09(5): Origin of the Thomas Precession in Spacetime Torsion

Consider the definition of linear velocity $\underline{v}$ in plane polar coordinates:

$$\underline{v} = \frac{d\underline{r}}{dt} = \frac{dr}{dt}\underline{e}_r + r\frac{d\underline{e}_r}{dt}$$

$$= \frac{dr}{dt}\underline{e}_r + r\dot{\phi}\,\underline{e}_\theta \quad -(1)$$

$$= \frac{dr}{dt}\underline{e}_r + \omega r\,\underline{e}_\theta$$

In the Cartesian frame the position vector is:

$$\underline{r} = X\underline{i} + Y\underline{j} + Z\underline{k} \quad -(2)$$

and

$$\underline{v} = \dot{X}\underline{i} + \dot{Y}\underline{j} + \dot{Z}\underline{k} \quad -(3)$$

The plane polar coordinates are defined by a rotating frame of reference such that:

$$\frac{d\underline{e}_r}{dt} = \dot{\phi}\,\underline{e}_\theta = \omega\underline{e}_\theta \quad -(4)$$

where

$$\omega = \frac{d\phi}{dt} \quad -(5)$$

is the magnitude of the angular velocity. The latter originates in rotating a frame of reference.

From eq. (1):

$$v^2 = \dot{r}^2 + \omega^2 r^2 \quad -(6)$$

in plane polar coordinates. In Cartesian coordinates the axes are fixed, so in the plane (X, Y):

$$v^2 = \dot{X}^2 + \dot{Y}^2 \quad -(7)$$

It follows that:

$$\dot{X}^2 + \dot{Y}^2 = \dot{r}^2 + \omega^2 r^2 \quad -(8)$$

In plane polar coordinates:

$$\underline{v} = \underline{v}_r + \underline{v}_\theta \quad -(9)$$

where

$$\underline{v}_r = \dot{r}\,\underline{e}_r \quad -(10)$$

$$\underline{v}_\theta = \omega r\,\underline{e}_\theta \quad -(11)$$

The unit vectors of the plane polar system are:

$$\underline{e}_r = \underline{i}\cos\phi + \underline{j}\sin\phi \quad -(12)$$

$$\underline{e}_\theta = -\underline{i}\sin\phi + \underline{j}\cos\phi \quad -(13)$$

so

$$\underline{e}_\theta = \underline{k} \times \underline{e}_r. \quad -(14)$$

Defining:

$$\underline{\omega} = \omega\,\underline{k} \quad -(15)$$

and

$$\underline{r} = r\,\underline{e}_r \quad -(16)$$

it follows that:

$$\underline{v}_\theta = \underline{\omega} \times \underline{r} \quad -(17)$$

and

$$\underline{v} = \frac{dr}{dt}\underline{e}_r + \underline{\omega} \times \underline{r} \quad -(18)$$

In the Cartesian system:

$$\underline{v} = \dot{X}\,\underline{i} + \dot{Y}\,\underline{j} \quad -(19)$$

so

$$\dot{X}\,\underline{i} + \dot{Y}\,\underline{j} = \frac{dr}{dt}\underline{e}_r + \underline{\omega} \times \underline{r} \quad -(20)$$

The acceleration in the plane polar system is

$$\underline{a} = \frac{d\underline{v}}{dt} = \frac{d}{dt}\left(\dot{r}\,\underline{e}_r + r\dot{\phi}\right)\underline{e}_\theta \quad -(21)$$

$$= \frac{d\underline{v}}{dt} + \underline{\omega} \times \underline{v}$$

$$= \frac{d}{dt}\left(\frac{dr}{dt}\underline{e}_r + \underline{\omega}\times\underline{r}\right) + \underline{\omega}\times(\underline{\omega}\times\underline{r})$$

$$+ \underline{\omega}\times\left(\frac{dr}{dt}\underline{e}_r\right) \quad -(22)$$

This can be expressed as:

$$\underline{g} = (\ddot{r} - r\dot{\phi}^2)\underline{e}_r + (r\ddot{\phi} + 2\dot{r}\dot{\phi})\underline{e}_\phi \quad -(23)$$

In Cartesian coordinates:

$$\underline{g} = \ddot{X}\,\underline{i} + \ddot{Y}\,\underline{j} \quad -(24)$$

In ECE2 theory, the acceleration due to gravity originates in spacetime torsion, and is:

$$\underline{g} = -\underline{\nabla}\,\underline{\Phi} + \underline{\Omega}\,\underline{\Phi} \quad -(25)$$

where $\underline{\Phi}$ is the gravitational potential and $\underline{\Omega}$ is the spin connection. From eqs. (23) and (25):

$$\underline{g} = -\underline{\nabla}\,\underline{\Phi} + \underline{\Omega}\,\underline{\Phi}$$

$$= (\ddot{r} - r\omega^2)\underline{e}_r + (r\dot{\omega} + 2\dot{r}\omega)\underline{e}_\phi \quad -(26)$$

so it is clear that ω originates in spacetime torsion, and inter alia, spacetime torsion gives rise to ω.

