411(6): Rotating Frame and Vacuum Energy

As shown in 411(4) the frame rotation:

$$\phi' = \phi + \omega_1 t \quad -(1)$$

produces the orbit:

$$r = \frac{\alpha'}{1+\epsilon' \cos(\phi + \omega_1 t)} \quad -(2)$$

and the shrinkage equation

$$r^2 = \frac{L'}{m\left(\omega + \omega_1 + T \frac{d\omega_1}{dt}\right)} \quad -(3)$$

The precession from eq. (2) can be defined as:

$$\Delta \phi = \omega_1 T \quad -(4)$$

where T is the time taken for one orbit and ω_1 is the angular velocity of frame rotation. In these equations the rotating frame is marked as $'$. The classical hamiltonian in the rotating frame is:

$$H' = \frac{1}{2} \mu \dot{r}^2 + \frac{1}{2} \frac{L'^2}{mr^2} + U(r) \quad -(5)$$

in which:

$$L' = \mu r^2 \omega' = \text{constant} \quad -(6)$$

The hamiltonian H' and the angular momentum L' are constants of motion in the rotating frame, and

$$\omega' = \frac{d\phi'}{dt} = \frac{d\phi}{dt} + \frac{d}{dt}(\omega t) \quad -(7)$$

$$:= \omega + \omega_1 + t\frac{d\omega_1}{dt}.$$

It follows that

$$L' = \mu r^2 \left(\omega + \omega_1 + t \frac{d\omega_1}{dt}\right) = \text{constant} \quad -(8)$$

Eq. (3) follows from Eq. (8) given an orbital time T.

The kinetic energy in eq. (5) increases by rotating the frame according to eq. (1). The frame rotates because of the underlying geometrical torsion, so the frame rotation is caused by geometry, and this is the geometry of spacetime, or "aether" or "vacuum".

<u>So</u> <u>frame rotation</u> <u>is a</u> <u>source of</u> <u>vacuum</u> <u>effects</u> <u>such</u> <u>as</u> <u>energy</u> <u>and</u> <u>precession.</u>

The kinetic energy of the rotating frame is:

$$T' = \frac{1}{2}\mu \dot{r}^2 + \frac{1}{2}\frac{\mu r^4}{\mu r^2}\left(\omega + \omega_1 + T\frac{d\omega_1}{dt}\right)^2$$

$$= \frac{1}{2}\mu\left(\dot{r}^2 + r^2\left(\omega + \omega_1 + T\frac{d\omega_1}{dt}\right)^2\right) \quad -(9)$$

The kinetic energy of the static frame is

$$T = \frac{1}{2}\mu\left(\dot{r}^2 + r^2\omega^2\right) \quad -(10)$$

so

$$T' > T \quad -(11)$$

Q.E.D.

It is well known that eq. (10) leads to the Newtonian orbital velocity:

$$v_N^2 = \frac{mMG}{\mu}\left(\frac{2}{r} - \frac{1}{a}\right) \quad -(12)$$

where the reduced mass is:

$$\mu = \frac{mM}{m+M} \quad -(13)$$

and the semi major axis is:

$$a = \frac{\alpha}{1-\epsilon^2} = \frac{mMG}{2|H|} \quad -(14)$$

The orbit is characterized by the half right latitude α and the eccentricity ϵ as is well known.

Assuming that the frame rotation (1) does not change r, the orbital velocity in the rotating frame is:

$$v'^2 = \frac{mMG}{\mu}\left(\frac{2}{r} - \frac{1}{a'}\right) = MG\left(\frac{2}{r} - \frac{1}{a'}\right) \quad -(15)$$

if

$$\mu \sim m \quad -(16)$$

i.e.

$$M \gg m \quad -(17)$$

as in the solar system.

In the rotating frame:

$$a' = \frac{\alpha'}{1-\epsilon'^2} = \frac{mMG}{2|H_1|} \quad -(15)$$

so

$$a' < a \quad -(16)$$

and the orbit shrinks, Q.E.D. This is self consistent with eq. (3). The orbit also precesses on the classical level according to eq. (4). These are the main features of the Hulse Taylor binary pulsar.

The original de Sitter rotation (1) was applied to the Schwarzschild metric, so was a relativistic idea. The relativistic infinitesimal line element is

$$ds^2 = c^2 d\tau^2 = (c^2 - v_N^2)\, dt^2 \quad -(17)$$

here $d\tau$ is the infinitesimal of proper time. If frame K' is moved at a constant velocity with respect to frame K, then τ is the time in frame K'. The time in frame K is t. A clock in frame K' does not move with respect to an observer in frame K'.

