215(7): Experimental Measurement of $n(r)$ Using the Sagnac Effect

With reference to papers such as 4FT45 and 4FT46, the Sagnac effect can be derived theoretically from the infinitesimal line element:

$$ds^2 = c^2 dt^2 - dr^2 - r^2 d\phi^2 \quad -(1)$$

when:

$$v = c \quad -(2)$$

and in a plane:

$$dr = 0 \quad -(3)$$

using the frame rotation:

$$d\phi \rightarrow d\phi + \omega_1 dt \quad -(4)$$

or

$$d\phi \rightarrow d\phi - \omega_1 dt. \quad -(5)$$

Using these equations:

$$dt = \frac{r}{c}\left(d\phi + \omega_1 t\right) \quad -(6)$$

from eq. (4), or

$$dt = \frac{r}{c}\left(d\phi - \omega_1 t\right) \quad -(7)$$

from eq. (5). Define the angular frequency of the beam of light in a Sagnac interferometer as:

$$\omega = \frac{c}{r} = \kappa c \quad -(8)$$

and it follows that:

$$dt = \frac{d\phi}{\omega \pm \omega_1}. \quad -(9)$$

The time T taken for the light to go around the Sagnac interferometer is:

$$\boxed{T = \frac{2\pi}{\omega \pm \omega_1}} \quad -(10)$$

depending on whether the platform is rotated clockwise or anticlockwise at an angular frequency ω_1.

In the metric of m space:

$$ds^2 = m(r)c^2 dt^2 - \frac{dr^2}{m(r)} - r^2 d\phi^2 \quad -(11)$$

It follows from eqs. (2) and (3) that:

$$m(r)c^2 dt^2 = r^2 d\phi^2 \quad -(12)$$

and using

$$d\phi \rightarrow d\phi \pm \omega_1 dt \quad -(13)$$

it follows that:

$$m(r)c^2 dt^2 = r^2 (d\phi \pm \omega_1 dt)^2 \quad -(14)$$

so

$$dt = \frac{r}{m^{1/2}(r)\, c} (d\phi \pm \omega_1 dt) \quad -(15)$$

and

$$m^{1/2}(r)\, \omega\, dt = d\phi \pm \omega_1 dt \quad -(16)$$

or

$$dt = \frac{d\phi}{m^{1/2}(r)\,\omega \pm \omega_1} \quad -(17)$$

and for one revolution:

$$\boxed{T_1 = \frac{2\pi}{m^{1/2}(r)\,\omega \pm \omega_1}} \quad -(18)$$

This method can be used to measure $m^{1/2}(r)$ [when] the Sagnac interferometer is placed in a strong gravitational field. The Sagnac effect in a satellite orbiting the earth in conditions [of] zero gravitation is given by eq. (10), but on [the] earth's surface is given by eq. (18). This method also shows that ω_1 can be measured in [the] Sagnac interferometer.