407(1): Thomas Precession of Planetary Orbits and H Atom Orbitals

As shown in UFT110 and in immediately preceding papers the Thomas precession in radians per revolution of 2π is:

$$\Delta \phi_T = 2\pi \left(\left(1 - \frac{v^2}{c^2} \right)^{-1/2} - 1 \right) \quad -(1)$$

$$\sim \pi \left(\frac{v}{c} \right)^2 \left(1 + \frac{3}{4} \left(\frac{v}{c} \right)^2 + \frac{35}{64} \left(\frac{v}{c} \right)^6 + \frac{5}{8} \left(\frac{v}{c} \right)^4 + \dots \right)$$

where v is the magnitude of the linear orbital velocity.

Eq. (1) is obtained by rotating the ECE2 infinitesimal line element:

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - dr^2 - r^2 d\phi^2 \quad -(2)$$

in plane polar coordinates r and ϕ. The rotation is defined by

$$\phi' = \phi + \omega_0 t \quad -(3)$$

where ω_0 is the angular velocity of the rotation. The rotating line element is:

$$ds'^2 = \left(1 - \frac{v^2}{c^2} \right) \left(c^2 dt^2 - 2r^2 \Omega \, d\phi \, dt \right) - dr^2 - r^2 d\phi^2 \quad -(4)$$

in which the ECE2 covariant angular velocity is:

$$\Omega = \omega_0 \left(1 - \frac{v^2}{c^2} \right)^{-1} \quad -(5)$$

The same rotation produces:

$$dt'^2 = \left(1 - \frac{v^2}{c^2} \right) dt^2 \quad -(6)$$

so

$$\Delta \phi_T = \Omega \, dt' - \omega \, dt \quad -(7)$$

$$= \omega \, dt \left(\left(1 - \frac{v^2}{c^2} \right)^{-1/2} - 1 \right)$$

For a rotation of 2π, Eq. (1) is obtained.

) For a nearly circular orbit, v is the orbital linear speed
to a good approximation and for:

$$v << c \quad - (8)$$

the Thomas precession is:

$$\Delta\phi \sim \pi \left(\frac{v}{c}\right)^2 \quad - (9)$$

which is the first term in the binomial expansion (1).
The precession rate in radians per radian is:

$$\frac{\Delta\phi_T}{2\pi} = \frac{1}{2}\left(\frac{v}{c}\right)^2 \quad - (10)$$

This means that for each radian of rotation, there is an additional advance defined by eq. (10). The factor 1/2 in eq. (10) is the origin of the Thomas factor. This factor was later given by the Dirac equation, and later by the fermion equation. Eq. (10) has been derived for constant travel around a circle. The Thomas factor is observed for spin-orbit interaction in spectra. In planetary motion a mass m orbits a mass M. In the Newtonian approximation

$$v^2 = MG\left(\frac{2}{r} - \frac{1}{a}\right) \quad - (11)$$

and in a nearly circular orbit:

$$a \sim r \quad - (12)$$

where a is the semi major axis of an ellipse, so:

$$v^2 \rightarrow \frac{MG}{r} \quad - (13)$$

The Thomas precession of the planet is therefore:

$$\boxed{\Delta\phi_T \sim \pi\left(\frac{v}{c}\right)^2 = \frac{\pi}{r}\left(\frac{MG}{c^2}\right)} \quad - (14)$$

The Thomas precession frequency in radians per second

) is

$$\omega_T = \frac{1}{2}\frac{v^2}{c^2}\omega_0 \quad -(15)$$

where ω_0 is a fundamental frequency of the atom or molecule such as the rest frequency:

$$\omega_0 = \frac{mc^2}{\hbar} \quad -(16)$$

It would seem that a second order relativistic correction of $\frac{1}{2}(v^2/c^2)$ would not have a very large effect, but Llewellyn Thomas in 1925 showed that it can have an effect of changing the spin orbit hamiltonian by a factor of two.

