412(3): Calculation of the Newtonian Velocity in the Rotating Frame.

In the usual Newtonian theory:

$$v_N^2 = \left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\phi}{dt}\right)^2 \quad -(1)$$

in plane polar coordinates r and ϕ. For the conic section:

$$r = \frac{\alpha}{1+\epsilon\cos\phi} \quad -(2)$$

where α is the half right latitude and ϵ the eccentricity. From Lagrangian dynamics the constant angular momentum is:

$$L = mr^2\frac{d\phi}{dt}. \quad -(3)$$

Now use:

$$\frac{dr}{dt} = \frac{dr}{d\phi}\frac{d\phi}{dt} \quad -(4)$$

and

$$\frac{dr}{d\phi} = \frac{\epsilon r^2}{\alpha}\sin\phi \quad -(5)$$

to find that

$$v_N^2 = \frac{L^2}{m}\left(\frac{1}{r^2} + \left(\frac{\epsilon}{\alpha}\right)\sin^2\phi\right) \quad -(6)$$

where

$$\sin^2\phi = 1 - \cos^2\phi \quad -(7)$$

and

$$\cos\phi = \frac{1}{\epsilon}\left(\frac{\alpha}{r} - 1\right) \quad -(8)$$

so

$$v_N^2 = \frac{L^2}{m}\left(\frac{1}{r^2} + \left(\frac{\epsilon}{\alpha}\right)^2\left(1 - \frac{1}{\epsilon^2}\left(\frac{\alpha}{r} - 1\right)^2\right)\right)$$

$$= \frac{L^2}{m}\left(\frac{\epsilon^2-1}{\alpha^2} + \frac{2}{\alpha r}\right) \quad -(9)$$

Finally, use:

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$$= \frac{L^2}{m}\left(\frac{\epsilon^2-1}{\alpha^2} + \frac{2}{\alpha r}\right) \quad -(9)$$

Finally, use:

2)

$$L^2 = m^2 M G \alpha \quad -(10)$$

to find that:

$$v_N^2 = mG\left(\frac{2}{r} + \frac{\epsilon^2 - 1}{\alpha}\right)$$

$$= mG\left(\frac{2}{r} - \frac{1}{a}\right) \quad -(11)$$

where

$$\frac{1}{a} = \frac{1-\epsilon^2}{\alpha} \quad -(12)$$

is the semi major axis.

In the rotating frame:

$$v_N^2 = mG\left(\frac{2}{r} - \frac{1}{a'}\right) \quad -(13)$$

where

$$a' = \frac{\alpha'}{1-\epsilon'^2}. \quad -(14)$$

Here:

$$\alpha' = \frac{L'^2}{m^2 M G} \quad -(15)$$

where:

$$L' = \mu r^2 \omega'$$

$$= \mu r^2 \frac{d\phi'}{dt} = \mu r^2 \frac{d}{dt}\left(\phi + \omega_1 t\right)$$

$$= \mu r^2\left(\omega + \omega_1 + t\frac{d\omega_1}{dt}\right) \quad -(16)$$

= constant of motion.

Also,

$$\epsilon'^2 = 1 + \frac{2H'L'^2}{\mu(mMG)^2} \quad -(17)$$

) where the hamiltonian H' in the rotating frame is:

$$H' = \frac{1}{2}\mu\left(\frac{dr}{dt}\right)^2 + \frac{1}{2}\frac{L'}{mr^2} - \frac{mMG}{r} \quad -(18)$$

= constant of motion.

The hamiltonian is related to a' as follows:

$$H' = \frac{1}{2}mv'^2 - \frac{mMG}{r} \quad -(19)$$

$$= \frac{1}{2}mMG\left(\frac{2}{r} - \frac{1}{a'}\right)$$

$$= -\frac{mMG}{2a'}$$

so a' is also a constant of motion.

In order to compute the hamiltonian use:

$$r = \frac{\alpha'}{1+\epsilon'\cos\phi'} \quad -(20)$$

and

$$\frac{dr}{dt} = -\frac{\epsilon'\alpha'\frac{d}{dt}\cos\phi'}{(1+\epsilon'\cos\phi')^2} \quad -(21)$$

$$= \frac{\epsilon'\alpha'\sin\phi'}{(1+\epsilon'\cos\phi')^2}\frac{d\phi'}{dt}$$

in eq. (18). Here:

$$\phi' = \phi + \omega_1 t, \quad -(22)$$

and

$$\frac{d\phi'}{dt} = \frac{d\phi}{dt} + \omega_1 + t\frac{d\omega_1}{dt} \quad -(23)$$

4) These equations allow H' and L' to be calculated in terms of ω and ω_1. For one orbit:

$$t = T' \quad -(24)$$

where:

$$T'^2 = \frac{4\pi^2 \mu}{m M G} a'^3 \quad -(25)$$

in the rotating frame. Eq. (25) is Kepler's third law.

Having calculated the hamiltonian H' the semi major axis in the rotating frame is calculated from eq. (19). This is a'. Knowing a', the time taken for one orbit in the rotating frame, T', can be calculated from Kepler's third law in the rotating frame, eq. (25).

In the rotating frame, theory, α' and ϵ' are the observed α' and ϵ'. For the Hulse Taylor binary pulsar and the S2 star, α' and ϵ' are known and have been used in previous UFT papers.

So v_N' can be calculated in the rotating frame and compared with v_N in the usual Newton theory. This gives the effect of rotation on the Newtonian linear velocity, i.e. the orbital linear velocity.

Similarly, the effect of frame rotation can be found for α, ϵ, T, L and H, and also for the lagrangian $\mathcal{L}$. Every quantity in classical dynamics is changed by ω_1

5) Note carefully that in the universal law of precession:

$$\Delta\phi = \frac{2\pi}{c^2}\left(v_N^2 + r^2\left(\omega_1^2 + 2\omega\omega_1\right)\right) \quad -(26)$$

the Newtonian velocity v_N is that of the unrotated frame, i.e. by:

$$v_N^2 = mG\left(\frac{2}{r} - \frac{1}{a}\right) \quad -(27)$$

with:

$$a = \frac{\alpha}{1-\epsilon^2} \quad -(28)$$

and

$$\alpha = \frac{L^2}{m^2 MG} \quad -(29)$$

$$\epsilon^2 = 1 + \frac{2HL^2}{\mu(mMG)^2} \quad -(30)$$

$$H = \frac{1}{2}\mu\left(\frac{dr}{dt}\right)^2 + \frac{1}{2}\frac{L}{mr^2} - \frac{mMG}{r} \quad -(31)$$

and

$$\frac{dr}{dt} = \frac{\epsilon\alpha\sin\phi}{(1+\epsilon\cos\phi)^2}\frac{d\phi}{dt} \quad -(32)$$

with

$$\omega = \frac{d\phi}{dt}.$$