420(3): Cartesian Representation of m Theory

Consider the infinitesimal line element of m space:

$$ds^2 = c^2 d\tau^2 = m(r)\, c^2 dt^2 - \frac{dr^2}{m(r)} - r^2 d\phi^2 \quad -(1)$$

in plane polar coordinates. The plane polar coordinates are defined by

$$X = r\cos\phi \quad -(2)$$

$$Y = r\sin\phi \quad -(3)$$

so

$$r^2 = X^2 + Y^2 \quad -(4)$$

$$\phi = \tan^{-1}\frac{X}{Y} \quad -(5)$$

$$-(6)$$

In m space:

$$\underline{r} = \frac{r}{m(r)^{1/2}}\,\underline{e}_r = \frac{1}{m(r)^{1/2}}\left(X\underline{i} + Y\underline{j}\right)$$

$$-(7)$$

where

$$m(r) = m\left(\left(X^2+Y^2\right)^{1/2}\right)$$

Therefore:

$$v^2 = \frac{\dot{r}^2 + r^2\dot{\phi}^2}{m(r)} \quad -(8)$$

$$= \frac{\dot{X}^2 + \dot{Y}^2}{m\left(\left(X^2+Y^2\right)^{1/2}\right)}$$

The generalized Lorentz factor is:

$$\gamma = \left(m(r) - \frac{\dot{r}^2 + r^2\dot{\phi}^2}{m(r)\,c^2}\right)^{-1/2} \quad -(9)$$

$$= \left(m(r) - \frac{\dot{X}^2 + \dot{Y}^2}{m(r)\,c^2}\right)^{-1/2}$$

where

$$r = \left(X^2+Y^2\right)^{1/2} \quad -(10)$$

The equations of motion in Cartesian coordinates

2) are:

$$\frac{dH}{dt} = 0 \quad -(11)$$

$$\frac{dL}{dt} = 0 \quad -(12)$$

where

$$H = \gamma m(r) mc^2 - m(r)^{1/2} \frac{mMG}{r} \quad -(13)$$

$$= \gamma m(r) mc^2 - m(r)^{1/2} \frac{mMG}{(X^2+Y^2)^{1/2}}$$

and

$$L = \frac{\gamma m r^2 \dot{\phi}}{m(r)} \quad -(14)$$

In Eq. (14):

$$\dot{\phi} = \frac{d}{dt}\left(\tan^{-1}\frac{X}{Y}\right) \quad -(15)$$

$$r^2 = X^2 + Y^2 \quad -(16)$$

Now use:

$$\tan\phi = \frac{X}{Y} \quad -(17)$$

so

$$\frac{d}{dt}\tan\phi = \frac{d}{dt}\left(\frac{X}{Y}\right) = \frac{\dot{X}Y - \dot{Y}X}{Y^2} \quad -(18)$$

and

$$\frac{d}{dt}\tan\phi = \frac{d}{d\phi}\tan\phi \frac{d\phi}{dt} = \frac{\dot{\phi}}{\cos^2\phi} \quad -(19)$$

So

$$\dot{\phi} = \left(\frac{\dot{X}Y - \dot{Y}X}{Y^2}\right)\cos^2\phi \quad -(20)$$

$$= \left(\frac{\dot{X}Y - \dot{Y}X}{Y^2}\right)\frac{X^2}{X^2+Y^2}$$

It follows that:

$$L = \frac{\gamma m r^2 \dot{\phi}}{m(r)} = \frac{\gamma m}{m(r)} \frac{X^2}{Y^2} \left(\dot{X} Y - \dot{Y} X \right) \quad -(21)$$