It is well known in classical dynamics that:

$$\left(\frac{d\underline{Q}}{dt}\right)_{Cartesian} = \left(\frac{d\underline{Q}}{dt} + \underline{\omega}\times\underline{Q}\right)_{plane\ polar} \quad -(27)$$

where $\underline{Q}$ is any vector, (Marion and Thornton,

) chapter 9). Therefore eq. (27) can be seen to be a general result of spacetime torsion, which generates the angular velocity vector $\underline{\omega}$. For example the torque is

$$\underline{Tq} = \left(\frac{d\underline{L}}{dt}\right)_{\text{Cartesian}} = \left(\frac{d\underline{L}}{dt} + \underline{\omega} \times \underline{L}\right)_{\text{plane polar}} \quad -(28)$$

and is the root cause of gyroscope precession.

Consider now the infinitesimal line element of ECE2 covariant unified field theory:

$$ds^2 = c^2 d\tau^2 = (c^2 - v^2)\, dt^2 \quad -(29)$$

where

$$v^2 dt^2 = dr^2 + r^2 d\phi^2 \quad -(30)$$

It follows that:

$$v^2 = \left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\phi}{dt}\right)^2 \quad -(31)$$

$$= \left(\frac{dr}{dt}\right)^2 + \omega^2 r^2 \quad -(32)$$

which is eq. (6), Q.E.D.

Note carefully that the above results are obtained by spinning the coordinate system to produce relations between unit vectors such as eq. (4).

From the definition (5):

$$d\phi = \omega\, dt \quad -(33)$$

and from eq. (11)

$$|\underline{v}_\theta| = v_\theta = \omega r \quad -(34)$$

The Thomas precession originates in:

$$\omega \rightarrow \omega + \omega_T \quad -(35)$$

where

$$\omega_T = \frac{d\phi_T}{dt} \quad -(36)$$

Eq. (36) produces the Thomas velocity:

$$v_T = \omega_T r \quad -(37)$$

i.e.

$$\underline{v}_T = r\omega_T \underline{e}_\theta \quad -(38)$$

and the Thomas acceleration:

$$\underline{g}_T = -r\omega_T^2 \underline{e}_r + (r\dot{\omega}_T + 2\dot{r}\omega_T)\underline{e}_\phi \quad -(39)$$

so it is clear that ω_T originates in spacetime torsion because $\underline{g}_T$ originates in spacetime torsion, Q.E.D.

If it is assumed that:

$$\dot{\omega}_T = 0, \quad \dot{r} = 0 \quad -(40)$$

then:

$$\underline{g}_T = -r\omega_T^2 \underline{e}_r \quad -(41)$$

In general, eq. (40) need not be assumed. The effect of the Thomas rotation is to produce:

$$ds^2 = c^2 dt^2 - (dr^2 + r^2 d\phi'^2) \quad -(41)$$

where

$$d\phi'^2 = d\phi^2 + \omega_T^2 dt^2 \quad -(42)$$

and to produce the Thomas precession:

$$\Delta\phi_T = 2\pi\left(\left(1 - \frac{v_T^2}{c^2}\right)^{-1/2} - 1\right) \quad -(43)$$

It is proposed that all precessions in cosmology

b) are due to the Thomas velocity v_T, which is fitted to the experimental result. It is proposed that the velocity v_T be named the ECE2 velocity in a generally covariant unified field theory in a space with finite torsion and curvature.

The original theory by Thomas was developed in a Minkowski space and no consideration was given to torsion and curvature.

In the special case:

$$v_T = v_N \quad -(44)$$

where v_N is the Newtonian orbital velocity, eq. (43) gives the Lorentz factor:

$$\gamma = \left(1 - \frac{v_N^2}{c^2}\right)^{-1/2} \quad -(45)$$

simply by rotating the frame of reference. It is well known that the same Lorentz factor is obtained from the Lorentz boost.

Finally, the acceleration (41) can be expressed in terms of an additional spin connection Ω_T:

$$\underline{g}_T = -\underline{\nabla}\Phi + \underline{\Omega}_T \Phi \quad -(46)$$

$$= -r\omega_T^2 \, \underline{e}_r$$

so the Thomas precession is understood in terms of the spin connection.