)$$

whose solutions were discovered by the Bernoulli brothers in 1739 and published by Euler in 1743 (see Marion & Thornton, Appendix C).

In the special case:

$$f(x) = 0 \quad -(4)$$

eqn (3) reduces to the linear homogeneous class

$$\frac{d^2 y}{dx^2} + a\frac{dy}{dx} + by = 0 \quad -(5)$$

II) The general solution of the class (5) is

$$y = c_1 \exp(r_1 x) + c_2 \exp(r_2 x) \quad - (6)$$
$$(r_1 \neq r_2)$$

with the auxiliary equation:

$$r^2 + ar + b = 0 \quad - (7)$$

If the roots of (7) are real and unequal, i.e. $r_1 \neq r_2$, eqn. (6) holds. If the roots of eqn. (7) are imaginary $(\alpha \pm i\beta)$ then:

$$y = e^{\alpha x}\left(c_1 \cos \beta x + c_2 \sin \beta x\right)$$
$$= \mu e^{\alpha x} \sin(\beta x + \delta) \quad - (8)$$

Now let $y = u$ be the general solution of

$$y'' + ay' + by = 0 \quad - (9)$$

and let $y = v$ be any solution of

$$y'' + ay' + by = f(x) \quad - (10)$$

then

$$y = u + v \quad - (11)$$

is a solution of eqn. (3). The function u is the complementary function and v is the particular integral. One must find by inspection or trial some function v that satisfies:

III)

$$v'' + av' + bv = f(x) \quad -(12)$$

Therefore this is also the general solution of the differential form equation (1). To gain more insight into this consider the equation:

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = A\cos\omega t \quad -(13)$$

(Marion and Thornton eqn. (3.51), page 114).

Eqn (13) is equivalent to:

$$m\ddot{x} + b\dot{x} + kx = F_0\cos\omega t. \quad -(14)$$

This is the equation of a <u>driven oscillation</u>. In eqn (14) the <u>external driving force</u> varies harmonically with time, and is applied to the oscillator. The total force on the particle is:

$$F = -kx - b\dot{x} + F_0\cos\omega t. \quad -(15)$$

There is <u>a linear restoring force</u>, $-kx$, (Hooke's law), and <u>a viscous damping force</u>, $-b\dot{x}$. So we see that the basic wave equation of ECE field theory, eqn (1), has all these basic features, but is a much more richly structured format of differential geometry.

<u>Resonance solutions</u> of eqn (14) are

IV) found from the complementary function $x_c(t)$ and the particular integral $x_p(t)$. The former is

$$x_c(t) = e^{-\beta t}\left(A_1 \exp\left((\beta^2-\omega_0^2)^{1/2} t\right) + A_2 \exp\left(-(\beta^2-\omega_0^2)^{1/2} t\right)\right) \quad -(16)$$

and the latter is:

$$x_p(t) = D\cos(\omega t - \delta). \quad -(17)$$

It follows that

$$x_p(t) = A\left((\omega_0^2-\omega^2)^2 + 4\omega^2\beta^2\right)^{-1/2}\cos(\omega t-\delta) \quad -(18)$$

where:

$$\delta = \tan^{-1}\left(\frac{2\omega\beta}{\omega_0^2-\omega^2}\right). \quad -(19)$$

The general solution is:

$$x(t) = x_c(t) + x_p(t) \quad -(20)$$

The term $x_c(t)$ represents <u>transient effects</u> that depend on the initial conditions. These damp out with time because of $e^{-\beta t}$. The term $x_p(t)$ represents <u>steady state effects</u>, which dominate for $t \gg 1/\beta$. The quantity δ is the <u>phase difference</u> between the driving force and the resultant motion, i.e. a delay between the

V) action of the force and the response of the system.

For a fixed ω_0, as ω increases from 0, the phase increases from $\delta = 0$ at $\omega = 0$ to $\delta = \pi/2$ at $\omega = \omega_0$ and to π as $\omega \to \infty$.

