404(2): Calculations of Precession due to a Rotating Object

Apsidal Method

As in Note 404(1) the force per unit mass $\underline{f}$ is:

$$\underline{f} = -\underline{\nabla}\phi - \frac{\partial \underline{Q}(\text{total})}{\partial t} \quad -(1)$$

$$= -\underline{\nabla}\phi + \underline{\omega}\,\phi$$

so the vacuum force is:

$$\underline{f}(\text{vac}) = \underline{\omega}\,\phi - \frac{\partial \underline{Q}(\text{total})}{\partial t} \quad -(2)$$

Therefore the rotation of the object, such as the earth, induces the vacuum force through the spin connection. In standard physics this is known as the frame dragging of the Lense Thirring effect. In ECE2 physics the development of the effect is much simpler, it relies only on the spin connection $\underline{\omega}$. Here ϕ is the Newtonian gravitational potential:

$$\phi = -\frac{MG}{r} \quad -(3)$$

The apsidal angle is:

$$\phi = \pi \left(3 + \frac{r f'(r)}{f(r)} \right)^{-1/2} \quad -(4)$$

As in UFT403, and the apsidal angle gives the precession due to the rotation of the earth, for example:

$$\Delta\phi = \frac{r^2}{2}\left(\frac{\omega}{r} - \frac{\partial \omega}{\partial r} \right) \quad -(5)$$

where

$$\omega = |\underline{\omega}| \quad -(6)$$

and

$$-\frac{\partial \underline{Q}(\text{total})}{\partial t} = \underline{\omega}\,\phi \quad -(7)$$

3) The magnitude of the spin connection is:

$$\omega = \frac{2}{3} \frac{\langle \delta \underline{r} \cdot \delta \underline{r} \rangle}{r^3} \quad -(8)$$

where $\langle \delta \underline{r} \cdot \delta \underline{r} \rangle$ is the isotropically averaged fluctuation of the vacuum (spacetime or aether). Therefore:

$$\Delta \phi = \frac{4}{3} \frac{\langle \delta \underline{r} \cdot \delta \underline{r} \rangle}{r^2} - \frac{1}{3r} \frac{\partial}{\partial r} \langle \delta \underline{r} \cdot \delta \underline{r} \rangle \quad -(9)$$

From eq. (7):

$$\omega \phi = - \frac{\partial}{\partial t} \left| \underline{Q}(\text{total}) \right| \quad -(10)$$

In previous papers of the UFT series the Lense-Thirring effect was developed with the gravitomagnetic field:

$$\underline{\Omega} = \frac{G}{c^2 r^3} \left(3 \underline{m}_g \cdot \frac{\underline{r}\,\underline{r}}{r^2} - \underline{m}_g \right) \quad -(11)$$

$$= \frac{G}{2c^2 r^3} \left(3 \underline{L} \cdot \frac{\underline{r}\,\underline{r}}{r^2} - \underline{L} \right)$$

where

$$\underline{m}_g = \frac{1}{2} \underline{L} \quad -(12)$$

is the gravitomagnetic dipole moment and $\underline{L}$ the angular momentum of the earth.

By definition:

$$\underline{\Omega} = \underline{\nabla} \times \underline{Q}(\text{total}) \quad -(13)$$

So:

$$\underline{Q}(\text{total}) = \frac{G}{c^2}\frac{\underline{m_g} \times \underline{r}}{r^3} \qquad -(14)$$

$$= \frac{G}{2c^2}\frac{\underline{L} \times \underline{r}}{r^3}$$

Therefore:

$$|\underline{Q}(\text{total})| = \frac{G}{2c^2 r^3} |\underline{L} \times \underline{r}|$$

$$= \frac{G}{2c^2 r^3}\left(\underline{L}\times\underline{r} \cdot \underline{L}\times\underline{r}\right)^{1/2} \qquad -(15)$$

