2) Second Method of Development

Consider the hamiltonian of m theory:

$$H = m(r_1)\gamma mc^2 + U \quad -(1)$$

where

$$E = m(r_1)\gamma mc^2 \quad -(2)$$

is the total relativistic energy. In UFT 428 it was shown that:

$$E^2 = m(r_1)(c^2 p_1^2 + m^2 c^4) \quad -(3)$$

in the (r_1, ϕ) coordinate system of m theory.

From eq. (3):

$$E^2 - m(r_1) m^2 c^4 = m(r_1) c^2 p_1^2 \quad -(4)$$

i.e.

$$E = H - U = \frac{m(r_1) c^2 p_1^2}{E + m(r_1)^{1/2} mc^2} + m(r_1)^{1/2} mc^2 \quad -(5)$$

so:

$$H = \frac{m(r_1) c^2 p_1^2}{E + m(r_1)^{1/2} mc^2} + m(r_1)^{1/2}(mc^2 + U_0) \quad -(6)$$

In the hydrogen atom:

$$U_0 = \frac{-e^2}{4\pi\epsilon_0 r} \quad -(7)$$

is the Coulomb potential between the electron and proton in the H atom.

In the (r, ϕ) coordinate system:

$$H = \frac{c^2 p^2}{E + m(r)^{1/2} mc^2} + m(r)^{1/2}(mc^2 + U_0) \quad -(8)$$

2) In Eq. (8):

$$E = m(r)\gamma mc^2 = \hbar\omega \quad -(9)$$

using the Einstein / de Broglie equations, so:

$$\gamma = \frac{\hbar\omega}{m(r)mc^2} \quad -(10)$$

and

$$m(r)\gamma = \frac{\hbar\omega}{mc^2} \quad -(11)$$

Therefore:

$$H = \frac{p^2}{m\left(\frac{\hbar\omega}{mc^2} + m(r)^{1/2}\right)} + m(r)^{1/2}\left(mc^2 + U_0\right) \quad -(12).$$

For

$$m(r) = 1 \quad -(13)$$

this reduces to:

$$H = \frac{p^2}{m\left(\frac{\hbar\omega}{mc^2} + 1\right)} + \left(mc^2 + U_0\right) \quad -(14)$$

which is the theory of UFT 338, Q.E.D.

In the SU(2) basis eq. (12) is:

$$H = \frac{1}{m}\,\underline{\sigma}\cdot\underline{p}\,\frac{1}{\left(\frac{\hbar\omega}{mc^2} + m(r)^{1/2}\right)}\,\underline{\sigma}\cdot\underline{p} + m(r)^{1/2}\left(mc^2 + U_0\right) \quad -(15)$$

and in the presence of a magnetic field:

$$H = \frac{1}{m}\,\underline{\sigma}\cdot\left(\underline{p} - e\underline{A}\right)\frac{1}{\left(\frac{\hbar\omega}{mc^2} + m(r)^{1/2}\right)}\,\underline{\sigma}\cdot\left(\underline{p} - e\underline{A}\right) + m(r)^{1/2}\left(mc^2 + U_0\right) \quad -(16)$$

On quantization:

$$H\phi = \frac{ie\hbar}{m}\underline{\sigma}\cdot\underline{\nabla}\left(\left(\frac{1}{\left(\frac{\hbar\omega}{mc^2}+m(r)\right)^{1/2}}\right)\underline{\sigma}\cdot\underline{A}\,\phi\right)+\dots \quad -(17)$$

$$= -\frac{e\hbar}{m}\left(\left(\frac{1}{\left(\frac{\hbar\omega}{mc^2}+m(r)\right)^{1/2}}\right)\underline{\sigma}\cdot\underline{\nabla}\times\underline{A}\,\phi\right.$$

$$\left.+\,\underline{\sigma}\cdot\underline{\nabla}\left(\frac{1}{\left(\frac{\hbar\omega}{mc^2}+m(r)\right)^{1/2}}\right)\underline{\sigma}\cdot\underline{A}\,\phi\right)$$

Now use:

$$\underline{S} = \frac{\hbar}{2}\underline{\sigma} \quad -(18)$$

to find that

$$H\phi = -2\frac{e}{m}\left(\frac{1}{\left(\frac{\hbar\omega}{mc^2}+m(r)\right)^{1/2}}\right)\underline{S}\cdot\underline{B}\,\phi + \dots \quad -(19)$$

The g factor of the electron, g, is defined by:

$$H\phi = -g\frac{e}{2m}\underline{S}\cdot\underline{B}\,\phi \quad -(20)$$

from eqs. (19) and (20):

$$\frac{g}{2} = \frac{2}{\left(\frac{\hbar\omega}{mc^2}+m(r)\right)^{1/2}} \quad -(21)$$

d :

$$\boxed{g = \frac{4}{\frac{\hbar\omega}{mc^2} + m(r)^{1/2}}} \quad -(22)$$

In the limit:

$$m(r) = 1 \quad -(23)$$

$$g = \frac{4}{\frac{\hbar\omega}{mc^2} + 1} \quad -(24)$$

For the rest electron:

$$\hbar\omega_0 = mc^2 \quad -(25)$$

so

$$g \rightarrow 2 \quad -(26)$$

This is the result from the Dirac theory. For the rest electron with finite $m(r)$:

$$g = \frac{4}{1 + m(r)^{1/2}} \quad -(27)$$

By observation:

$$g = 2.002319314 \quad -(28)$$

so

$$1 + m(r)^{1/2} = \frac{4}{2.002319314} \quad -(29)$$
$$= 1.997683$$

and

$$m(r)^{1/2} = 0.997683 \quad -(30)$$

$$m(r) = 0.99884 \quad -(31)$$