The Amplitude Resonance Frequency

This is the frequency ω_R at which the amplitude D is a maximum. It is defined by:

$$\left.\frac{dD}{d\omega}\right|_{\omega=\omega_R} = 0 \qquad (21)$$

i.e.

$$\boxed{\omega_R = \left(\omega_0^2 - 2\beta^2\right)^{1/2}} \qquad (22)$$

We see that for an eqn such as (2) there is no resonance, Q.E.D.

The degree of damping in an oscillating system is described by the quality factor:

$$Q = \frac{\omega_R}{2\beta} \qquad (23)$$

The resonance and phase curves for different values of Q are given in Figure 3-14, page 117 of Marion & Thornton, and are sketched below:

VI)

Fig (1)

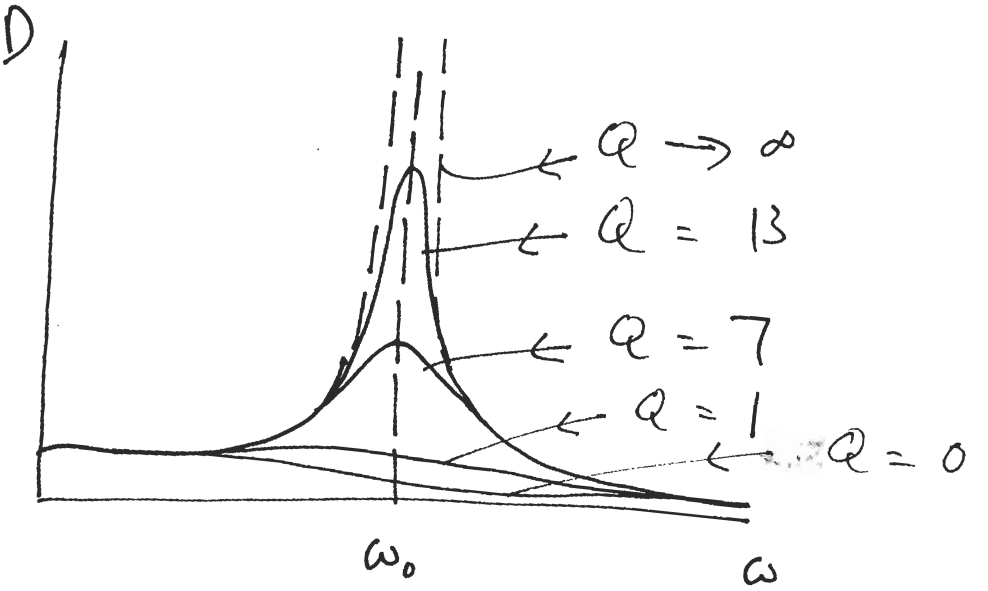

Fig (2)

