433(5): Calculation of the Rest Energy of Any Particle

These are given by:

$$m^2c^4 = \left\langle \frac{E^2}{m(r)} \right\rangle - c^2 \left\langle \frac{p^2}{m(r)} \right\rangle \quad -(1)$$

where:

$$\left\langle \frac{E^2}{m(r)} \right\rangle = -\hbar^2 \int \phi^* \frac{\partial^2}{\partial t^2}\left(\frac{\phi}{m(r)}\right) d\tau \quad -(2)$$

$$\left\langle \frac{p^2}{m(r)} \right\rangle = -\hbar^2 \int \phi^* \nabla^2 \left(\frac{\phi}{m(r)}\right) d\tau \quad -(3)$$

For a plane wave:

$$\phi = \exp\left(-i\left(\omega t - \kappa r\right)\right) \quad -(4)$$

and assuming that $m(r)$ is independent of time (stationary metric assumption), then:

$$\left\langle \frac{E^2}{m(r)} \right\rangle = -\hbar^2 \int \phi^* \frac{1}{m(r)} \frac{\partial^2 \phi}{\partial t^2} d\tau \quad -(5)$$

where:

$$\frac{\partial^2 \phi}{\partial t^2} = -\omega^2 \phi \quad -(6)$$

so

$$\left\langle \frac{E^2}{m(r)} \right\rangle = \hbar^2 \omega^2 \int \phi^* \frac{1}{m(r)} \phi \, d\tau \quad -(7)$$

The Laplacian in spherical polar coordinates

$$\nabla^2 f = \frac{1}{r}\frac{\partial^2}{\partial r^2}(rf) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\phi^2} \qquad -(8)$$

The relevant part is:

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) \qquad -(9)$$

so for a plane wavefunction the square of the rest energy of any elementary particle is given by:

$$m^2c^4 = \hbar^2\omega^2\int\phi^*\frac{1}{m(r)}\phi\, d\tau - \hbar^2\int\phi^*\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\left(\frac{\phi}{m(r)}\right)\right)d\tau \qquad -(10)$$

For the rest particle the momentum term is zero, and if the rest frequency is ω_0, then

$$\boxed{m^2c^4 = \hbar^2\omega_0^2\int\phi^*\frac{1}{m(r)}\phi\, d\tau} \qquad -(11)$$

Here ω_0 is the de Broglie rest frequency of

3) the particle.

For example, the interaction between a proton and a neutron is mediated by particles defined in the following table:

Particle	Antiparticle	Rest Energy (MeV)
π^0	π^0	134.977
π^+	π^-	139.570
ρ^+	ρ^-	775.11
ρ^0	ρ^0	775.26
ω^0	ω^0	782.65

Table 1

So in this case, eq. (11) must produce the results of Table 1: a triplet of pions, a triplet of rho mesons and an omega meson singlet. The energies of π^+ and π^- are degenerate, and so are those of ρ^+ and ρ^-. The particles π^0, ρ^0 and ω^0 are their own antiparticles.

More generally the wave function is the solution of:

$$\left(\Box + m(r) \left(\frac{mc}{\hbar} \right)^2 \right) \phi = 0 \quad - (12)$$

So the results of Table 1 are given by a

Choice of ϕ and $m(r)$.

For the interaction between an electron and another electron, the mediating particle is the photon of mass m. In this case:

$$m(r) = 1 \qquad (13)$$

So eq. (11) reduces to:

$$m^2 c^4 = \hbar^2 \omega_0^2 \int \phi^* \phi \, d\tau \qquad (14)$$

This is the de Broglie equation for the photon with mass:

$$\hbar \omega_0 = mc^2 \qquad (15)$$

Q.E.D. In this case there is one photon and one rest mass.

Proceeding in this way it is possible to build up the entire subject of particle physics. Eq. (7) can give the masses of all the known mesons using ϕ and a function $m(r)$.