By vector algebra:

$$\underline{L}\times\underline{r}\cdot\underline{L}\times\underline{r} = L^2 r^2 - (\underline{L}\cdot\underline{r})^2 \qquad -(16)$$

$$= (\underline{L}\cdot\underline{L})(\underline{r}\cdot\underline{r}) - (\underline{L}\cdot\underline{r})(\underline{L}\cdot\underline{r})$$

So

$$|\underline{Q}(\text{total})| = \frac{G}{2c^2 r^3}\left(L^2 r^2 - (\underline{L}\cdot\underline{r})^2\right)^{1/2} \qquad -(17)$$

So

$$\frac{d}{dt}|\underline{Q}(\text{total})| = \frac{G}{2c^2}\frac{d}{dt}\left(\frac{1}{r^3}\left(L^2 r^2 - (\underline{L}\cdot\underline{r})^2\right)^{1/2}\right) \qquad -(18)$$

In Cartesian coordinates:

$$\underline{v} = \frac{d\underline{r}}{dt} \qquad -(19)$$

is the orbital velocity. So

$$v = \frac{dr}{dt} \qquad -(20)$$

If:

$$f(r) := \frac{1}{r^3}\left(L^2 r^2 - (\underline{L}\cdot\underline{r})^2\right)^{1/2} \quad -(21)$$

then

$$\frac{\partial f(r)}{\partial t} = \frac{dr}{dt}\frac{\partial f(r)}{\partial r}$$

$$= v\,\frac{d}{dr}\left(\frac{1}{r^3}\left(L^2 r^2 - (\underline{L}\cdot\underline{r})^2\right)^{1/2}\right) \quad -(22)$$

and

$$\frac{\partial}{\partial t}\left|\underline{a}(total)\right| = \frac{Gv}{2c^2}\frac{df(r)}{dr} \quad -(23)$$

From eqs (10) and (23):

$$\omega\phi = -\frac{G}{2c^2}\,v\,\frac{df(r)}{dr} \quad -(24)$$

where

$$\phi = -\frac{MG}{r} \quad -(25)$$

so

$$\boxed{\omega = -\frac{rv}{2Mc^2}\frac{df(r)}{dr}} \quad -(26)$$

If the sign of $\underline{\omega}$ is reversed in eq. (1), then:

$$\underline{g} = -\underline{\nabla}\phi - \underline{\omega}\phi \quad -(27)$$

and

$$\omega\phi = \frac{\partial}{\partial t}\left|\underline{a}(total)\right| \quad -(28)$$

so

$$\omega = \frac{rv}{2Mc^2}\frac{df(r)}{dr} \quad -(29)$$

5) The precession in radians per second is given by

$$\Delta \phi = \frac{r^2}{2} \left(\frac{\omega}{r} - \frac{\partial \omega}{\partial r} \right) \quad -(30)$$

In Gravity Probe B for example, r is the distance from the centre of the earth to the spacecraft, and v is the orbital velocity of the spacecraft. Here M is the mass of the earth and c is the speed of light. The angular momentum of the Earth is taken from UFT 117 - UFT 119 and UFT 345:

$$L = \frac{2}{5} M r_E^2 \omega_E \quad -(31)$$

where M is the mass of the earth, r_E is the earth's radius, ω_E is its angular velocity:

$$\left. \begin{aligned} M &= 5.98 \times 10^{24} \text{ kg} \\ r_E &= 6.37 \times 10^{6} \text{ m} \\ \omega_E &= 7.292 \times 10^{-5} \text{ rad s}^{-1} \end{aligned} \right\} \quad -(32)$$

For gravity Probe B:

$$r = 7.02 \times 10^{6} \text{ m} \quad -(33)$$

on average, but varies in general. The constants are:

$$c = 2.998 \times 10^{8} \text{ m s}^{-1} \quad -(34)$$

$$G = 6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2} \quad -(35)$$

Therefore $\Delta \phi$ can be evaluated with computer algebra for a given r and v, and the result adjusted to give agreement with experiment.

