32(3): Resonance Condition in a Born Landé Lattice

The calculation is developed in the (r_1, ϕ) frame of reference so the Born Landé potential energy is:

$$U = -\frac{Z_1 Z_2 e^2 M}{4\pi\epsilon_0 r_1} + \frac{B}{r_1^n} \quad -(1)$$

and the lattice force is:

$$F_1 = -\frac{\partial U}{\partial r_1} \quad -(2)$$

$$= -\frac{Z_1 Z_2 e^2 M}{4\pi\epsilon_0 r_1^2} + \frac{nB}{r_1^{n+1}}$$

From UFT 427, the m force in the frame (r_1, ϕ) is:

$$F_1 = -\frac{E}{2m(r_1)} \frac{\partial m(r_1)}{\partial r_1} \quad -(3)$$

where

$$E^2 = m(r_1)\left(p_1^2 c^2 + m^2 c^4\right) \quad -(4)$$

Now equate the force of attraction from eqs. (2) and (3):

$$\boxed{\frac{Z_1 Z_2 e^2 M}{4\pi\epsilon_0 r_1^2} = \frac{E}{2m(r_1)} \frac{\partial m(r_1)}{\partial r_1}} \quad -(5)$$

Now back transform to frame (r, ϕ) to find that

$$\frac{Z_1 Z_2 e^2 M m(r)}{4\pi\epsilon_0 r^2} = \frac{E}{2m(r)} \frac{\partial m(r)}{\partial r} \frac{\partial r}{\partial r_1} \quad -(6)$$

where:

$$r_1 = \frac{r}{m(r)^{1/2}} \quad -(7)$$

So:

$$\frac{dr_1}{dr} = \frac{d}{dr}\left(\frac{r}{m(r)^{1/2}}\right) \quad -(8)$$

$$= \frac{1}{m(r)}\left(m(r)^{1/2} - \frac{r}{2}\frac{dm(r)}{dr}m(r)^{-1/2}\right)$$

$$= \frac{1}{2m(r)^{3/2}}\left(2m(r) - r\frac{dm(r)}{dr}\right)$$

Therefore the m force in frame (r, ϕ) is:

$$F = -\frac{dm(r)}{dr}\frac{m^{1/2}(r)\,E}{2m(r) - r\frac{dm(r)}{dr}} \quad -(9)$$

as in previous papers, Q.E.D.

Therefore: $\quad -(10)$

$$\frac{Z_1 Z_2 e^2 M}{4\pi\epsilon_0}\frac{m(r)}{r^2} = m^{1/2}(r)\frac{dm(r)}{dr}\frac{E}{2m(r) - r\frac{dm(r)}{dr}}$$

In frame (r, ϕ)

$$E^2 = (p^2c^2 + m(r)m^2c^4) \quad -(11)$$

If one considers a Born Landé lattice immersed in hydrogen gas, then:

$$E = m(r)^{1/2} mc^2 \quad -(12)$$

, and from eqs. (10) and (12):

$$\frac{Z_1 Z_2 e^2 M}{4\pi \epsilon_0} \frac{1}{r^2} = \frac{dm(r)}{dr} \frac{mc^2}{2m(r) - r\frac{dm(r)}{dr}} \quad -(13)$$

i.e.

$$\left(2m(r) - r\frac{dm(r)}{dr}\right) A = r^2 \frac{dm(r)}{dr} \quad -(14)$$

where

$$A = \frac{Z_1 Z_2 e^2 M}{4\pi \epsilon_0 mc^2} \quad -(15)$$

where M is the Madelung constant.

Using rock salt as an example:

$$M = \pm (1.748 - 3.495) \quad -(16)$$

$$Z_1(Na^+) = 11, \quad Z(Cl^-) = 17 \quad -(17)$$

So

$$A = \pm 7.524 \times 10^{-16} \text{ m} \quad -(18)$$

From eq. (14):

$$2m(r) = \frac{dm(r)}{dr}\left(\frac{r^2}{A} - r\right) \quad -(19)$$

$$= \frac{dm(r)}{dr}\, r \left(\frac{r - A}{A}\right)$$

So:

$$\boxed{\frac{dm(r)}{dr} = \frac{2Am(r)}{r(r-A)}} \quad -(20)$$

When

$$r = A = 7.524 \times 10^{-16} \text{ m} \quad -(21)$$

the attractive m force (9) becomes infinite, and the electron is drawn into the lattice, Q.E.D.