415(1): The Lagrangian and Hamiltonian of m Theory

The infinitesimal line element of m theory is:

$$ds^2 = c^2 d\tau^2 = m(r)c^2 dt^2 - \frac{dr^2}{m(r)} - r^2 d\phi^2 \quad -(1)$$

in plane polar coordinates (r, ϕ). Here $m(r)$ is a function of r. Eq. (1) is the most general spherically symmetric line element apart from the Crothers line element used in one of the UFT papers. The line element (1) can be written as:

$$c^2 d\tau^2 = (c^2 - v_N'^2)\, dt'^2 \quad -(2)$$

where:

$$v_N'^2 dt'^2 = \frac{dr^2}{m(r)} + r^2 d\phi^2 \quad -(3)$$

and

$$dt'^2 = m(r)\, dt^2 \quad -(4)$$

It follows that the Lorentz factor of the m theory is calculated from:

$$d\tau^2 = \left(1 - \frac{v_N'^2}{c^2}\right) dt'^2 \quad -(5)$$

to give:

$$\gamma' = \left(\frac{dt'}{d\tau}\right)^2 = \left(1 - \frac{v_N'^2}{c^2}\right)^{-1/2} \quad -(6)$$

Therefore the hamiltonian of m theory is:

$$H = \gamma' mc^2 - \frac{mMG}{r} \quad -(7)$$

and the Lagrangian is:

$$\mathcal{L} = -\frac{mc^2}{\gamma'} + \frac{mMG}{r} \quad -(8)$$

in which the Lorentz factor is:

$$\gamma' = \left(1 - \frac{1}{c^2}\left(\frac{1}{m(r)}\left(\frac{dr}{dt'}\right)^2 + r^2\left(\frac{d\phi}{dt'}\right)^2\right)\right)^{-1/2} \quad -(9)$$

Now use:

$$\frac{dr}{dt'} = \frac{dr}{dt}\frac{dt}{dt'} = \frac{1}{m^{1/2}(r)}\frac{dr}{dt} \quad -(10)$$

and

$$\frac{d\phi}{dt'} = \frac{1}{m^{1/2}(r)}\frac{d\phi}{dt} \quad -(11)$$

It follows that:

$$\gamma' = \left(1 - \frac{1}{m(r)c^2}\left(\frac{1}{m(r)}\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\phi}{dt}\right)^2\right)\right)^{-1/2} \quad -(12)$$

The field equations of m theory are obtained from the Lagrangian (8) wrt Lagrange variables r and ϕ.

$$\frac{\partial \mathcal{L}}{\partial r} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{r}} \quad -(13)$$

and

$$\frac{\partial \mathcal{L}}{\partial \phi} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\phi}} \quad -(14)$$

In the limit

$$m(r) \rightarrow 1 \quad -(15)$$

field equations from eqs. (13) and (14) are:

$$\gamma^3 \frac{dv}{dt} = -\frac{MG}{r^2} \quad -(16)$$

and

$$\frac{dL}{dt} = 0 \quad -(17)$$

) where:

$$L = \gamma m r^2 \dot{\phi} \quad -(18)$$

is the relativistic angular momentum, a constant of motion.

In eq. (16):

$$\frac{dv}{dt} = \ddot{r} - r\dot{\phi}^2 \quad -(19)$$

In the limit (15), simultaneous solution of eqns. (16) and (17) gives the relativistic orbit of m about M.

This orbit is developed into the orbit of m theory by using the Lorentz factor (12).

As in Note 414(4) write:

$$f = 1 - \frac{1}{m(r)c^2}\left(\frac{1}{m(r)}\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\phi}{dt}\right)^2\right), \quad -(20)$$

then

$$\frac{\partial \mathcal{L}}{\partial f} = -\frac{1}{2} m c^2 f^{-1/2} = -\frac{1}{2}\gamma' m c^2. \quad -(21)$$

It follows that:

$$\frac{\partial \mathcal{L}}{\partial \dot{\phi}} = \frac{\partial \mathcal{L}}{\partial f}\frac{\partial f}{\partial \dot{\phi}} = \frac{1}{2}\gamma' m c^2 . \frac{2\dot{\phi} r^2}{m(r)c^2}$$

$$= \frac{\gamma' m r^2 \dot{\phi}}{m(r)} \quad -(22)$$

From eq. (14):

$$\frac{dL}{dt} = 0 \quad -(23)$$

So

$$\boxed{L = \frac{\gamma'}{m(r)} m r^2 \dot{\phi}} \quad -(24)$$

the constant angular momentum of m theory.