The precession is experimentally very tiny, so the Newtonian theory is an excellent approximation for

6) r and v of Gravity Probe B orbiting the Earth:

$$r = \frac{\alpha}{1+\epsilon\cos\phi} \quad -(36)$$

and

$$v^2 = MG\left(\frac{2}{r} - \frac{1}{a}\right) \quad -(37)$$

where

$$a = \frac{\alpha}{1-\epsilon^2}. \quad -(38)$$

Here α is the half right latitude and ϵ is the ellipticity of the orbit of Gravity Probe B.

If $\underline{r}$ is perpendicular to $\underline{L}$ experimentally, then

$$\underline{L} \cdot \underline{r} = 0 \quad -(39)$$

and

$$\omega = \pm \frac{rv}{2Mc^2} \frac{d}{dr} \frac{L}{r^2}$$

$$= \mp \frac{vL}{Mc^2r^2} \quad -(40)$$

which has the correct units of inverse metres. The velocity in eq. (40) is given by eq. (37) and the angular momentum by eq. (31). If it is assumed that

$$r_E \sim r \quad -(41)$$

then:

$$\omega \sim \mp \frac{2}{5} \frac{\omega_E v}{c^2} \quad -(42)$$

where the velocity of rotation of the earth about its axis is:

$$v \sim 4.60 \times 10^2 \, m s^{-1} \quad -(43)$$

Therefore the magnitude of the spin connection is,

1) in this rough approximation:

$$\omega \sim \mp \frac{2}{5} \times \frac{7.292 \times 10^{-5} \times 4.60 \times 10^{2}}{(2.998 \times 10^{8})^2}$$

$$= 3.0 \times 10^{-13} \, m^{-1} \quad -(44)$$

From eqs. (30) and (40) the precession in radians per second is:

$$\Delta\phi = \frac{r^2}{2}\left(\frac{\omega}{r} - \frac{\partial\omega}{\partial r}\right). \quad -(45)$$

Taking the negative value of ω in eq. (40) then:

$$\frac{\omega}{r} = -\frac{L}{mc^2}\frac{v}{r^3} \quad -(46)$$

and

$$\frac{\partial\omega}{\partial r} = \frac{L}{mc^2}\frac{\partial}{\partial r}\left(\frac{v}{r^2}\right)$$

$$= \frac{L}{mc^2}\left(\frac{1}{r^2}\frac{\partial v}{\partial r} + v\frac{\partial}{\partial r}\left(\frac{1}{r^2}\right)\right) \quad -(47)$$

$$= \frac{L}{mc^2}\left(-\frac{2v}{r^3} + \frac{1}{r^2}\frac{\partial v}{\partial r}\right)$$

So

$$\Delta\phi = \frac{Lr^2}{2mc^2}\left(\frac{v}{r^3} + \frac{1}{r^2}\frac{\partial v}{\partial r}\right) \quad -(48)$$

$$= \frac{L}{2mc^2}\left(\frac{v}{r} + \frac{\partial v}{\partial r}\right)$$

If v is approximately constant:

$$v \sim 4.60 \times 10^{2} \, ms^{-1} \quad -(49)$$

then

$$\frac{\partial r}{\partial r} \sim 0 \quad \text{--- (50)}$$

and

$$\Delta\phi \sim \frac{1}{5} \cdot \frac{\omega_E v r}{c^2} \quad \text{- (51)}$$

$$= 5.24 \times 10^{-13} \text{ rad per year}$$

2) Geodetic Method

This method is given in UFT 345, and gives

$$\Delta\phi \sim 3.18 \times 10^{-13} \text{ rad per year}$$

The experimental result from Gravity Probe B is claimed to be

$$\Delta\phi\,(\text{exp}) \sim 1.016 \times 10^{-13} \text{ rad per year}$$
$$\text{-- (52)}$$

An averaging method was used in UFT 345 to give precise agreement with experimental data.

The approximate result (51) from the apsidal method can be made exact using computer algebra, and the averaging procedure used to give precise agreement with experimental data.