411(4): Comparison of Precession Theories

The relativistic theory of UFT 310 gives the results:

$$|\Delta\phi| = \omega t - \omega_0 \tau = \frac{2\pi}{c^2}\left(v_N^2 + 3\omega_+^2 r^2\right) \quad -(1)$$

for

$$\phi' \rightarrow \phi + \omega_+ t \quad -(2)$$

and

$$|\Delta\phi| = \omega t - \omega_0 \tau = \frac{2\pi}{c^2}\left(v_N^2 - \omega_-^2 r^2\right) \quad -(3)$$

for

$$\phi' \rightarrow \phi - \omega_- t \quad -(4)$$

For rotation (2), the Newtonian velocity is changed to:

$$v_N'^2 = v_N^2 + 3\omega_+^2 r^2. \quad -(5)$$

For rotation (4), the Newtonian velocity is changed to:

$$v_N'^2 = v_N^2 - \omega_-^2 r^2 \quad -(6)$$

The infinitesimal line element is changed to:

$$ds^2 = c^2 d\tau^2 = \left(c^2 - v_N^{2\,\prime}\right) dt^2 \quad -(7)$$

$$= c^2 dt^2 - dr^2 - r^2 d\phi'^2$$

From eq. (7) it follows that:

$$d\tau^2 = \left(1 - \frac{v_N'^2}{c^2}\right) dt^2 \quad -(8)$$

$$:= \frac{dt^2}{\gamma'^2}$$

where

$$\gamma' = \left(1 - \frac{v_N'^2}{c^2}\right)^{-1/2} \quad -(9)$$

) In the unrotated frame corresponding to:

$$ds^2 = c^2 d\tau^2 = (c^2 - v_N^2) dt^2 \quad -(10)$$

The angular velocity Ω of a matter wave is defined by the de Broglie Einstein equation:

$$E = \hbar\Omega = \gamma m c^2 \quad -(11)$$

The rest frequency is defined by:

$$\gamma \rightarrow 1 \quad -(12)$$

so

$$\Omega_0 = \frac{m c^2}{\hbar} . \quad -(13)$$

This is the de Broglie rest frequency and is frame independent.

The rest phase is defined by:

$$\Phi_0 = \Omega_0 d\tau \quad -(14)$$

where τ is the proper time, the time in the frame in which the particle is at rest.

In the rotated frame corresponding to eq. (7):

$$\hbar\Omega' = \gamma' m c^2 \quad -(15)$$

where γ' is defined by eq. (9). So:

$$\Omega' = \gamma' \frac{m c^2}{\hbar} = \gamma' \Omega_0 . \quad -(16)$$

The interval of time t in the rotated frame is dt. The rest frame is defined by:

$$ds^2 = c^2 d\tau^2 \quad -(17)$$

and the rotated moving frame is defined by

$$ds^2 = (c^2 - v_N^{\prime 2})\, dt^2 \quad -(18)$$

The phase in the rotated moving frame is:

$$\Phi = \Omega\, dt \quad -(19)$$

so the phase change is:

$$\Delta\Phi = \Omega' dt - \Omega_0 d\tau$$
$$= \Omega' dt \left(1 - \frac{1}{\gamma'^2}\right) \quad -(20)$$

In one orbit:

$$\Omega' dt = 2\pi \quad -(21)$$

So the phase change is

$$\Delta\Phi = 2\pi \left(1 - \frac{1}{\gamma'^2}\right)$$
$$= 2\pi \frac{v_N'^2}{c^2} \quad -(22)$$

as first derived in a slightly different way in UFT410.

The precession is the phase change per radian:

$$\Delta\phi = \frac{\Delta\Phi}{2\pi} = 1 - \frac{1}{\gamma'^2}. \quad -(23)$$
$$= \frac{v_N'^2}{c^2}$$

In UFT410, the phase change (22) was used in the universal law of precession. However, the phase change per radian can be used, so t)

$$\boxed{\Delta\phi = \frac{v_N'^2}{c^2}} \quad -(24)$$

The type of law used depends on the way in which the precession is measured!

In UFT 410 the universal law of precession in the form:

$$\Delta\Phi = 2\pi \frac{v_N'^2}{c^2} \quad -(25)$$

was used to show that the criterion (2) applied to the four inner planets, Mercury, Venus, Earth and Mars, while the criterion (4) applied to the outer planets starting with Jupiter.

For Earth, the modulus of the experimentally observed precession is

$$|\Delta\phi_T| = 5.551 \times 10^{-4} \text{ radians per second} \quad -(26)$$

and it was found that:

$$\omega_+ = 1.628 \times 10^{-7} \text{ rad s}^{-1} \quad -(27)$$

using

$$|\Delta\phi_T| = 2\pi \frac{v_N'^2}{c^2} \quad -(28)$$

From the classical method of preceding notes, the modulus of the phase change was found to be

$$|\Delta\phi_T| = \omega_1 T \quad -(29)$$

5) where ω_1 is the classical angular velocity of frame rotation, and T is the time taken for one orbit. For the earth:

$$T = \text{one year} = 3.1557 \times 10^{7} \text{ secs,} \quad -(30)$$

so

$$\omega_1 = \frac{|\Delta \phi_T|}{T} = 1.76 \times 10^{-11} \text{ rad s}^{-1} \quad -(31)$$

whereas the relativistic ω_T is:

$$\omega_T = 1.628 \times 10^{-7} \text{ rad s}^{-1} \quad -(32)$$

The classical theory of precession relies on

$$\phi' \rightarrow \phi \pm \omega_1 t \quad -(33)$$

to produce the results of Note 411(2) and the orbit:

$$r = \frac{\alpha'}{1 + \epsilon' \cos(\phi + \omega_1 t)} \quad -(34)$$

which is a shrinking and precessing orbit. The orbit shrinks according to:

$$r^2 = \frac{L'}{m\left(\omega + \omega_1 + t\,\frac{d\omega_1}{dt}\right)} \quad -(35)$$

In a complete orbit of 2π, the time taken for the orbit is T, so in one orbit:

$$r^2 = \frac{L'}{m\left(\omega + \omega_1 + T\,\frac{d\omega_1}{dt}\right)} \quad -(36)$$

giving the shrinkage per orbit.

The classical method depends on meaning the angle ϕ from the position of the perihelion at $t = 0$. Successive appearance of the perihelion appears when the argument of the cosine term increases to 2π, 4π and so on. An increase of the argument by 2π requires:

$$\phi + \omega_1 t = 2\pi \quad -(37)$$

so

$$\phi = 2\pi - \omega_1 t \quad -(38)$$

and

$$\Delta\phi = 2\pi - (2\pi - \omega_1 t)$$
$$= \omega_1 t \quad -(39)$$

For one orbit:

$$t = T \quad -(40)$$

so

$$\boxed{\Delta\phi = \omega_1 T} \quad -(41)$$

This is the classical law of universal precession. There is only one experimentally observable precession, so by comparison of eqs. (1) and (41):

$$\Delta\phi_T = \frac{2\pi}{c^2}\left(v_N^2 + 3\omega_+^2 r^2\right) = \omega_1 T \quad -(42)$$

and by comparison of eqs. (3) and (41)

$$\Delta\phi_T = \frac{2\pi}{c^2}\left(v_N^2 - \omega_-^2 r^2\right) = \omega_1 T \quad -(43)$$

Eq. (42) applies to:

$$\phi' = \phi + \omega_1 t \quad -(44)$$

and Eq. (43) applies to:

$$\phi' = \phi - \omega_1 t \quad -(45)$$