Similarly, from eq. (20):

4)

$$\frac{\partial f}{\partial r} = -\frac{1}{c^2}\left(\left(\frac{dr}{dt}\right)^2 \frac{d}{dr}\left(\frac{1}{m^2(r)}\right) + \left(\frac{d\phi}{dt}\right)^2 \frac{d}{dr}\left(\frac{r^2}{m(r)}\right)\right)$$

$$= -\frac{1}{c^2}\left(\dot{r}^2 \frac{d}{dr}\left(\frac{1}{m^2(r)}\right) + \dot{\phi}^2 \frac{d}{dr}\left(\frac{r^2}{m(r)}\right)\right) \quad -(25)$$

and

$$\frac{\partial f}{\partial \dot{r}} = -\frac{2\dot{r}}{c^2 m^2(r)} \quad -(26)$$

So

$$\frac{\partial \mathcal{L}}{\partial r} = \frac{\partial \mathcal{L}}{\partial f}\frac{\partial f}{\partial r} \quad -(27)$$

$$= \frac{1}{2}\gamma' m\left(\dot{r}^2 \frac{d}{dr}\left(\frac{1}{m^2(r)}\right) + \dot{\phi}\frac{d}{dr}\left(\frac{r^2}{m(r)}\right)\right) - \frac{mMG}{r^2}$$

and

$$\frac{\partial \mathcal{L}}{\partial \dot{r}} = \frac{\partial \mathcal{L}}{\partial f}\frac{\partial f}{\partial \dot{r}} = \frac{m\gamma'\dot{r}}{m^2(r)} \quad -(28)$$

Therefore:

$$\boxed{\frac{d}{dt}\left(\frac{\gamma'\dot{r}}{m^2(r)}\right) - \frac{1}{2}\gamma'\left(\dot{r}^2\frac{d}{dr}\left(\frac{1}{m^2(r)}\right) + \dot{\phi}\frac{d}{dr}\left(\frac{r^2}{m(r)}\right)\right) = -\frac{MG}{r^2} \quad -(29)}$$

, the Leibniz equation of m theory.

In the limit

$$m(r) \to 1 \quad -(30)$$

t goes to

$$\frac{d\gamma}{dt}\dot{r} + \gamma(\ddot{r} - r\dot{\phi}^2) = -\frac{MG}{r^2} \quad -(31)$$

ich is

$$\gamma^3 \dot{v} = -\frac{MG}{r^2} \quad -(32)$$

Q.E.D.

5) A model for $m(r)$ in eq. (29) is needed in order to solve eqs. (29) and (24) simultaneously to give the orbits of m theory. In the limit:

$$m(r) \rightarrow 1 \quad -(25)$$

Eqs. (16) and (18) must be solved simultaneously. It is known from previous UFT papers that the simultaneous solution of eqs. (16) and (18) gives a precessing orbit. With reference to UFT108 the function:

$$m = 1 - \frac{r_0}{r} - \frac{a}{r^2} \quad -(26)$$

gives a shrinking orbit. So the use of this function in Eqs. (29) and (24) should give a shrinking and precessing orbit. In UFT190, the m theory was used to deduce:

$$\frac{dr}{d\phi} = r^2 \left(\frac{1}{b^2} - m(r) \left(\frac{1}{a^2} + \frac{1}{r^2} \right) \right)^{1/2} \quad -(27)$$

from the line element (1). In UFT190 it was also shown that in ECE cosmology:

$$m(r) = 2 - \exp\left(2 \exp\left(-\frac{r}{R} \right) \right) \quad -(28)$$

can be deduced.