Eq. (17) can be expressed as:

$$ds^2 = c^2 d\tau^2 = \left(1 - \frac{v_N^2}{c^2}\right) c^2 dt^2 \quad -(18)$$

so $$d\tau = \left(1 - \frac{v_N^2}{c^2}\right)^{1/2} dt \quad -(19)$$

and $$d\tau < dt \quad -(20)$$

The Lorentz factor is defined as:

$$\gamma = \left(1 - \frac{v_N^2}{c^2}\right)^{-1/2} \quad -(21)$$

so $$dt = \gamma d\tau \quad -(22)$$

Using: $$v_N^2 = \left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\phi}{dt}\right)^2 \quad -(23)$$

the infinitesimal line element (17) becomes:

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - dr^2 - r^2 d\phi^2 \quad -(24).$$

The de Sitter rotation (1) changes this to:

$$ds'^2 = c^2 d\tau'^2 = c^2 dt^2 - dr^2 - r^2 d\phi'^2 \quad -(25)$$

so that r is unchanged, Q.E.D. $\quad -(26)$

Eq. (25) is:

$$ds'^2 = c^2 dt^2 - dr^2 - r^2 d\phi^2 - 2\omega_1 r^2 d\phi\, dt - \omega_1^2 r^2 dt^2$$

$$= (c^2 - \omega_1^2 r^2) dt^2 - 2\omega_1 r^2 d\phi\, dt - dr^2 - r^2 d\phi^2$$

Now use:

$$\omega = \frac{d\phi}{dt} \quad -(27)$$

[where] ω is the orbital angular velocity of m around [M]. In general this is different from ω_1, which is [the] angular velocity of frame rotation.

) It follows that:

$$ds'^2 = (c^2 - \omega_1^2 r^2)\,dt^2 - 2\omega\omega_1 r^2 dt^2 - v_N^2\,dt^2$$

$$= \left(1 - \frac{v^2}{c^2}\right) c^2 dt^2 \quad -(19)$$

where:

$$\boxed{v^2 = v_N^2 + r^2(\omega_1^2 + 2\omega\omega_1)} \quad -(20)$$

This result generalizes the theory of UFT410, in which it was assumed that:

$$\omega = \omega_1 \quad -(21)$$

So

$$v^2 = v_N^2 + 3\omega^2 r^2. \quad -(22)$$

Similarly, for:

$$\phi' = \phi - \omega_1 t \quad -(23)$$

it follows that:

$$\boxed{v^2 = v_N^2 + r^2(\omega_1^2 - 2\omega\omega_1)} \quad -(24)$$

which generalizes the result of UFT410 for eq. (23):

$$v^2 = v_N^2 - r^2\omega^2 \quad -(25)$$

Therefore ω^2 of UFT410 is replaced

by:

$$\boxed{\begin{aligned} \omega^2 &\rightarrow \omega_1^2 + 2\omega\omega_1 \quad &-(26) \\ \phi' &= \phi + \omega_1 t \quad &-(27) \end{aligned}}$$

for

and by:

$$\boxed{\begin{aligned} \omega^2 &\rightarrow \omega_1^2 - 2\omega\omega_1 \quad &-(28) \\ \phi' &= \phi - \omega_1 t \quad &-(29) \end{aligned}}$$

for

6) Therefore if $\phi' = \phi + \omega_1 t$ - (30)

the relativistic phase change is:

$$\Delta\Phi = \frac{2\pi}{c^2}\left(v_N^2 + r^2\left(\omega_1^2 + 2\omega\omega_1\right)\right) \quad - (31)$$

and if $\phi' = \phi - \omega_1 t$ - (32)

the relativistic phase change is:

$$\Delta\Phi = \frac{2\pi}{c^2}\left(v_N^2 + r^2\left(\omega_1^2 - 2\omega\omega_1\right)\right) \quad - (33)$$

The modulus of the classical phase change is

$$|\Delta\Phi|_0 = \omega_1 T \quad - (34)$$

The classical phase change is obtained from the classical hamiltonian (5), and the relativistic phase change from the infinitesimal line elements corresponding to the relativistic hamiltonian:

$$H' = \gamma' mc^2 + U(r) \quad - (35)$$

The classical lagrangian in the rotating frame is

$$\mathcal{L}' = \frac{1}{2}\mu\left(\dot{r}^2 + r^2\dot{\phi}'^2\right) - U(r) \quad - (36)$$

and the relativistic lagrangian is:

$$\mathcal{L}' = -\frac{mc^2}{\gamma'} - U(r) \quad - (37)$$

Therefore for self consistency, eqs. (35) and (37) must give the results (31) and (33).

In Cartesian coordinates:

1)

$$X = r\cos\phi \quad -(38)$$
$$Y = r\sin\phi \quad -(39)$$

If the frame is rotated by

$$\phi' = \phi + \omega_1 t \quad -(40)$$

then

$$X' = r\cos(\phi + \omega_1 t) \quad -(41)$$
$$Y' = r\sin(\phi + \omega_1 t) \quad -(42)$$

The ellipse in the static frame is:

$$\frac{X^2}{a^2} + \frac{Y^2}{b^2} = 1 \quad -(43)$$

where a and b are the semi major and minor axes.

