408(6): Occurrences of the Thomas Half in Fundamental Theory

Adopt the notation:

$$\beta = \frac{\Delta\phi_T}{2\pi} = \gamma - 1, \quad -(1)$$

it is seen that the Thomas half is ubiquitous because:

$$\boxed{\gamma = 1 + \beta} \quad -(2)$$

and

$$\beta = \left(1 - \frac{v_N^2}{c^2}\right)^{-1/2} - 1 \quad -(3)$$

where v_N is the Newtonian velocity. The Thomas half is defined by:

$$\beta_0 = \left(1 - \frac{v_N^2}{c^2}\right)^{-1/2} - 1, \quad v \ll c \quad -(4)$$

$$= \frac{1}{2}\frac{v_N^2}{c^2}$$

The Lorentz factor γ is defined by the ECE2 covariant metric:

$$ds^2 = c^2 d\tau^2 = (c^2 - v_N^2)\, dt^2 \quad -(5)$$

so

$$\gamma = \frac{dt}{d\tau} = \left(1 - \frac{v_N^2}{c^2}\right)^{-1/2} \quad -(6)$$

and β is defined by the rotation of the metric (5) to give the Thomas precession:

$$\Delta\phi_T = 2\pi\beta \quad -(7)$$

It follows that all the results of ECE2 covariant general relativity can be described in terms of β. For example:

1) Relativistic velocity $\underline{v} = (1+\beta)\,\underline{v}_N$, $\quad -(8)$

2) Relativistic total energy $E = (1+\beta)\,mc^2 \quad -(9)$

3) Relativistic kinetic energy $T = \beta mc^2 \quad -(10)$

4) Relativistic hamiltonian $H = (1+\beta)\,mc^2 + U \quad -(11)$

5) Relativistic lagrangian $\mathcal{L} = -\dfrac{mc^2}{1+\beta} - U \quad -(12)$

6) Hamiltonian for Dirac equation:

$$H_0 = H - mc^2 = \frac{p^2}{m(\beta+2)} + U \quad -(13)$$

7) Einstein energy equation:

$$E = H - U = (1+\beta)mc^2 = (c^2p^2 + m^2c^4)^{1/2} \quad -(14)$$

so

$$E = \frac{p^2c^2}{E+mc^2} + mc^2$$

$$= \frac{m^2(1+\beta)^2v^2c^2}{E+mc^2} + mc^2 \quad -(15)$$

$$= \left(\frac{(1+\beta)^2}{2+\beta}\right)mv^2 + mc^2$$

So the relativistic kinetic energy is:

$$T = E - mc^2 = \left(\frac{(1+\beta)^2}{2+\beta}\right)mv^2 \quad -(16)$$

$$\xrightarrow[v<<c]{} \frac{1}{2}mv^2$$

8) In the Dirac approximation:

$$T = E - mc^2 = \frac{p^2}{m(2+\beta)} \rightarrow \frac{p_N^2}{2m}\left(1 - \frac{U}{2mc^2}\right)^{-1} \quad -(17)$$

where

$$p^2 = (1+\beta)^2 p_N^2 \quad -(18)$$

so

$$\frac{(1+\beta)^2}{2+\beta} = \frac{1}{2}\left(1 - \frac{U}{2mc^2}\right)^{-1} \quad -(19)$$

$$= \frac{1}{2} + \frac{U}{4mc^2}$$

so β can be found in terms of U. In the H atom:

$$U = -\frac{e^2}{4\pi\epsilon_0 r} \quad -(20)$$

The Thomas half is denoted β_0 and is defined by:

$$\beta_0 = \frac{1}{2}\frac{v^2}{c^2} = \underset{v \ll c}{Lt}\ \beta = \underset{v \ll c}{Lt}\ \gamma - 1 \quad -(21)$$

In the H atom, as shown in UFT407:

$$\beta_0 = \frac{1}{2}\frac{\alpha^2}{n^2} \quad -(22)$$

where α is the fine structure constant and n the principal quantum number. The energy levels of the H atom are given by:

$$E(H) = -\beta_0 mc^2 \quad -(23)$$

i.e.

$$E = \langle H_0 \rangle = \left\langle \frac{p^2}{2m} \right\rangle + \langle U \rangle \quad -(24)$$

where

$$\left\langle \frac{p^2}{2m} \right\rangle = \beta_0 mc^2 \quad -(25)$$

$$\langle U \rangle = -2\beta_0 mc^2 \quad -(26)$$

in which

$$\beta_0 = \frac{1}{2}\frac{v^2}{c^2} = \frac{1}{2}\frac{\alpha^2}{n^2} \quad -(27)$$

From eq. (25) the classical kinetic energy is:

$$T = \left\langle \frac{p^2}{2m} \right\rangle = \beta_0 mc^2 \quad -(28)$$

where in general:

$$\beta_0 = \frac{1}{2}\frac{v^2}{c^2} \quad -(29)$$

The relativistic kinetic energy is given by Eq. (16):

$$T = E - mc^2 = \left(\frac{(1+\beta)^2}{2+\beta}\right) mv^2 \xrightarrow[\beta \to 0]{} \frac{1}{2}mv^2 \quad -(30)$$

