420(3): Hamiltonian Formulation of m Theory

In the (r_1, ϕ) coordinate system the hamiltonian of m theory is:

$$H = m(r_1)\gamma m c^2 - \frac{mMG}{r_1} \quad -(1)$$

where

$$r_1 = \frac{r}{m(r)^{1/2}} \quad -(2)$$

The equation of motion is:

$$\frac{dH}{dt} = 0 \quad -(3)$$

because the hamiltonian is a constant of motion. It follows that:

$$mc^2 \frac{d}{dt}\left(m(r_1)\gamma\right) = \frac{d}{dt}\frac{mMG}{r_1} \quad -(4)$$

where

$$\gamma = \left(m(r_1) - \frac{v_1^2}{c^2}\right)^{-1/2} \quad -(5)$$

and

$$v_1^2 = \dot{r}_1^2 + r_1^2\dot{\phi}^2 \quad -(6)$$

In the Cartesian system of coordinates:

$$v_1 = \frac{dr_1}{dt} \quad -(7)$$

Eq. (4) can be written as:

$$\frac{dH}{dt} = mc^2 \frac{d}{dv_1}\left(m(r_1)\gamma\right)\frac{dv_1}{dt} + \frac{mMG}{r_1^2}v_1 = 0 \quad -(8)$$

i.e

$$mc^2\left(m(r_1)\frac{d\gamma}{dv_1} + \gamma\frac{dm(r_1)}{dv_1}\right)\frac{dv_1}{dt} = -\frac{mMG}{r_1^2}v_1 \quad -(9)$$

Now use:

$$\frac{d\gamma}{dv_1} = \frac{v_1}{c^2}\gamma^3 \quad -(10)$$

It follows that

$$m\, m(r_1)\, v_1 \gamma^3 \frac{dv_1}{dt} + mc^2 \gamma \frac{dm(r_1)}{dv_1}\frac{dv_1}{dt} = -\frac{mMG}{r_1^2} v_1 \quad -(11)$$

Note that:

$$\frac{d}{dt}(\gamma v_1) = \frac{d}{dv_1}(\gamma v_1)\frac{dv_1}{dt}$$

$$= \left(\gamma + v_1 \frac{d\gamma}{dv_1}\right)\frac{dv_1}{dt}$$

$$= \left(\gamma + \frac{v_1^2}{c^2}\gamma^3\right)\frac{dv_1}{dt} \quad -(12)$$

$$= \gamma^3 \frac{dv_1}{dt}\left(\frac{1}{\gamma^2} + \frac{v_1^2}{c^2}\right)$$

$$= m(r)\gamma^3 \frac{dv_1}{dt} \quad -(13)$$

Therefore eq. (11) becomes:

$$\frac{d}{dt}(\gamma m v_1) = -\frac{mMG}{r_1^2} - \frac{mc^2}{v_1}\frac{dv_1}{dt}\frac{dm(r_1)}{dv_1}\gamma$$

Now use:

$$\frac{dm(r_1)}{dr_1} = \frac{dm(r_1)}{dt}\frac{dt}{dr_1} = \frac{1}{v_1}\frac{dm(r_1)}{dt} \quad -(14)$$

3) and
$$\frac{dm(r_1)}{dt} = \frac{dm(r_1)}{dv_1}\dot{v}_1 \quad -(15)$$
to find that:
$$\frac{1}{v_1}\frac{dv_1}{dt}\frac{dm(r_1)}{dv_1} = \frac{dm(r_1)}{dr_1} \quad -(16)$$
It follows that:
$$\boxed{\frac{d}{dt}(\gamma m v_1) = -\frac{mMG}{r_1^2} - mc^2\gamma\frac{dm(r_1)}{dr_1}} \quad -(17)$$
The vacuum force is
$$F(vac) = -mc^2\gamma\frac{dm(r_1)}{dr_1}. \quad -(18)$$
It follows that:
$$F(vac) = -\frac{\partial E}{\partial r_1} \quad -(19)$$
where
$$E = \gamma m(r_1) mc^2 \quad -(20)$$
is the relativistic energy.
The conventional inverse square force is:
$$F = -\frac{\partial U}{\partial r_1} \quad -(20)$$
where
$$U = -\frac{mMG}{r_1} \quad -(21)$$
is the gravitational potential energy.
Note carefully that the Lagrangian

4) method of UFT417 produce the result:

$$\frac{d}{dt}\left(\gamma m v_1\right) = -\frac{mMG}{r_1^2} - \frac{mc^2}{2}\gamma\frac{dm(r_1)}{dr_1} \quad -(22)$$

from the lagrangian:

$$\mathcal{L} = -mc^2\left(m(r_1) - \frac{1}{c^2}\dot{\underline{r}}_1\cdot\dot{\underline{r}}_1\right)^{1/2} + \frac{mMG}{r_1} \quad -(23)$$

$$= -\frac{mc^2}{\gamma} + \frac{mMG}{r_1} \quad -(24)$$

and the Euler Lagrange equation:

$$\frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{\underline{r}}_1} = \frac{\partial\mathcal{L}}{\partial\underline{r}_1} = \underline{\nabla}\mathcal{L} = \frac{\partial\mathcal{L}}{\partial r_1}\underline{e}_r.$$

i.e.

$$\frac{d}{dt}\left(\gamma m v_1\right) = \frac{\partial\mathcal{L}}{\partial r_1} \quad -(25)$$

There is a factor of 2 difference between eqs. (17) and eq. (22).

The hamiltonian method is more accurate than the lagrangian method, in which the lagrangian has to be chosen to give the results of the hamiltonian method. The lagrangian method is therefore correct if and only if:

$$\mathcal{L} = -mc^2\left(x + m(r_1)\right)^{1/2} + \frac{mMG}{r_1} \quad -(26)$$

$$x = m(r_1) - \frac{\dot{\underline{r}}_1\cdot\dot{\underline{r}}_1}{c^2} \quad -(27)$$