1)

Notes for Paper 46, Part 2

To Prove

That

$$\underline{\nabla} \times \underline{E}^a + \frac{\partial \underline{B}^a}{\partial t} = \mu_0 \underline{\tilde{j}}^a \quad -(1)$$

results in a frequency shift of the Sagnac effect.

Proof

It is first shown that eq. (1) is the same as:

$$\underline{\nabla} \times \underline{D}^a + \frac{1}{c^2}\frac{\partial \underline{H}^a}{\partial t} = \underline{0} \quad -(2)$$

where:

$$\underline{D}^a = \epsilon_0 \underline{E}^a + \underline{P}^a \quad -(3)$$

$$\underline{H}^a = \frac{1}{\mu_0}\underline{B}^a - \underline{M}^a \quad -(4)$$

From eqs. (3) and (4) in (2):

$$\underline{\nabla} \times \left(\epsilon_0 \underline{E}^a + \underline{P}^a\right) + \mu_0 \epsilon_0 \frac{\partial}{\partial t}\left(\frac{1}{\mu_0}\underline{B}^a - \underline{M}^a\right) = \underline{0} \quad -(5)$$

where:

$$\epsilon_0 \mu_0 = \frac{1}{c^2} \quad -(6)$$

$$\therefore \epsilon_0\left(\underline{\nabla} \times \underline{E}^a + \frac{\partial \underline{B}^a}{\partial t}\right) + \underline{\nabla} \times \underline{P}^a - \frac{1}{c^2}\frac{\partial \underline{M}^a}{\partial t} = \underline{0} \quad -(7)$$

2)

i.e.
$$\underline{\nabla} \times \underline{E}^a + \frac{\partial \underline{B}^a}{\partial t} = \frac{1}{\epsilon_0}\left(\epsilon_0 \mu_0 \frac{\partial \underline{M}^a}{\partial t} - \underline{\nabla} \times \underline{P}^a\right)$$
$$= \frac{1}{\epsilon_0}\left(\frac{1}{c^2}\frac{\partial \underline{M}^a}{\partial t} - \underline{\nabla} \times \underline{P}^a\right)$$
$$= \mu_0 \tilde{\underline{j}}^a \qquad -(8)$$

i.e.
$$\tilde{\underline{j}}^a = c^2\left(\frac{1}{c^2}\frac{\partial \underline{M}^a}{\partial t} - \underline{\nabla} \times \underline{P}^a\right)$$
$$\boxed{\tilde{\underline{j}}^a = \frac{\partial \underline{M}^a}{\partial t} - c^2 \underline{\nabla} \times \underline{P}^a} \qquad -(9)$$

Eqn. (9) expresses the $\tilde{j}^a$ current in terms of polarization and magnetization. The $\tilde{\underline{j}}^a$ current is due to "EMG coupling", this is the interaction between gravitation and electromagnetism that is being sought. This changes the Einstein Cartan Evans (ECE) spacetime:
$$\tilde{j}^a = \frac{A^{(0)}}{\mu_0}\left(R^a{}_b \wedge q^b - \omega^a{}_b \wedge T^b\right)$$

By comparison of eqns (9) and (10) it is seen that $\underline{P}^a$ and $\underline{M}^a$ have a geometrical origin as demanded by general relativity.

<u>Weak</u> EMG <u>Coupling</u>

Here:

$$\boxed{\underline{\tilde{j}}^a \rightarrow \underline{0}}. \quad -(11)$$

This means that:

$$\frac{\partial \underline{M}^a}{\partial t} - c^2 \underline{\nabla} \times \underline{P}^a \sim \underline{0} \quad -(12)$$

or

$$\frac{\partial \underline{M}^a}{\partial t} - \frac{1}{\epsilon_0 \mu_0} \underline{\nabla} \times \underline{P}^a \sim \underline{0}. \quad -(13)$$

Now introduce <u>the permeability</u> μ <u>and</u> <u>permittivity</u> ϵ <u>of the ECE spacetime</u>. In the simplest case:

$$\underline{P}^a = \epsilon \underline{E}^a \quad -(14)$$

$$\underline{M}^a = \frac{1}{\mu} \underline{B}^a \quad -(15)$$

to find:

$$\boxed{\frac{\partial \underline{B}^a}{\partial t} - \frac{\epsilon \mu}{\epsilon_0 \mu_0} \underline{\nabla} \times \underline{E}^a \sim \underline{0}}$$

(16)

4)

i.e.:

$$\boxed{\frac{\partial \underline{B}^a}{\partial t} - n^2 \underline{\nabla} \times \underline{E}^a \sim \underline{0}} \quad -(17)$$

where n is the refractive index of ECE spacetime.

The phase velocity of the wave in the weak EMF coupling limit is:

$$v = \frac{c}{n}. \quad -(18)$$

The frequency and wavenumber are related by:

$$\omega = \kappa v \quad -(19)$$

and the phase of the Sagnac effect is shifted to:

$$\boxed{\Delta \phi = \frac{4 \Omega A_r}{\lambda v}} \quad -(20)$$

from:

$$\Delta \phi = \frac{4 \Omega A_r}{\lambda c}. \quad -(21)$$

The time delay is changed to:

$$\boxed{\Delta t = 4 A_r / v^2} \quad -(22)$$

from:

$$\Delta t = 4 A_r / c^2. \quad -(23)$$