404(3): Calculation of the $\langle \delta r \cdot \delta r \rangle$ Responsible for Precession due to Rotation.

The vacuum fluctuation is related to the modulus of the spin connection by:

$$\omega = \frac{2}{3} \frac{\langle \delta \underline{r} \cdot \delta \underline{r} \rangle}{r^3} \quad -(1)$$

Define the spin connection as in Note 404(2):

$$\underline{\omega} \, \phi = - \frac{\partial \underline{Q}}{\partial t}(\text{total}) \quad -(2)$$

where

$$\underline{Q}(\text{total}) = \frac{G}{2c^2} \frac{\underline{L} \times \underline{r}}{r^3} \quad -(3)$$

is the total ECE2 gravitational vector potential, assumed to be a dipole potential.

Taking magnitudes on left side of eq. (2):

$$\omega \phi = \frac{\partial}{\partial t} |\underline{Q}(\text{total})| \quad -(4)$$

where

$$|\underline{Q}(\text{total})| = \frac{G}{2c^2 r^3} \left(\underline{L} \times \underline{r} \cdot \underline{L} \times \underline{r} \right)^{1/2}$$

$$= \frac{G}{2c^2 r^3} \left(L^2 r^2 - (\underline{L} \cdot \underline{r})^2 \right)^{1/2} \quad -(5)$$

In eq. (4):

$$\frac{\partial}{\partial t} |\underline{Q}(\text{total})| = \frac{dr}{dt} \frac{d}{dr} |\underline{Q}(\text{total})|, \quad -(6)$$

$$= \frac{G}{2c^2} \frac{dr}{dt} \frac{d}{dr} \left(\frac{\left(L^2 r^2 - (\underline{L} \cdot \underline{r})^2 \right)^{1/2}}{r^3} \right)$$

$$= \omega \phi = - \frac{MG\omega}{r}$$

It follows that:

$$\omega = -\frac{r}{Mc^2}\frac{dr}{dt}\frac{d}{dr}\left(\frac{(L^2r^2-(\underline{L}\cdot\underline{r})^2)^{1/2}}{r^3}\right) \quad -(7)$$

$$= \frac{2}{3}\frac{\langle \delta\underline{r}\cdot\delta\underline{r}\rangle}{r^3}$$

The magnitude of the Earth's angular momentum is:

$$L = \frac{2}{5}M r_E^2 \omega_E \quad -(8)$$

as in Note 404(2).

If it is arranged experimentally that:

$$\underline{L} \perp \underline{r} \quad -(9)$$

then

$$\underline{L}\cdot\underline{r} = 0 \quad -(10)$$

and for

$$r \sim r_E \quad -(11)$$

the spin connection is:

$$\omega \sim \frac{2}{5}\frac{\omega_E v_E}{c^2} \quad -(12)$$

$$= 3.0\times10^{-13}\ m^{-1}.$$

Here v_E is the velocity of the Earth about its own axis, surface velocity:

$$v_E = 4.60\times10^2\ ms^{-1} \quad -(13)$$

Under the condition (11):

$$\omega = \frac{2}{3}\frac{\langle\delta\underline{r}\cdot\delta\underline{r}\rangle}{r_E^3} = \frac{2}{5}\frac{\omega_E v_E}{c^2} \quad -(14)$$

so

$$\langle\delta\underline{r}\cdot\delta\underline{r}\rangle = 1.163\times10^8\ m^2 \quad -(15)$$

) and

$$\boxed{\langle \delta \underline{r} \cdot \delta \underline{r} \rangle^{1/2} = 1.078 \times 10^{4} \, m} \quad -(16)$$

It follows that:

$$\frac{\langle \delta \underline{r} \cdot \delta \underline{r} \rangle^{1/2}}{r_E} = 1.69 \times 10^{-3} \sim 0.2\% \quad -(17)$$

and the vacuum fluctuation, the root mean square vacuum fluctuation $\langle \delta \underline{r} \cdot \delta \underline{r} \rangle^{1/2}$, is roughly 0.2% of the earth's radius r_E.

The Lense Thirring precession of the standard model is replaced by an ECE 2 covariant theory based on vacuum fluctuation.

The above calculation can be refined and made more precise using computer algebra, and the averaging procedure of UFT 345.