The classical development of spin orbit interaction considers an electron travelling with velocity $\underline{v}$ in an electric field strength $\underline{E}$ in volts per metre. It produces the magnetic flux density:

$$\underline{B} = \frac{1}{c^2}\underline{E}\times\underline{v} \quad -(17)$$

where

$$\underline{E} = -\underline{\nabla}\phi = -\frac{d\phi}{dr}\underline{e}_r \quad -(18)$$

and

$$\underline{e}_r = \frac{\underline{r}}{r} \quad -(19)$$

So:

$$\underline{B} = -\frac{d\phi}{dr}\frac{1}{c^2 r}\underline{r}\times\underline{v} \quad -(20)$$

$$= -\frac{1}{mc^2 r}\frac{d\phi}{dr}\underline{L}$$

here $\underline{L}$ is the orbital angular momentum:

$$\underline{L} = m\,\underline{r}\times\underline{v} \quad -(21)$$

So:

$$\underline{B} = -\frac{1}{mc^2 r}\frac{d\phi}{dr}\underline{L} \quad -(22)$$

The energy of interaction of a magnetic dipole

) moment $\underline{m}_s$ with $\underline{B}$ is:

$$E_n = - \underline{m_s} \cdot \underline{B} \quad -(23)$$

and the scalar potential energy is:

$$\phi = - \frac{e^2}{4\pi\epsilon_0 r} \quad -(24)$$

so

$$\frac{\partial \phi}{\partial r} = \frac{e^2}{4\pi\epsilon_0 r^2} \quad -(25)$$

Therefore: $$H = \frac{e^2}{4\pi\epsilon_0 m c^2 r^3} \underline{m_s} \cdot \underline{L} \quad -(26)$$

and is the classical spin-orbit hamiltonian. The spin magnetic dipole moment has been assumed to be classical. However, the Dirac equation gives:

$$H = \frac{e^2}{8\pi\epsilon_0 m c^2 r^3} \underline{m_s} \cdot \underline{L} \quad -(27)$$

which is the Thomas factor of 1/2 multiplied by the classical result.

Eq. (27) is observed with great accuracy and is one of the reasons for accepting ECE2 covariance.

The effective energy of interaction has been reduced by half, to

$$H = - \frac{1}{2} \underline{m_s} \cdot \underline{B} \, . \quad -(28)$$

To show how this can happen in the spectra of atoms and molecules, consider as in UFT 329 ff. the energy levels of the hydrogen atom. Its classical hamiltonian is:

$$H = \frac{p^2}{2m} + U \quad -(29)$$

and its energy levels are:

$$E_n = \left\langle \frac{p^2}{2m} \right\rangle + \langle u \rangle$$

$$= -\frac{me^4}{32\pi^2\epsilon_0^2\hbar^2 n^2} = -\frac{1}{2}\frac{mc^2\alpha^2}{n^2} \quad -(30)$$

Here n is the principal quantum number and α is the fine structure constant:

$$\alpha = \frac{e^2}{4\pi\hbar c\epsilon_0} \quad -(31)$$

The individual expectation values are:

$$\left\langle \frac{p^2}{2m} \right\rangle = \frac{1}{2}m\langle v^2\rangle = \frac{1}{2}\frac{m\alpha^2c^2}{n^2} \quad -(32)$$

and

$$\langle u \rangle = -\frac{m\alpha^2c^2}{n^2} \quad -(33)$$

The frequency corresponding to the energy (30) is:

$$\omega_n = \frac{|E_n|}{\hbar} = \frac{1}{2}\frac{\alpha^2}{n^2}\frac{mc^2}{\hbar} \quad -(34)$$

so

$$\boxed{\omega_n = \frac{1}{2}\frac{\alpha^2}{n^2}\omega_0} \quad -(35)$$

where

$$\omega_0 = \frac{mc^2}{\hbar} \quad -(36)$$

the rest frequency.