The Dirac atom is therefore given by Eq. (17):

$$T = \frac{p_N^2}{2m}\left(1 - \frac{U}{2mc^2}\right)^{-1} \quad -(31)$$

$$= \beta_0 mc^2 \left(1 - \frac{U}{2mc^2}\right)^{-1}$$

$$\doteq \beta_0 mc^2 \left(1 + \frac{U}{2mc^2}\right)$$

In the SU(2) basis:

$$T = \frac{1}{2m} \underline{\sigma} \cdot \underline{p}_N \left(1 + \frac{U}{2mc^2}\right) \underline{\sigma} \cdot \underline{p}_N \quad -(32)$$

The Thomas half is defined by:

$$\beta_0 = \frac{1}{2}\frac{v^2}{c^2} = \frac{1}{2}\frac{p_N^2}{m^2c^2} \quad -(33)$$

Quantization of Eq. (32) takes place with:

$$\underline{p}_N \phi = -i\hbar \underline{\nabla} \phi \quad -(34)$$

and

$$p_N^2 \phi = -\hbar^2 \nabla^2 \phi \quad -(35).$$

In these equations:

$$\beta_0 = \frac{1}{2}\left(\frac{p_N}{mc}\right)^2 \quad -(36)$$

so

$$\beta_0^{1/2} = \frac{1}{\sqrt{2}}\frac{p_N}{mc}. \quad -(37)$$

Now express:

$$\alpha_0 := \beta_0^{1/2} = \frac{1}{\sqrt{2}mc} p_N \quad -(38)$$

so:

$$\underline{\alpha}_0 = \frac{1}{\sqrt{2}mc} \underline{p}_N \quad -(39)$$

and

$$|\underline{\alpha}_0| = (\alpha_0^2)^{1/2} = \alpha_0 = \beta_0^{1/2} \quad -(40)$$

Therefore the Dirac action (32) can be expressed as:

$$\boxed{T = mc^2 \, \underline{\sigma} \cdot \underline{\alpha}_0 \left(1 + \frac{U}{2mc^2}\right) \underline{\sigma} \cdot \underline{\alpha}_0 \quad -(41)}$$

5) Quantization of the Thomas field is defined by

$$\underline{p}_N \phi = \sqrt{2}\, mc\, \underline{\alpha}_0 \phi = -i\hbar \underline{\nabla} \phi \quad -(42)$$

i.e

$$\boxed{\underline{\alpha}_0 \phi = -\frac{i}{\sqrt{2}} \frac{\hbar \nabla \phi}{mc}} \quad -(43)$$

Therefore the Thomas field:

$$\beta_0 = \alpha_0^2 \quad -(44)$$

is the basis of Schroedinger quantization. In eq. (43):

$$\frac{\hbar}{mc} = \frac{1}{2\pi} \frac{h}{mc} = \frac{\lambda_c}{2\pi} \quad -(45)$$

where λ_c is the Compton wavelength.

Squaring eq. (43) gives:

$$p_N^2 \phi = 2m^2c^2 \alpha_0^2 \phi = -\hbar^2 \nabla^2 \phi \quad -(46)$$

so

$$\frac{p_N^2}{2m} \phi = mc^2 \alpha_0^2 \phi = -\frac{\hbar^2 \nabla^2}{2m} \phi \quad -(47)$$

and

$$\alpha_0^2 \phi = \beta_0 \phi = -\frac{\hbar^2 \nabla^2}{2m^2c^2} \phi \quad -(48)$$

i.e

$$\boxed{\beta_0 \phi = -\frac{1}{2} \left(\frac{\hbar}{mc} \right)^2 \nabla^2 \phi} \quad -(49)$$

So the Thomas field is the foundational cause of energy quantization, eq. (49), and momentum quantization, eq. (43).

Energy Quantization

$$T\phi = mc^2 \beta_0 \phi = -\frac{\hbar^2 \nabla^2}{2m} \phi \quad -(50)$$

i.e.

$$\beta_0 \phi = -\frac{1}{2}\left(\frac{\hbar}{mc}\right)^2 \nabla^2 \phi \quad -(51)$$

Momentum Quantization

$$\underline{p}_N \phi = \sqrt{2}\, mc\, \underline{\alpha_0}\, \phi = -i\hbar \underline{\nabla} \phi \quad -(52)$$

i.e.

$$\underline{\alpha_0} \phi = -\frac{i}{\sqrt{2}}\left(\frac{\hbar}{mc}\right) \underline{\nabla} \phi \quad -(53)$$

where

$$|\underline{\alpha_0}| = \alpha_0 = \beta_0^{1/2} \quad -(54)$$

The Thomas precession is also responsible for the Einstein / de Broglie equations:

$$E = \gamma mc^2 = (1+\beta) mc^2 = \hbar\omega \quad -(55)$$

and

$$\underline{p} = \gamma m \underline{v} = (1+\beta) m \underline{v} = \hbar \underline{\kappa} \quad -(56)$$

Therefore β is responsible for Planck / Einstein quantization, the de Broglie wave particle dualism, and for quantum and relativistic quantum mechanics

Conclusion Essentially all of physics is based on the Thomas half. The EGR is refuted by the rest of physics, because as shown in UFT408(5) Thomas precession and the Thomas half in the presence of EGR incorrectly change the Schwarzschild metric produce gravitation. For example, the change:

$$\beta_0 = \frac{1}{2}\frac{v^2}{c^2} \rightarrow \frac{1}{2}\frac{1}{c^2}\left(v^2 + \frac{2MG}{r}\right) \quad -(57)$$

by de Sitter precession. The change (57) is never observed in classical and relativistic physics, or quantum