# 432(1): The Theory of the Nuclear Force and the Strong Force

The nuclear force is that between protons and neutrons in a nucleus. It is powerfully attractive at 1 fm (femtometre), and rapidly decreases to zero at 2.5 fm. At distance of 0.7 fm it is repulsive, and determines the size of a nucleon. By 1935 it was known that the nuclear force is transmitted by mesons. The neutron was discovered in the Rutherford group by Chadwick in 1932. The interaction between neutrons and protons was first considered by Heisenberg and Dirac shortly after the discovery of the neutron. Heisenberg considered protons and neutrons to be different quantum states of the same particle, nucleons with different isospin quantum numbers.

The liquid drop model explained the spherical shape of the nucleus, and its nuclear binding energy. In the Yukawa model, massive bosons mediate the interaction between two nucleons. The Yukawa potential is:

$$U_1 = -g^2 \frac{\exp(-\mu r)}{r} \quad -(1)$$

where $\mu$ is the Yukawa particle mass and $g$ a scaling factor. So in the Yukawa potential:

$$U_1 = -g^2 \frac{\exp(-\alpha m r)}{r} \quad -(2)$$

where $g$ is a magnitude scaling constant, and $\alpha$ is another scaling constant so that the range is $1/(\alpha m)$. The potential consists of an exponential decay term ($\exp(-\alpha m r)$) and an electromagnetic term ($1/r$). The Yukawa potential is the result of an exchange particle with mass, called the meson. Its mass is about two hundred times that of the electron. The meson mass is related to the range of the interaction, $1/(\alpha m)$. The meson was discovered experimentally in 1947, and

2) became known as the pion.

The units of the Yukawa potential are J, so the factor $g^2$ is Jm. If the mass m is zero, the exchange particle is the photon, which in standard quantum field theory is considered to be massless, but in finite photon mass theory the range is not infinite and the photon is a massive boson. So the Coulomb potential is:

$$U_1(m=0) = -\frac{g^2}{r} = -\frac{e^2}{4\pi\epsilon_0 r} \quad -(3)$$

so

$$g^2 = \frac{e^2}{4\pi\epsilon_0} \quad -(4)$$

In gauge theory of the standard model, g is the gauge coupling constant.

In the frame of the m theory, the Yukawa potential becomes:

$$U_1(r_1) = -g^2 \frac{\exp(-\mu r_1)}{r_1} \quad -(5)$$

where

$$r_1 = \frac{r}{m(r)^{1/2}} \quad -(6) \quad -(7)$$

so:

$$U_1(r) = -\frac{g^2 m^{1/2}(r)}{r} \exp\left(-\frac{\mu r}{m^{1/2}(r)}\right)$$

Therefore the Yukawa potential in m space is

$$U_1(r) = -g^2 \frac{\exp(-\mu r_1)}{r_1} \quad -(8)$$

$$= -g^2 \frac{m^{1/2}(r)}{r} \exp\left(-\frac{\mu r}{m^{1/2}(r)}\right)$$

where

$$\mu = \alpha m \quad \text{--- (9)}$$

This is the attractive force between protons and neutrons.

The repulsive between protons is: — (10)

$$U_2 = \frac{Z_1 Z_2 e^2}{r_1} = Z_1 Z_2 \frac{m^{1/2}(r) e^2}{r}$$

So the total potential is: — (11)

$$U = U_1 + U_2$$

$$= m^{1/2}(r) \left( \frac{Z_1 Z_2 e^2}{r} - \frac{g^2 m^{1/2}(r)}{r} \exp\left( - \frac{\alpha m r}{m^{1/2}(r)} \right) \right)$$

The total force is:

$$F = - \frac{\partial U}{\partial r} \quad \text{--- (12)}$$

The attractive force is:

$$F_1 = - \frac{\partial U_1}{\partial r} \quad \text{--- (13)}$$

and the repulsive force is:

$$F_2 = - \frac{\partial U_2}{\partial r} \quad \text{--- (14)}$$

To eliminate human error the forces can be worked out with computer algebra and graphed. The range of the Yukawa potential is changed and is proportional to $m^{1/2}(r) / (\alpha m)$.

5) From UFT 431, the alternative force is:

$$F = -\frac{dm(r)}{dr}\left(\frac{m(r)}{2m(r) - r\frac{dm(r)}{dr}}\right)^{1/2} E \quad -(15)$$

where

$$E^2 = p^2c^2 + m(r)\,m^2c^4 \quad -(16)$$

So from eqs. (8) and (15): $\quad -(17)$

$$-\frac{\partial}{\partial r}\left(\frac{g^2}{r}\exp\left(-\frac{dmr}{m^{1/2}(r)}\right)\right) = \frac{dm(r)}{dr}\left(\frac{E}{2m(r) - r\frac{dm(r)}{dr}}\right)$$

This equation simplifies if we use a frame of reference with coordinate $r_1$. In that case:

$$\boxed{-\frac{\partial}{\partial r_1}\left(\frac{g^2}{r_1}\exp\left(-dmr_1\right)\right) = \frac{mc^2}{2}\,\gamma\,\frac{dm(r_1)}{dr_1}} \quad -(18)$$

where:

$$\gamma = \left(m(r_1) - \frac{1}{c^2}\,\underline{\dot{r}_1}\cdot\underline{\dot{r}_1}\right)^{-1/2} \quad -(19)$$

The total relativistic energy is:

$$E = m(r_1)\,\gamma\, mc^2 \quad -(20)$$

So

$$\gamma = \frac{E}{m(r_1)\,mc^2} \quad -(21)$$

From eqs. (18) and (21):

$$-\frac{\partial}{\partial r_1}\left(\frac{g^2}{r_1}\exp\left(-dmr_1\right)\right) = \frac{E}{2m(r_1)}\frac{dm(r_1)}{dr_1} \quad -(20)$$

Here $E$ is the total relativistic energy

) of a particle of mass m. By wave particle dualism:

$$E = n(r_1)\gamma mc^2 = \hbar\omega \quad -(21)$$

so

$$\boxed{-\frac{\partial}{\partial r_1}\left(\frac{g^2}{r_1}\exp(-\alpha m r_1)\right) = \frac{\hbar\omega}{2m(r_1)}\frac{dn(r_1)}{dr_1}} \quad -(22)$$

Eq. (22) in frame $(r_1, \phi)$ is equivalent to Eq. (17) in frame $(r, \phi)$. Therefore the nuclear force may be expressed as:

$$U_1(r_1) = -\frac{\hbar\omega}{2m(r_1)}\frac{dn(r_1)}{dr_1} \quad -(23)$$

x as:

$$U_1(r) = -\left(\frac{\hbar\omega}{2n(r) - r\frac{dn(r)}{dr}}\right)\frac{dn(r)}{dr} \quad -(24)$$

These equations give the nuclear force in terms of $\hbar\omega$, $m(r)$ and $dn(r)/dr$.

Any empirical force may be expressed by the force derived from eqs. (23) and (24). For example the Reid potential:

$$U_{Reid}(r) = -10.463\frac{e^{-\mu r}}{\mu r} - 1650.6\frac{e^{-4\mu r}}{\mu r} + 6484.2\frac{e^{-7\mu r}}{\mu r} \quad -(25)$$

or the force derived from the Woods Saxon potential.