420(4): The Sagnac Effect in m Theory

Consider the infinitesimal line element of m theory:

$$ds^2 = m(r)c^2dt^2 - \frac{dr^2}{m(r)} - r^2 d\phi^2 \quad -(1)$$

For a null geodesic in a plane:

$$ds^2 = 0, \quad dr^2 = 0 \quad -(2)$$

so

$$m(r)\,c^2dt^2 = r^2 d\phi^2 \quad -(3)$$

The Sagnac effect is:

$$d\phi \rightarrow d\phi + \Omega\, dt \quad -(4)$$

where Ω is the angular frequency of the rotation of the platform.

So:

$$m(r)^{1/2}\, dt = \frac{r}{c}\left(d\phi + \Omega\, dt\right) \quad -(5)$$

Define the angular frequency of the light traversing the Sagnac interferometer by:

$$\omega = \frac{c}{r} \quad -(6)$$

and define:

$$\Omega = \frac{v}{r} \quad -(7)$$

It follows that:

$$dt = \frac{\frac{1}{\omega}\, d\phi}{m(r)^{1/2} - \frac{\Omega}{\omega}} \quad -(8)$$

So

$$dt = \frac{d\phi}{m(r)^{1/2}\omega - \Omega} \quad -(9)$$

Integrating over the 2π orbit of the light around the platform:

$$t_1 = \frac{2\pi}{n(r)^{1/2}\omega - \Omega} \quad -(10)$$

For light traversing in the opposite direction

$$t_2 = \frac{2\pi}{n(r)^{1/2}\omega + \Omega} \quad -(11)$$

So:

$$\Delta t = t_1 - t_2$$

$$= 2\pi\left(\frac{1}{n(r)^{1/2}\omega - \Omega} - \frac{1}{n(r)^{1/2}\omega + \Omega}\right)$$

$$= \frac{4\pi\Omega}{n(r)\omega^2 - \Omega^2} \quad -(12)$$

If

$$\omega \gg \Omega \quad -(13)$$

then

$$\Delta t = \frac{4\pi\Omega}{n(r)\omega^2}$$

$$= \frac{4\pi r^2\Omega}{n(r)c^2}$$

$$\boxed{\Delta t = \frac{4Ar\Omega}{n(r)c^2}} \quad -(14)$$

Therefore $n(r)$ can be measured experimentally a Sagnac interferometer.

3) In order to maximize the time difference Δt the area A_r and the frequency of rotation of the platform must be maximized. This is achieved with a fibre optic Sagnac interferometer, in which as many fibre optic loops as possible are used. This is a compact apparatus so Ω can be made a maximum by spinning the platform as fast as possible.

In general $n(r)$ can be any function of r.

In UFT145 and UFT146 r in Eq. (14) was interpreted as the distance of the interferometer from the centre of the earth, R_0, so in the absolute Einstein theory:

$$m = 1 - \frac{2MG}{c^2 R_0} \quad -(15)$$

where M is the mass of the earth. So by measuring Δt at different R_0, the Einstein theory can be tested with Eq. (14). For example the Δt can be measured on the surface of the earth and in a satellite. Another example is to measure Δt on the surface of the earth at sea level and on top of a high mountain.

More generally m is any function of R_0, so the interferometer could be used to measure the dependence of m on R_0. This is the principle of the gyrogravimeter suggested in UFT145 - UFT147.