404(1): Summary of ECE2 Precession due to a Rotating Object

In the standard model this is known as Lense-Thirring precession but it is derived from the incorrect Einstein field equation. In ECE2 theory the derivation of the precession is based on the gravitational vector potential $\underline{a}$ total defined by:

$$\underline{\Omega} = \underline{\nabla} \times \underline{a}_{total} \quad -(1)$$

where $\underline{\Omega}$ is the gravitomagnetic field. The dipole potential is

$$\underline{a}_{total} = \frac{G}{c^2} \frac{\underline{m}_g \times \underline{r}}{r^3} \quad -(2)$$

where the gravitational magnetic dipole moment is:

$$\underline{m}_g = \frac{1}{2} \underline{L} \quad -(3)$$

where $\underline{L}$ is the angular momentum of a rotating mass such as the Earth. From eqs (1) and (2):

$$\underline{\Omega} = \frac{G}{c^2 r^3} \left(3 \underline{m}_g \cdot \frac{\underline{r}\,\underline{r}}{r^2} - \underline{m}_g \right) \quad -(4)$$

and the Lense Thirring precession is:

$$\Omega_{LT} = \frac{1}{2} | \underline{\Omega} | \quad -(5)$$

in units of s^{-1}. In this theory it is seen as a Larmor precession. The precession in radians is

$$\Delta \phi = \Omega_{LT}\, t \quad -(6)$$

where t is a time interval such as the time taken to complete an orbit.

The gravitational force due to a rotating object such as the earth is, in ECE2 theory:

$$\underline{g} = -\underline{\nabla}\phi - \frac{\partial \underline{Q}_{total}}{\partial t} \quad -(7)$$

$$= -\underline{\nabla}\phi + \underline{\omega}\phi = -\frac{\partial \underline{Q}}{\partial t} - \omega_0 \underline{Q}$$

The vector potential $\underline{Q}$ is defined by:

$$\underline{\Omega} = \underline{\nabla} \times \underline{Q} - \underline{\omega} \times \underline{Q} \quad -(8)$$

Therefore the spin connection is defined by:

$$\underline{\omega}\phi = -\frac{\partial \underline{Q}_{total}}{\partial t} \quad -(9)$$

$$= -\frac{1}{2}\frac{G}{c^2}\frac{\partial}{\partial t}\left(\frac{\underline{L} \times \underline{r}}{r^3}\right)$$

In eq. (7),

$$\phi = -\frac{mG}{r} \quad -(10)$$

and the units of $\underline{Q}_{total}$ are $m s^{-1}$.

Therefore

$$\underline{F} = m\underline{g} = -m\underline{\nabla}\phi - m\frac{\partial \underline{Q}_{total}}{\partial t} \quad -(11)$$

$$= -m\underline{\nabla}\phi + m\underline{\omega}\phi$$

The vacuum force is:

$$\boxed{\underline{F}(vac) = m\underline{\omega}\phi = -m\frac{\partial \underline{Q}_{total}}{\partial t}} \quad -(12)$$

3) So the Lense-Thirring precession is a vacuum effect. It occurs aboard a spacecraft or in a pendulum.

The near circular apsidal method can now be applied to eq. (11) to evaluate the precession in another way. The near circular approximation considers the Leibnitz equation of an orbit:

$$\ddot{r} - \frac{L^2}{m^2 r^3} = f(r) = \frac{F(r)}{m} \quad -(13)$$

for small departures from circularity:

$$r = r_c + x \quad -(14)$$

where r_c is the circular radius. Therefore:

$$\ddot{r} = \ddot{r}_c + \ddot{x} = \ddot{x} \quad -(15)$$

because the circular radius does not change with time. So:

$$\ddot{x} - \frac{L^2}{m^2 (r_c + x)^3} = F(r_c + x) \quad -(16)$$

i.e.

$$\ddot{x} - \frac{L^2}{m^2 r_c^3 \left(1 + \frac{x}{r_c}\right)^3} = F(r_c + x) \quad -(17)$$

