407(3): A Simple Refutation of de Sitter Precession

In Note 407(1) it was shown that the energy levels of the hydrogen atom are due to Thomas precession:

$$E_n = \hbar\omega_n = \frac{1}{2}\frac{\alpha^2}{n^2}mc^2 \quad -(1)$$

where

$$\frac{1}{2}\frac{\alpha^2}{n^2} = \frac{1}{2}\frac{v^2}{c^2} = \frac{1}{2\pi}\lim_{v \ll c} 2\pi\left(\left(1-\frac{v^2}{c^2}\right)^{-1/2}-1\right) \quad -(2)$$

the low velocity limit of the Thomas precession:

$$\Delta\phi_T = 2\pi\left(\left(1-\frac{v^2}{c^2}\right)^{-1/2}-1\right) = 2\pi(\gamma-1) \quad -(3)$$

Eq. (3) is obtained by rotating the infinitesimal line element of ECE2 theory.

The geodetic de Sitter precession is:

$$\Delta\phi_g = 2\pi\left(\left(1-\frac{v_1^2}{c^2}\right)^{-1/2}-1\right) \quad -(4)$$

where:

$$v_1^2 = v^2 + \frac{2MG}{r} \quad -(5)$$

If the H atom is located on the surface of the earth, then:

$$M = 5.98 \times 10^{24}\ \text{kg} \quad -(6)$$

$$r = 6.378 \times 10^{6}\ \text{km} \quad -(7)$$

$$G = 6.67 \times 10^{-11}\ \text{m}^3\,\text{kg}^{-1}\,\text{s}^{-2} \quad -(8)$$

Here r is the radius of the Earth. The result (4) is obtained from rotating the Schwarzschild metric. It is clear that eq. (5) must be incorrect, because it is based on a geometry without torsion.

In the low velocity limit (eq. (2)), eq. (5) gives for the energy levels of the H atom (see 4FT266, 329 ff):

$$E_n = \hbar\omega_n = -\frac{\hbar^2}{2m}\int \psi^* \nabla^2 \psi \, d\tau + \int \psi^* U \psi \, d\tau \quad -(9)$$

where:

$$U = U_c + U_{grav} \quad -(10)$$

where:

$$U_c = -\frac{e^2}{4\pi\epsilon_0 r} \quad -(11)$$

and

$$U_{grav} = \frac{2mMG}{r} \quad -(12)$$

So

$$E_n = \hbar\omega_n = \left\langle \frac{p^2}{2m} \right\rangle + \langle U_c \rangle + \langle U_{grav.} \rangle \quad -(13)$$

As in 4FT266, 4FT329 ff and Note 407(1).

$$\left\langle \frac{p^2}{2m} \right\rangle = \frac{1}{2} m \langle v^2 \rangle = \frac{1}{2}\left(\frac{\alpha}{n}\right)^2 mc^2 \quad -(14)$$

$$\langle U_c \rangle = -\left(\frac{\alpha}{n}\right)^2 mc^2 \quad -(15)$$

So

$$E_n = \hbar\omega_n = -\frac{1}{2}\left(\frac{\alpha}{n}\right)^2 mc^2 + \langle U_{grav} \rangle \quad -(16)$$

From eq. (14), the Thomas half is:

$$\frac{1}{2}\frac{v^2}{c^2} = \frac{1}{2}\frac{\alpha^2}{n^2} \quad -(17)$$

Now note that:

$$mMG = \alpha\hbar c \left(\frac{mMG}{\alpha\hbar c}\right) = x\alpha\hbar c \quad -(18)$$

where

$$x = \frac{mMG}{\alpha\hbar c} \quad -(19)$$

3) It follows that:

$$\left\langle \frac{mMG}{r} \right\rangle = x \left\langle \frac{\alpha \hbar c}{r} \right\rangle \quad -(20)$$

$$= x \left(\frac{\alpha}{n}\right)^2 mc^2$$

The energy levels of the H atom from de Sitter precession are therefore:

$$E_n = \frac{1}{2}\left(\frac{\alpha}{n}\right)^2 mc^2 - \left(\frac{\alpha}{n}\right)^2 mc^2 + x\left(\frac{\alpha}{n}\right)^2 mc^2$$

$$= \left(x - \frac{1}{2}\right)\left(\frac{\alpha}{n}\right)^2 mc^2 \quad -(21)$$

The energy levels of the H atom, from Thomas (i.e. ECE2) precession are:

$$E_n = -\frac{1}{2}\left(\frac{\alpha}{n}\right)^2 mc^2 \quad -(22)$$

The values in eq. (22) are well observed experimentally. The factor x is given by eq. (19). For an H atom on the earth's surface:

$$\left.\begin{aligned} m &= 9.10953 \times 10^{-31}\ \text{kg} \\ M &= 5.98 \times 10^{24}\ \text{kg} \\ G &= 6.67 \times 10^{-11}\ \text{m}^3\ \text{kg}^{-1}\ \text{s}^{2} \\ \alpha &= 0.0073 \\ \hbar &= 1.054 \times 10^{-34}\ \text{Js} \\ c &= 2.998 \times 10^{8}\ \text{ms}^{-1} \end{aligned}\right\} \quad -(23)$$

so

$$x = 1.5752 \times 10^{10} \quad -(24)$$

Clearly, such a large effect of the earth on the H atom is never observed, so the de Sitter precession is refuted, Q.E.D.