437(3): Fluctuating m Space Theory

In order to forge a self consistent theory of the Lamb shift, assume that the potential energy in the presence of fluctuations is:

$$U = -\frac{e^2}{4\pi\epsilon_0(r+\delta r)} = -\frac{m(r)^{1/2}e^2}{4\pi\epsilon_0 r} \quad -(1)$$

where δr is the vacuum fluctuation used in Lamb shift theory, e is the charge on the proton, ϵ_0 is the vacuum permittivity, and r is the distance between the electron and proton.

It follows that:

$$\boxed{m(r)^{1/2} = \frac{r}{r+\delta r}} \quad -(2)$$

<u>This</u> <u>is defined as the</u> <u>fluctuating</u> <u>$m(r)$</u> <u>function.</u>

From eq. (2):

$$m(r)^{1/2} = \frac{1}{1+\frac{\delta r}{r}} \sim 1 - \frac{\delta r}{r} \quad -(3)$$

if

$$\delta(r) \ll r, \quad -(4)$$

and

$$m(r) = \left(1 - \frac{\delta r}{r}\right)^2 \sim 1 - 2\frac{\delta r}{r} \quad -(5)$$

The Lamb shift is given immediately by eq. (5) using the calculations given in preceding UFT papers:

$$\Delta U = U(\underline{r}+\delta\underline{r}) - U(\underline{r}) \quad -(6)$$

$$= \delta\underline{r}\cdot\underline{\nabla}U(\underline{r}) + \frac{1}{2}(\delta\underline{r}\cdot\underline{\nabla})^2 U(\underline{r}) + \ldots$$

So:

a)
$$\langle \Delta U \rangle = \frac{1}{6} \langle (\delta \underline{r} \cdot \delta \underline{r}) \rangle_{vac} \left\langle \nabla^2 \left(\frac{-e^2}{4\pi \epsilon_0 r} \right) \right\rangle \quad -(7)$$

in which:

$$\langle \delta \underline{r} \cdot \delta \underline{r} \rangle_{vac} = \frac{1}{2\epsilon_0 \pi^2} \left(\frac{e^2}{\hbar c} \right) \left(\frac{\hbar}{mc} \right)^2 \int \frac{d\kappa}{\kappa} \quad -(8)$$

where κ is a wavenumber.

Eq. (8) is obtained by assuming that:

$$\delta r(t) = \delta r(0) \left(e^{-i\omega t} + e^{i\omega t} \right) \quad -(9)$$

and using:

$$m \frac{d^2}{dt^2} (\delta \underline{r})_{\kappa} = -e \underline{E}_{\kappa} \quad -(10)$$

where $\underline{E}_{\kappa}$ are vacuum electric fields.

Eq. (8) diverges, but limits are imposed to keep the integral finite:

$$\langle \delta \underline{r} \cdot \delta \underline{r} \rangle_{vac} = \frac{1}{2\epsilon_0 \pi^2} \left(\frac{e^2}{\hbar c} \right) \left(\frac{\hbar}{mc} \right)^2 \int_{\pi / a_0}^{mc/\hbar} \frac{d\kappa}{\kappa} \quad -(11)$$

where a_0 is the Bohr radius. So: $\quad -(12)$

$$\langle \delta \underline{r} \cdot \delta \underline{r} \rangle_{vac} \sim \frac{1}{2\epsilon_0 \pi^2} \left(\frac{e^2}{\hbar c} \right) \left(\frac{\hbar}{mc} \right)^2 \log_e \left(\frac{4\epsilon_0 \hbar c}{e^2} \right)$$

<u>This expression</u> is <u>made</u> up <u>entirely of fundamental constants.</u>

To complete the calculation of the Lamb shift:

$$\left\langle \nabla^2 \left(\frac{-e^2}{4\pi\epsilon_0 r} \right) \right\rangle = -\frac{e^2}{4\pi\epsilon_0} \int d\underline{r}\, \psi^*(\underline{r}) \nabla^2 \left(\frac{1}{r}\right) \psi(\underline{r})$$

$$= \frac{e^2}{\epsilon_0} |\psi(0)|^2 \quad -(13)$$

because:

$$\nabla^2 \left(\frac{1}{r}\right) = -4\pi\, \delta(\underline{r}) \quad -(14)$$