Eq. (43) is equivalent to:

$$r = \frac{\alpha}{1+\epsilon\cos\phi} \quad -(44)$$

The ellipse in the rotating frame is:

$$\frac{X'^2}{a'^2} + \frac{Y'^2}{b'^2} = 1 \quad -(45)$$

and this is equivalent to:

$$r = \frac{\alpha'}{1+\epsilon'\cos(\phi+\omega_1 t)} \quad -(46)$$

If

$$\phi' = \phi - \omega_1 t \quad -(47)$$

then

$$r = \frac{\alpha'}{1+\epsilon'\cos(\phi-\omega_1 t)} \quad -(48)$$

With these preliminaries we consider the Newtonian force in the static frame:

)

$$\underline{F} = m\underline{g} = -\frac{mMG\,\underline{r}}{r^3} \quad -(49)$$

which leads to:

$$\underline{r} = \frac{\alpha}{1+\epsilon\cos\phi}. \quad -(50)$$

In Cartesian coordinates:

$$\ddot{X}\,\underline{i} + \ddot{Y}\,\underline{j} = -\frac{MG(X\underline{i}+Y\underline{j})}{(X^2+Y^2)^{3/2}} \quad -(51)$$

so

$$\ddot{X} = -\frac{MG\,X}{(X^2+Y^2)^{3/2}} \quad -(52)$$

$$\ddot{Y} = -\frac{MG\,Y}{(X^2+Y^2)^{3/2}} \quad -(53)$$

These equations, when solved simultaneously, lead to:

$$\frac{X^2}{a^2} + \frac{Y^2}{b^2} = 1 \quad -(54)$$

It is now assumed that the frame rotation (40) leads to:

$$\underline{F}' = m\underline{g}' = -\frac{mMG}{r^3}\,\underline{r} + \underline{\Omega}\,\underline{\Phi} \quad -(55)$$

$$= -\frac{mMG}{r^3}\,\underline{r} - \underline{\Omega}\,\frac{mMG}{r^2}\,\underline{r}$$

so

$$\ddot{X}' = -MG\left[\frac{X'}{(X'^2+Y'^2)^{3/2}} + \frac{\Omega_X X'}{(X'^2+Y'^2)}\right]$$

$$= -\frac{MG\,X'}{X'^2+Y'^2}\left[\frac{1}{(X'^2+Y'^2)^{1/2}} + \Omega_X\right]$$

9)

$$\ddot{y}' = -\frac{mG}{x'^2+y'^2}\, y' \left[\frac{1}{(x'^2+y'^2)^{1/2}} + \Omega_y\right] \quad -(57)$$

It has been assumed that the rotation of the frame is a vacuum effect due to the spin connection.

Eqs. (56) and (57) are solved simultaneously by computer. They are ECE2 covariant so must give a precession of the type (31) and (32). So Ω_x and Ω_y are functions of $\omega_1 t$. In computing $\ddot{x}'$ and $\ddot{y}'$ care must be taken to apply the Leibniz theorem correctly. This is best left to computer algebra:

For example:

$$x' = r\cos(\phi + \omega_1 t) \quad -(58)$$

so

$$\dot{x}' = \frac{dr}{dt}\cos(\phi+\omega_1 t) + r\frac{d}{dt}\cos(\phi+\omega_1 t) \quad -(59)$$

and so on.

These concepts can be extended to plane polar coordinates. In the static frame the acceleration is:

$$\underline{a} = (\ddot{r} - r\dot{\phi}^2)\,\underline{e}_r + (r\ddot{\phi} + 2\dot{r}\dot{\phi})\,\underline{e}_r \quad -(60)$$

so the Newtonian force is:

$$\underline{F} = m\underline{g} = -mMG\frac{\underline{r}}{r^3} \quad -(61)$$

in the static frame. Now use:

$$\underline{r} = r\,\underline{e}_r \quad -(62)$$

to find that:

$$F = \ddot{r} - r\dot{\phi}^2 = -\frac{mG}{r^2} \quad -(63)$$

$$r\ddot{\phi} + 2\dot{r}\dot{\phi} = 0 \quad -(64)$$

which give

$$r = \frac{\alpha}{1+\epsilon\cos\phi} \quad -(65)$$

Simultaneous solution of eqs. (63) and (64) give eq. (65). In these equations the angular momentum:

$$L = mr^2\dot{\phi} \quad -(66)$$

is a constant of motion, and eq. (64) follows from eq. (66). Eq. (63) is therefore:

$$\ddot{r} - \frac{rL^2}{m^2r^4} = -\frac{mG}{r^2} \quad -(67)$$

i.e.

$$\ddot{r} - \frac{L^2}{m^2r^3} = -\frac{mG}{r^2} \quad -(68)$$

In the rotating frame:

$$\underline{F}' = m\underline{g}' = -mMG\frac{\underline{r}}{r^3} + \underline{\Omega}\, r \quad -(69)$$

so:

$$F' = \ddot{r} - \frac{L'^2}{m^2r^3} = -\frac{mG}{r^2} + \Omega r \quad -(70)$$

in which

$$L' = mr^2\frac{d}{dt}(\phi + \omega_1 t) \quad -(71)$$

so Ω is due to ω_1, Q.E.D.