From eq. (32):

$$\frac{1}{2}\frac{v^2}{c^2} = \frac{1}{2}\frac{\alpha^2}{n^2} \quad -(37) \quad ; \boxed{\frac{v}{c} = \frac{\alpha}{n}} ;$$

so

$$\boxed{\omega_T = \omega_n = \frac{1}{2}\frac{v^2}{c^2}\omega_0} \quad -(38)$$

So the frequencies corresponding to the energy levels of the H atom are Thomas frequencies. The precession in each orbital in terms of radians per radian is:

$$\Delta\phi_H\,(\text{radian per radian}) = \frac{1}{2}\frac{v^2}{c^2} = \frac{1}{2}\frac{\alpha^2}{n^2} \quad -(39)$$

The Thomas precession in radians in each orbital of the H atom is:

$$\Delta\phi_H\,(\text{radians}) = \pi\frac{v^2}{c^2} = \pi\frac{\alpha^2}{n^2} \quad -(40)$$

For $n = 1$:

$$\Delta\phi_H(n=1) = \pi\alpha^2 \text{ radians} \quad -(41)$$

$$= 1.67 \times 10^{-4} \text{ radians}$$

This is much larger than in a planetary orbit. The above treatment is treated as non-relativistic, but nevertheless, the result (38) can be viewed as containing a relativistic facet. This is a remarkable property of the H atom as developed by the Schrödinger equation. For:

$$n = 1 \quad -(42)$$

eq. (40) gives

$$\frac{v}{c} = \alpha = 0.0073 \quad -(43)$$

The correct way of developing the H atom is to use the QED Schrödinger equation with radiative corrections. The above treatment shows that the Thomas precession in radians per radian

$$\Delta\phi_T = \frac{1}{2}\frac{v^2}{c^2} \quad -(44)$$

gives the energy levels of the H atom when multiplied

7) by the rest frequency of the electron:

$$\omega_0 = \frac{mc^2}{\hbar} \quad -(45)$$

Without the Thomas precession there would be no H atom.

It is therefore a very fundamental phenomenon of physics, and it follows that the Thomas precession of planets is also very fundamental and is given by:

$$\Delta\phi_T = \frac{\pi}{r}\left(\frac{mG}{c^2}\right) \quad -(46)$$

The vacuum fluctuations of the Thomas precession can be calculated from:

$$\Delta\phi_T = \frac{2}{3}\frac{\langle \delta \underline{r} \cdot \delta \underline{r} \rangle}{r^2} = \frac{\pi}{r}\left(\frac{mG}{c^2}\right) \quad -(47)$$

The Thomas precession (46) is a third of the value of the standard model geodetic precession:

$$\Delta\phi_g = \frac{3\pi}{r}\left(\frac{mG}{c^2}\right) \quad -(48)$$

The standard model gives the result:

$$\Delta\phi = \Delta\phi_E + \Delta\phi_g + \Delta\phi_{LT} \quad -(49)$$

where

$$\Delta\phi_E = \frac{6\pi mG}{c^2 a(1-\epsilon^2)} \quad -(50)$$

is the Einstein precession due to the fact that the force law of EGR is no longer inverse square. The standard model claim that:

$$\Delta\phi \stackrel{?}{=} \Delta\phi_E \quad -(51)$$

is obviously wrong.

8) In view of the fact that EGR has been effectively abandoned by the ECE2 school of thought, the only correct theoretical calculation is that of the Thomas precession (46). This is part of the experimentally observed precession, and that is all that can be claimed

Conclusions

1) The standard model geodetic precession $\Delta\phi_g$ is three times the Thomas precession:

$$\Delta\phi_g = 3\Delta\phi_T \quad - (52)$$

This is a purely numerical result because $\Delta\phi_g$ is calculated incorrectly, by rotating the incorrect Schwarzschild metric. On the other hand $\Delta\phi_T$ is calculated without the use of the EGR.

2) The Thomas precession defines the energy levels of the H atom, a hitherto unknown and remarkable result.

3) The standard model itself gives the result (49), which is obviously not the incorrect claim (51), endlessly repeated in the dogmatic literature