For the 2S wave function of H: $-(15)$

$$\left\langle \nabla^2 \left(-\frac{e^2}{4\pi\epsilon_0 r} \right) \right\rangle = \frac{e^2}{\epsilon_0} |\psi_{2S}(0)|^2 = \frac{e^2}{8\pi\epsilon_0 a_0^3}$$

So the Lamb shift for the 2S state is:

$$\langle \Delta U \rangle = \frac{4}{3} \frac{e^2}{4\pi\epsilon_0} \frac{e^2}{4\pi\epsilon_0 \hbar c} \left(\frac{\hbar}{mc}\right)^2 \frac{1}{8\pi a_0^3} \log_e \frac{4\epsilon_0 \hbar c}{e^2}$$

$$= \frac{\alpha^5 mc^2}{6\pi} \log_e \frac{1}{\pi\alpha} \quad -(16)$$

For the 2P state: $\langle \Delta U \rangle = 0 \quad -(17)$

From eq. (2):

$$\delta \underline{r} = \underline{r} \left(\frac{1}{m(r)^{1/2}} - 1 \right) \quad -(18)$$

so $$\langle \delta\underline{r} \cdot \delta\underline{r} \rangle = \left\langle \underline{r} \cdot \underline{r} \left(\frac{1}{m(r)^{1/2}} - 1 \right)^2 \right\rangle \quad -(19)$$

Assume that:

4)

$$\left\langle \underline{r} \cdot \underline{r} \left(\frac{1}{m(r)^{1/2}} - 1 \right)^2 \right\rangle$$

$$= \langle \underline{r} \cdot \underline{r} \rangle \left\langle \left(\frac{1}{m(r)^{1/2}} - 1 \right)^2 \right\rangle \quad -(20)$$

From UFT 340:

$$\langle r \rangle (2s) = 6 a_0 , \quad -(21)$$

$$\langle r \rangle (3s) = \frac{27}{2} a_0 , \quad -(22)$$

$$\langle r \rangle (1s) = \frac{3}{2} a_0 \quad -(23)$$

where a_0 is the Bohr radius. Therefore in the theory:

$$\boxed{\langle \delta \underline{r} \cdot \delta \underline{r} \rangle_{vac} = 36 a_0^2 \left\langle \left(\frac{1}{m(r)^{1/2}} - 1 \right)^2 \right\rangle} \quad -(24)$$

From eq. (12):

$$\langle \delta \underline{r} \cdot \delta \underline{r} \rangle_{vac} = \frac{1}{2 \epsilon_0 \pi^2} \frac{e^2}{\hbar c} \left(\frac{\hbar}{mc} \right)^2 \log_e \left(\frac{4 \epsilon_0 \hbar c}{e^2} \right)$$

$$= 36 a_0^2 \left\langle \left(\frac{1}{m(r)^{1/2}} - 1 \right) \right\rangle \quad -(25)$$

5) i.e.:

$$\langle \underline{\delta r} \cdot \underline{\delta r} \rangle_{vac} = \frac{2\alpha}{\pi} \left(\frac{\hbar}{mc} \right)^2 \log_e \frac{1}{\pi \alpha} \quad -(26)$$

$$= 36\, a_0^2 \left\langle \left(\frac{1}{n(r)^{1/2}} - 1 \right)^2 \right\rangle \quad -(28)$$

Therefore:

$$2.623 \times 10^{-27} = 9.80 \times 10^{-20} \left\langle \left(\frac{1}{n(r)^{1/2}} - 1 \right)^2 \right\rangle$$

i.e.

$$\boxed{\left\langle \left(\frac{1}{n(r)^{1/2}} - 1 \right) \right\rangle = 2.677 \times 10^{-8}} \quad -(29)$$

and from conventional Lamb shift theory:

$$\langle \underline{\delta r} \cdot \underline{\delta r} \rangle_{vac} = 2.623 \times 10^{-27}\, m^2 \quad -(30)$$

Both eqs. (29) and (30) are universal results. The fact that there is a Lamb shift in some states but not in others is due to the wavefunction. Using this method, m theory can explain the Lamb shift by combining m theory and vacuum fluctuation theory. From eq. (2):

$$\boxed{\frac{r}{n(r)^{1/2}} = r + \delta r} \quad -(31)$$