Now use

$$\left(1 + \frac{x}{r_c}\right)^{-3} \sim 1 - 3\frac{x}{r_c} \quad -(18)$$

for

$$\frac{x}{r_c} << 1, \quad -(19)$$

in eq. (17) in a Taylor and expand the right hand side of series:

$$F(r_c + x) = F(r_c) + x\left(\frac{dF}{dr}\right)_{r=r_c} + \ldots \quad -(20)$$

4) Therefore:

$$\ddot{x} - \frac{L^2}{m^2 r_c^3}\left(1 - \frac{3x}{r_c}\right) = f(r_c) + x f'(r_c) \quad -(21)$$

in which:

$$r \sim r_c \quad -(22)$$

From eq. (17):

$$f(r_c) \sim \frac{-L^2}{m^2 r_c^3} \quad -(23)$$

So

$$\ddot{x} + 3x \frac{L^2}{m^2 r^4} = x f'(r) \quad -(24)$$

where we have used eq. (22). Eq. (23) in eq. (24) gives:

$$\ddot{x} - \left(\frac{3 f(r)}{r} + f'(r)\right) x = 0 \quad -(25)$$

The apsis is the point in the orbit at which r is a maximum or a minimum. The perihelion and aphelion of planetary orbits are apsides. The apsidal angle is the angle through which r rotates between two consecutive apsides. For elliptical orbits it is π.

The period of oscillation from eq. (25) is:

$$T = \frac{2\pi}{\left(-\frac{3 f(r)}{r} - f'(r)\right)^{1/2}} \quad -(26)$$

The apsidal angle ϕ is the amount by which the polar angle ϕ increases in going from a maximum to a minimum of r. The time needed for this is $T/2$. From a Lagrangian analysis:

$$\dot{\phi} = \frac{L}{m r^2} \quad -(27)$$

) where L is the total angular momentum, a constant of motion. For nearly circular orbits r and $\dot{\phi}$ are nearly constants. Now use eq. (23) with

$$r \sim r_c \quad -(28)$$

So

$$f(r) = -\frac{L^2}{m^2 r^3} \quad -(29)$$

and it follows that:

$$\dot{\phi} = \frac{L}{mr^2} = \left(-\frac{f}{r}\right)^{1/2} \quad -(30)$$

The apsidal angle is therefore:

$$\phi = \frac{T}{2}\dot{\phi} = \pi \left(\frac{-\frac{f}{r}}{-3\frac{f}{r} - f'} \right)^{1/2}$$

$$= \pi \left(3 + \frac{r f'(r)}{f} \right)^{-1/2} \quad -(31)$$

i.e.

$$\boxed{\phi = \pi \left(3 + \frac{r F'(r)}{F(r)} \right)^{-1/2}} \quad -(32)$$

Q.E.D.

Here F is the modulus of force:

$$F = |\underline{F}| = mg \quad -(33)$$

and

$$F' = \frac{dF}{dr} \quad -(34)$$

6)

From eqs. (7) and (32) it is possible to calculate the apsidal angle for any particular scalar potential ϕ and vector potential $\underline{A}$. The latter is related to the spin connection through eq. (12). The Lense-Thirring effect is given by the particular choice (9).

Any precession can be calculated in general through a choice of spin connection. Therefore all precession originates in vacuum fluctuations. For example:

1) Planetary precession.
2) Lense - Thirring precession.
3) Thomas precession.
4) de Sitter precession.
5) Equinoctial precession.

This will be the subject of future work. The precessions can be calculated as a Larmor precession from eqs. (1) and (5), or as a deviation from

$$\psi = \pi. \quad -(35)$$

For the Newtonian inverse square law:

$$\underline{F} = -\frac{mMG}{r^3}\underline{r} = -\frac{mMG}{r^2}\underline{e}_r \quad -(36)$$

so

$$F = -\frac{mMG}{r^2} \quad -(37)$$

and

$$F' = \frac{2mMG}{r^3} \quad -(38)$$

It follows from eq. (32) that:

$$\psi = \pi \quad -(39)$$

Q.E.D., i.e the Newtonian orbit is an ellipse.