Therefore the Cartesian equations of motion are:

$$\frac{dH}{dt} = 0 \quad -(22)$$

$$\frac{dL}{dt} = 0 \quad -(23)$$

where:

$$H = \gamma m(r) m c^2 - m(r)^{1/2} \frac{m M G}{(X^2+Y^2)^{1/2}} \quad -(24)$$

$$L = \frac{\gamma m}{m(r)} \frac{X^2}{Y^2} \left(\dot{X} Y - \dot{Y} X \right) \quad -(25)$$

and

$$\gamma = \left(m(r) - \frac{\dot{X}^2 + \dot{Y}^2}{m(r) c^2} \right)^{-1/2} \quad -(26)$$

with

$$m(r) := m\left((X^2+Y^2)^{1/2} \right) \quad -(27)$$

As in UFT417 the relevant lagrangian is:

$$\mathcal{L} = -mc^2 \left(m(r) - \frac{1}{c^2} \underline{\dot{r}}_1 \cdot \underline{\dot{r}}_1 \right)^{1/2} + \frac{mMG}{r_1} \quad -(28)$$

with Euler Lagrange equation:

$$\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \underline{\dot{r}}_1} = \frac{\partial \mathcal{L}}{\partial \underline{r}_1} = \underline{\nabla} \mathcal{L} = \frac{\partial \mathcal{L}}{\partial r_1} \underline{e}_r$$

giving the force equation:

$$\underline{F}_1 = \frac{d\underline{p}_1}{dt} = \frac{\partial \mathcal{L}}{\partial r_1} \underline{e}_r \quad -(29)$$

where:

$$\underline{p}_1 = \gamma m \underline{\dot{r}}_1 = \frac{\gamma m \underline{\dot{r}}}{m(r)^{1/2}} \quad -(30)$$

In these equations:

$$\frac{\partial \mathcal{L}}{\partial r_1} = -\frac{mc^2}{2}\gamma \frac{dm(r)}{dr_1} - \frac{mMG}{r_1^2} \quad -(31)$$

so

$$\frac{d}{dt}\left(\frac{\gamma m \underline{\dot{r}}}{m(r)^{1/2}}\right) = \left(-\frac{mc^2}{2}\gamma \frac{dm(r)}{dr_1} - \frac{mMG}{r_1^2}\right)\underline{e}_r \quad -(32)$$

This equation gives the vacuum force, or force from m space:

$$\underline{F}(\text{vac}) = -\frac{mc^2}{2}\gamma \frac{dm(r)}{dr_1}\underline{e}_r \quad -(33)$$

$$= \left(\frac{\gamma mc^2 m(r)^{3/2}\frac{dm(r)}{dr}}{r\frac{dm(r)}{dr} - 2m(r)}\right)\underline{e}_r$$

Note that Eq. (32) contains the same information as

$$\frac{dH}{dt} = 0. \quad -(34)$$

So the Cartesian equations (22) to (27) can be used. Eq. (32) has the advantage of identifying the vacuum force (33). On the classical level for example:

$$H = \frac{1}{2}mv^2 - \frac{mMG}{r} \quad -(35)$$

and

$$\frac{dH}{dt} = \frac{1}{2}m\frac{dv^2}{dt} - mMG\frac{d}{dt}\left(\frac{1}{r}\right)$$

$$= \frac{1}{2}m\frac{dv^2}{dv}\frac{dv}{dt} - mMG\frac{d}{dr}\left(\frac{1}{r}\right)\frac{dr}{dt}$$

$$\gamma^3 \ddot{X} = -\frac{mG\,X}{(X^2+Y^2)^2} \quad -(51)$$

$$\gamma^3 \ddot{Y} = -\frac{mG\,Y}{(X^2+Y^2)^2} \quad -(52)$$

Finally consider the Hamiltonian of n theory:

$$H = n(r)\,\gamma m c^2 - \frac{n m G}{r} \quad -(53)$$

The equation of motion is:

$$\frac{dH}{dt} = 0 \quad -(54)$$

i.e.

