407(2): Thomas Precession from the Commutator of Lorentz Boosts

In UFT406 it was shown that the Einstein theory has been refuted entirely because of its neglect of geodetic and Lense Thirring precessions when considering planetary orbits. This leaves the Thomas precession as the only correct theory of planetary precession. In ECE2 theory it can be derived from the commutator of ECE2 covariant boosts. These have the same mathematical structure as the well known Lorentz boosts, but are defined in a space with finite curvature and torsion.

Boost Matrix in the X Direction

This is defined by:

$$B_X = \begin{bmatrix} \cosh\phi & \sinh\phi & 0 & 0 \\ \sinh\phi & \cosh\phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \gamma & \gamma\beta & 0 & 0 \\ \gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad -(1)$$

where

$$\beta = \frac{v}{c}, \quad \gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad -(2)$$

This connects two inertial frames, moving with relative speed v. If the relative motion is along the X axis, then:

$$\left.\begin{aligned} X' &= \gamma(X + vt) \\ Y' &= Y \\ Z' &= Z \\ t' &= \gamma(t + vX/c^2) \end{aligned}\right\} \quad -(3)$$

The boost generator along X is:

$$K_X = \frac{1}{i}\frac{\partial B_X}{\partial \phi}\Big|_{\phi=0} = -i\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad -(4)$$

Boost Matrix in the Y Direction

In this case the relative motion is along Y, so

$$\left.\begin{aligned} X' &= X \\ Y' &= \gamma(Y + vt) \\ Z' &= Z \\ t' &= \gamma(t + vY/c^2) \end{aligned}\right] - (5)$$

and

$$B_Y = \begin{bmatrix} \gamma & 0 & \gamma\beta & 0 \\ 0 & 1 & 0 & 0 \\ \gamma\beta & 0 & \gamma & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cosh\phi & 0 & \sinh\phi & 0 \\ 0 & 1 & 0 & 0 \\ \sinh\phi & 0 & \cosh\phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} - (6)$$

The boost generator is:

$$K_Y = \frac{1}{i}\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} - (7)$$

<u>Boost Matrix</u> <u>in the</u> Z <u>Direction</u>

In this case:

$$\begin{aligned} X' &= X \\ Y' &= Y \\ Z' &= \gamma(Z + vt) \\ t' &= \gamma(t + vZ/c^2) \end{aligned} \quad -(8)$$

The boost matrix is:

$$B_Z = \begin{bmatrix} \gamma & 0 & 0 & \gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \gamma\beta & 0 & 0 & \gamma \end{bmatrix} = \begin{bmatrix} \cosh\phi & 0 & 0 & \sinh\phi \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \sinh\phi & 0 & 0 & \cosh\phi \end{bmatrix} - (9)$$

and the boost generator is

$$K_Z = \frac{1}{i}\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} - (10)$$

It is shown in "The Enigmatic Photon" that:

3)

$$[K_x, K_y] = -i J_z \quad -(11)$$

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where the rotation generators are:

$$J_x = \frac{1}{i}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}, \quad J_y = \frac{1}{i}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \quad J_z = \frac{1}{i}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad -(12)$$

So the commutator of ECE2 covariant boosts is the ECE2 covariant rotation generator. This is the fundamental origin of the Thomas precession.

Proof

Consider the commutator:

$$B_x B_y - B_y B_x = \begin{bmatrix} 0 & \gamma\beta - \gamma^2\beta & \gamma^2\beta - \gamma\beta & 0 \\ \gamma^2\beta - \gamma\beta & 0 & \gamma^2\beta^2 & 0 \\ \gamma\beta - \gamma^2\beta & -\gamma^2\beta^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad -(13)$$

$$= \gamma\beta \begin{bmatrix} 0 & -(\gamma-1) & \gamma-1 & 0 \\ \gamma-1 & 0 & \gamma\beta & 0 \\ -(\gamma-1) & -\gamma\beta & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

For a rotation of 2π, the Thomas precession is:

$$\Delta\phi_T = 2\pi(\gamma - 1) \quad -(14)$$

obtained from the rotation about Z of the ECE2 infinitesimal line element, as shown in previous notes and papers.

From eq. (14):

4)

$$2\pi\left(B_X B_Y - B_Y B_X\right) = \gamma\beta \begin{bmatrix} 0 & -\Delta\phi_T & \Delta\phi_T & 0 \\ \Delta\phi_T & 0 & 2\pi\gamma\beta & 0 \\ -\Delta\phi_T & -2\pi\gamma\beta & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad -(15)$$

$$= \gamma\beta\left(\Delta\phi_T\left(\begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\right) + 2\pi\gamma\beta\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\right)$$

This result shows that the commutator of boost matrices B_X and B_Y produces rotations about X, Y and Z. The commutator of boost generators however produces a rotation only about Z.

Finally use the results:

$$\gamma^2\beta^2 = \gamma^2 - 1 = (\gamma - 1)(\gamma + 1) \quad -(16)$$

and

$$\gamma\beta = (\gamma^2 - 1)^{1/2} \quad -(17)$$

From eq. (14):

$$\gamma = \frac{\Delta\phi_T}{2\pi} + 1 \quad -(18)$$

so

$$\gamma^2\beta^2 = \left(\frac{\Delta\phi_T}{2\pi} + 1\right)^2 - 1 \quad -(19)$$

$$\gamma\beta = \left(\left(\frac{\Delta\phi_T}{2\pi} + 1\right)^2 - 1\right)^{1/2} \quad -(20)$$

Therefore the commutator can be expressed entirely in terms of the Thomas precession. Q.E.D.

The result for Z axis rotation from eq. (15)

) is :

$$(B_x B_y - B_y B_x)_z = \gamma^2 \beta^2 \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad -(21)$$

$$= \left(\left(\frac{\Delta \phi_T}{2\pi} + 1 \right)^2 - 1 \right) \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

1) If there is no Thomas precession:

$$\Delta \phi_T = 0 \quad -(22)$$

then there is no rotation:

$$(B_x B_y - B_y B_x)_z = 0 \quad -(23)$$

2) In the low velocity limit:

$$\frac{\Delta \phi_T}{2\pi} \longrightarrow \frac{1}{2} \left(\frac{v}{c} \right)^2 \quad -(24)$$

So

$$\boxed{(B_x B_y - B_y B_x)_z \longrightarrow 2 \left(\frac{1}{2} \left(\frac{v}{c} \right)^2 \right) \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}} \quad -(25)$$

which is the fundamental origin of the Thomas factor 1/2.

In deriving eq. (24) we have used:

$$\left(\frac{\Delta \phi_T}{2\pi} + 1 \right)^2 = \left(\frac{\Delta \phi_T}{2\pi} \right)^2 + 2 \left(\frac{\Delta \phi_T}{2\pi} \right) + 1 \quad -(26)$$

$$\sim 2 \left(\frac{1}{2} \left(\frac{v}{c} \right)^2 \right) + 1$$