433(4): Plane Wave Solutions

Consider first of all the d'Alembert wave equation:

$$\left(\Box + \left(\frac{mc}{\hbar} \right)^2 \right) \phi = 0 \quad -(1)$$

This has the solution:

$$\phi = \phi(0) \exp(-i(\omega t - \kappa Z)) \quad -(2)$$

provided that $\hbar^2 \omega^2 - c^2 \kappa^2 \hbar^2 = m^2 c^4 \quad -(3)$

Using the Einstein / de Broglie equations:

$$E = \hbar \omega, \quad \underline{p} = \hbar \underline{\kappa} \quad -(4)$$

eq. (3) is the Einstein energy equation:

$$E^2 = p^2 c^2 + m^2 c^4 \quad -(5)$$

Q.E.D.

The plane wave (2) is also a solution of

$$\left(\Box + m(r) \left(\frac{mc}{\hbar} \right)^2 \right) \phi = 0 \quad -(6)$$

provided that:

$$E^2 = p^2 c^2 + m(r) m^2 c^4 \quad -(7)$$

Q.E.D.

Now take expectation values of eq. (7):

$$\langle E^2 \rangle = c^2 \langle p^2 \rangle + m^2 c^4 \langle m(r) \rangle \quad -(8)$$

so the rest energies of the particles in the string field are:

$$m^2 c^4 = \left\langle \frac{E^2}{m(r)} \right\rangle - c^2 \left\langle \frac{p^2}{m(r)} \right\rangle \quad -(9)$$

Using
$$E\phi = i\hbar\frac{\partial\phi}{\partial t} \quad -(10)$$
and
$$\underline{p}\,\phi = -i\hbar\underline{\nabla}\phi \quad -(11)$$
the expectation values are:
$$\left\langle\frac{E^2}{m(r)}\right\rangle = -\hbar^2\int\phi^*\frac{\partial^2}{\partial t^2}\left(\frac{\phi}{m(r)}\right)d\tau \quad -(12)$$
and
$$\left\langle\frac{p^2}{m(r)}\right\rangle = -\hbar^2\int\phi^*\nabla^2\left(\frac{\phi}{m(r)}\right)d\tau \quad -(13)$$

It is known from previous work on the m theory of the Lamb shift that an integral of the type (13) causes a shift. So this produces the difference in energy between $\pi^{\pm}$ and π^0. Here:
$$\phi = \phi(0)\exp\left(-i(\omega t - \kappa Z)\right) \quad -(14)$$
$$\phi^* = \phi(0)\exp\left(i(\omega t - \kappa Z)\right) \quad -(15)$$