$$m c^2 \frac{d}{dt}\left(n(r)\gamma\right) + \frac{n m G}{r^2} v = 0. \quad -(55)$$

where

$$\gamma = \left(n(r) - \frac{v^2}{n(r)c^2}\right)^{-1/2} \quad -(56)$$

Now use:

$$\frac{d}{dt}\left(n(r)\gamma\right) = \frac{d}{dv}\left(n(r)\gamma\right)\frac{dv}{dt} \quad -(57)$$

$$= \left(\gamma \frac{dn(r)}{dv} + n(r)\frac{d\gamma}{dv}\right)\frac{dv}{dt}$$

Assume that $n(r)$ has no dependence on v in a steady state or static spherical spacetime, and use

$$\frac{d\gamma}{dv} = \frac{1}{n(r)}\frac{v}{c^2}\gamma^3 \quad -(58)$$

It follows that:

$$= mv\frac{dv}{dt} + \frac{mMG}{r^2}v \quad -(36)$$
$$= 0$$

It follows that:

$$m\dot{v} = -\frac{mMG}{r^2} \quad -(37)$$

which is the Newtonian force equation in Cartesian coordinates, i.e.

$$m\ddot{\underline{r}} = -mMG\frac{\underline{r}}{r^3} \quad -(38)$$

i.e.

$$\ddot{X} = -\frac{MGX}{(X^2+Y^2)^{3/2}} \quad -(39)$$

$$\ddot{Y} = -\frac{MGY}{(X^2+Y^2)^{3/2}} \quad -(40)$$

Eqs. (39) and (40) have been used in previous UFT papers.

Similarly, in special relativity:

$$H = \gamma mc^2 - \frac{mMG}{r} \quad -(41)$$

and

$$\frac{dH}{dt} = mc^2\frac{d\gamma}{dt} + \frac{mMG}{r^2}v$$
$$= mc^2\frac{d\gamma}{dv}\frac{dv}{dt} + \frac{mMG}{r^2}v \quad -(42)$$
$$= 0$$

where

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad -(43)$$

Therefore:

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$$\frac{d\gamma}{dv} = \gamma^3 \frac{v}{c^2} \quad -(44)$$

and it follows that:

$$\gamma^3 \dot{v} = -\frac{MG}{r^2} \quad -(45)$$

This is the force equation of special relativity in flat or Minkowski space. In Cartesian coordinates:

$$\gamma^3 \ddot{X} = -\frac{MGX}{(X^2+Y^2)^{3/2}} \quad -(46)$$

$$\gamma^3 \ddot{Y} = -\frac{MGY}{(X^2+Y^2)^{3/2}} \quad -(47)$$

It has been shown in previous UFT papers that eqs. (46) and (47) give precessing orbits. They can be expressed in terms of a spin connection.

Note carefully that the features of a whirlpool galaxy can be obtained by integrating:

$$m\ddot{\underline{r}} = -mMG\frac{\underline{r}}{r^4} \quad -(48)$$

i.e.

$$\ddot{X} = -\frac{MGX}{(X^2+Y^2)^2} \quad -(49)$$

$$\ddot{Y} = -\frac{MGY}{(X^2+Y^2)^2} \quad -(50)$$

These equations give the Cotes spirals. The special relativistic Cotes spirals are obtained from:

$$\gamma^3 \dot{v} = -\frac{mG}{r^2} \quad -(59)$$

In Cartesian coordinates:

$$\gamma^3 \ddot{x} = -\frac{mG\,x}{(x^2+y^2)^{3/2}} \quad -(60)$$

$$\gamma^3 \ddot{y} = -\frac{mG\,y}{(x^2+y^2)^{3/2}} \quad -(61)$$

in which:

$$\gamma^3 = \left(n(r) - \frac{\dot{x}^2 + \dot{y}^2}{n(r)c^2} \right)^{-3/2} \quad -(61)$$

and

$$r = (x^2+y^2)^{1/2} \quad -(62)$$

These equations give generalized orbits for any $n(r)$.

Generalized galactic orbits are found from:

$$\gamma^3 \ddot{x} = -\frac{mG\,x}{(x^2+y^2)^2} \quad -(63)$$

$$\gamma^3 \ddot{y} = -\frac{mG\,y}{(x^2+y^2)^2} \quad -(64)$$

for any $n(r)$.

This method can be generalized further by solving eqs. (11) and (12) simultaneously.