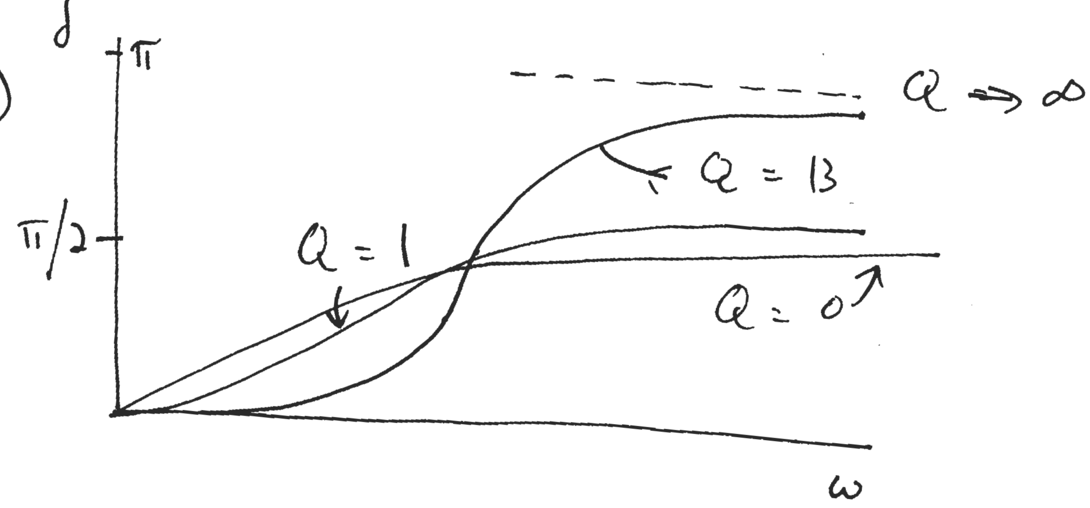

Fig (1) are <u>resonance curves</u>.

In loudspeakers, the value of Q may be a few hundred, in quartz crystal oscillators or tuning forks up to 10,000. Highly tuned electrical circuits or resonance cavities may have Q up to 10^5. The <u>oscillation</u> of <u>electrons within atoms</u> leads to optical radiation. The sharpness of the spectral lines is limited by the damping due to loss of energy by radiation (radiation damping). The minimum width of a line is, classically, about $\Delta\omega = 2 \times 10^{-8}\omega$. The Q of such an oscillation is therefore $\sim 5 \times 10^{7}$. The largest known Q's occur from radiation from a gas laser, up to $\sim 10^{14}$.

Therefore new energy from spacetime is also governed by such physical features. A current j is set up by eqn (1) and can set electrons with atoms into resonant motion. This is the reverse of the process of radiation from oscillating electrons.

Resonance in Kinetic Energy (T)

This is defined as the value of ω for which T is a maximum, where:

$$T = \frac{1}{2} m \dot{x}^2 \quad - (24)$$

It is found from:

$$\left.\frac{d\langle T\rangle}{d\omega}\right|_{\omega=\omega_E} = 0 \quad - (25)$$

and is

$$\boxed{\omega_E = \omega_0} \quad - (26)$$

Where:

$$\langle T \rangle = \frac{m A^2}{4} \frac{\omega^2}{(\omega_0^2 - \omega^2)^2 + 4\omega^2 \beta^2} \quad -(27)$$

The amplitude resonance occurs at $(\omega_0^2 - 2\beta^2)^{1/2}$, whereas the kinetic energy resonance occurs at ω_0. The potential energy is also proportional to the square of the amplitude, so also occurs at $(\omega_0^2 - 2\beta^2)^{1/2}$. The kinetic and potential energies resonate at different frequencies, because the damped oscillator is not a conservative system. Energy is continuously exchanged with the driving mechanism.

This is also what happens in energy from ECE spacetime in eqn. (1).

Atomic systems can also be represented classically as linear oscillators. When light falls on matter it causes the atoms and molecules to vibrate. A resonant frequency occurs at one of the spectral frequencies of the system. When light having one of the resonant frequencies of the atomic or molecular system falls on the material, electromagnetic energy is absorbed, causing the atom or molecule to oscillate with large amplitude. Large electromagnetic fields of the same frequency are produced by the oscillating electric charges.

ix)

Electric circuits are nonmechanical oscillations. Therefore resonance theory and electric circuit theory can be used to explain energy from spacetime. The mechanism is clear from eqn. (1) written as:

$$j = \frac{A^{(0)}}{\mu_0} \left(d \wedge (d \wedge q) + d \wedge (\omega \wedge q) \right) \quad -(28)$$

The current j is picked up from ECE spacetime as represented by q and ω of eqn (28). This is a driven damped oscillator equation. Amplitude, kinetic and potential energy resonances occur. The electrons in a circuit or material oscillate in constructive interference, producing a surge of current and electric power. This is observed experimentally in the reproducible and repeatable work of the Mexican group of AIAS.