FINAL VERSION OF NOTES FOR PAPER 43

In this version an explanation for the Faraday disc generator is derived which appears to be in agreement with points of view to date. We start with careful consideration of the homogeneous current:

$$j^a = \frac{A^{(0)}}{\mu_0} \left(R^a{}_b \wedge q^b + T^b \wedge \omega^a{}_b \right) \quad -(1)$$

This current vanishes when:

$$\omega^a{}_b = \kappa \epsilon^a{}_{bc} q^c , \quad -(2)$$

i.e when the spin connection is dual to the tetrad.

Proof Cartan's structure equations show that:

$$T^a = D \wedge q^a \quad -(3)$$

$$R^a{}_b = D \wedge \omega^a{}_b \quad -(4)$$

From eqns (2) and (4):

$$R^a{}_b = \kappa \epsilon^a{}_{bc} T^c . \quad -(5)$$

From eqs. (2) and (5) in eq. (1):

$$j^a = \frac{A^{(0)}}{\mu_0} \kappa \epsilon^a{}_{bc} \left(T^c \wedge q^b + T^b \wedge q^c \right) \quad -(6)$$

2)

For $a = 1$:

$$j^1 = \frac{A^{(0)}}{\mu_0} \kappa \left(\epsilon^1{}_{23} T^3 \wedge q^2 + \epsilon^1{}_{32} T^2 \wedge q^3 + \epsilon^1{}_{23} T^2 \wedge q^3 + \epsilon^1{}_{32} T^3 \wedge q^2 \right)$$

$$= 0 \qquad - (7)$$

because:

$$\epsilon^1{}_{23} = - \epsilon^1{}_{32}. \qquad - (8)$$

It is concluded that:

$$j^a = 0, \quad a = 0, 1, 2, 3 \qquad - (9)$$

Q.E.D.

For space indices:

$$\omega_{ab} = - \kappa \epsilon_{abc} q_c \qquad - (10)$$

$$= - \frac{\omega}{c} \epsilon_{abc} q_c .$$

The tetrad components are cartesian vector components within a factor $e^{i\phi}$ and so ω_{ab} are spin or rotation generator matrices. Since eq. (10) is always true for any type of rotation the intrinsic j^a is always zero. Therefore the intrinsic spin of the electromagnetic field

3) is always described by :

$$\boxed{d \wedge F^a = 0} \quad . \quad -(11)$$

An example of this intrinsic spin is circular polarization. In vector notation eqn. (11) is :

$$\underline{\nabla} \cdot \underline{B}^a = \underline{0} \quad -(12)$$

$$\underline{\nabla} \times \underline{E}^a + \frac{\partial \underline{B}^a}{\partial t} = \underline{0} \quad . \quad -(13)$$

The homogeneous current $\underline{j}^a$ is intrinsically non-zero if and only if $\omega^a{}_b$ is not the dual of q^c.

This can be true if and only if there is an effect of a central force such a gravitation on e/m in a gravitational field. For this effect to occur the $\omega^a{}_b$ must be asymmetric. If it is symmetric or antisymmetric then j^a vanishes.

Therefore in the Faraday disc generator the extra induction caused by mechanical rotation of the disc, magnet, or both must be

4) described by:

$$j^a = A^{(0)} \left(d \wedge T^a \right)_{\text{mechanical}} \quad -(14)$$

i.e by:

$$\nabla \cdot \underline{B}^a = \left(\underline{\nabla} \cdot \underline{B}^a \right)_{\text{mechanical}} \quad -(15)$$

$$\underline{\nabla} \times \underline{E}^a + \frac{\partial \underline{B}^a}{\partial t} = \left(\underline{\nabla} \times \underline{E}^a + \frac{\partial \underline{B}^a}{\partial t} \right)_{\text{mechanical}} \quad -(16)$$

The requirements for j^a to be non-zero are:

1) the existence of $A^{(0)}$ from the magnet;
2) the existence of a mechanically generated torsion T^a (mechanical):

$$\left(T^a = d \wedge q^a + \omega^a{}_b \wedge q^b \right)_{\text{mechanical}} . \quad -(17)$$

In Euclidean space:

$$\left(\underset{\sim}{T}^a = \underline{\nabla} \times q^a + \underline{\omega}^a{}_b \times q^b \right)_{\text{mechanical}} \quad -(18)$$

5)

Discussion

The mechanical rotation of the disc sets up an electric field $\underline{E}^a$ (mechanical) and magnetic field $\underline{B}^a$ (mechanical). If the disc is rotated about Z then there is an E_X and E_Y. These set up an emf or potential difference between the centre of the disk and its edge. If $\partial \underline{B}^a / \partial t$ is zero on the LHS of eq. (16) then there is a $\underline{\nabla} \times \underline{E}^a$ (mechanical) set up by rotation. This is true even if no lines of force from $\partial \underline{B}^a / \partial t$ cut the rotating disc or induction coil. The total derivatives of classical e/m (e.g. Jackson) are inherent in the structure of the Faraday law of induction. The mechanically induced current is:

$$\underline{j}^a = \frac{1}{\mu_0} \left(\underline{\nabla} \times \underline{E}^a + \frac{\partial \underline{B}^a}{\partial t} \right)_{\text{mechanical}} \qquad (19)$$