401(4): Calculation of the Angular Frequency of Gravitational Vacuum Fluctuations

As in Lamb shift theory, consider vacuum fluctuations of the type:

$$\delta \underline{r} = \delta \underline{r}(0) \exp(-i\Omega_0 t) \quad -(1)$$

$$\delta \underline{r}^* = \delta \underline{r}(0) \exp(i\Omega_0 t) \quad -(2)$$

It follows that:

$$\frac{d^2 \delta \underline{r}}{dt^2} = -\Omega_0^2 \delta \underline{r} \quad -(3)$$

$$\frac{d^2 \delta \underline{r}^*}{dt^2} = -\Omega_0^2 \delta \underline{r}^* \quad -(4)$$

so

$$\frac{d^2 \delta \underline{r}}{dt^2} \cdot \frac{d^2 \delta \underline{r}^*}{dt^2} = \Omega_0^4 \delta \underline{r} \cdot \delta \underline{r}^* \quad -(5)$$

The vacuum gravitational field is defined as:

$$g(\text{vac}) = \left\langle \frac{d^2 \delta \underline{r}}{dt^2} \cdot \frac{d^2 \delta \underline{r}^*}{dt^2} \right\rangle^{1/2} = \Omega_0^2 \langle \delta \underline{r} \cdot \delta \underline{r}^* \rangle^{1/2} \quad -(6)$$

where the angular brackets denote isotropic averaging. In eq. (6)

$$g(\text{vac}) = |\underline{g}(\text{vac})| = \left(g_X^2(\text{vac}) + g_Y^2(\text{vac})\right)^{1/2} \quad -(7)$$

From previous work:

$$\langle F(\text{vac}) \rangle = m \langle g(\text{vac}) \rangle$$

$$= |\underline{\omega}| \phi = (\omega_X^2 + \omega_Y^2)^{1/2} \phi$$

$$= \langle F(\text{vac}) \rangle^{(2)} + \langle F(\text{vac}) \rangle^{(4)} + \cdots \quad -(8)$$

Here

$$F = |\underline{F}| = \left(F_x^2 + F_y^2\right)^{1/2} \quad -(9)$$

and

$$\phi = -\frac{mMG}{r} = -\frac{mMG}{(x^2+y^2)^{1/2}} \quad -(10)$$

From the tensorial Taylor series:

$$\langle F(vac)\rangle^{(2)} = \frac{1}{6}\langle \delta\underline{r} \cdot \delta\underline{r}\rangle \nabla^2 F \quad -(11)$$

$$\langle F(vac)\rangle^{(4)} = \frac{1}{216}\langle(\delta\underline{r}\cdot\delta\underline{r})^2\rangle\left(\frac{\partial^4 F}{\partial x^4} + \frac{\partial^4 F}{\partial y^4} + 6\frac{\partial^4 F}{\partial x^2 \partial y^2}\right) \quad -(12)$$

and so on.

Here

$$F = \frac{mMG}{x^2+y^2} = \frac{mMG}{r^2} \quad -(13)$$

Therefore

$$\nabla^2 F = mMG\,\nabla^2\left(\frac{1}{r^2}\right)$$

$$= mMG\left(\frac{\partial^2}{\partial r^2}\left(\frac{1}{r^2}\right) + \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{1}{r^2}\right)\right) \quad -(14)$$

Here:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{1}{r^2}\right) = -\frac{2}{r^4} \quad -(15)$$

and

$$\frac{\partial^2}{\partial r^2}\left(\frac{1}{r^2}\right) = \frac{6}{r^4} \quad -(16)$$

so

$$\nabla^2 F = \frac{4mMG}{r^4} = \frac{4mMG}{(x^2+y^2)^2} \quad -(15)$$

3)

Assume that:

$$\langle F(vac)\rangle = \langle F(vac)\rangle^{(2)}$$

$$= \frac{2}{3}\, \frac{mMG \langle \delta \underline{r} \cdot \delta \underline{r} \rangle}{(x^2+y^2)^2} \quad - (16)$$

following the method of the Lamb shift, where only the term to order two of the Taylor series is used.

From eqs. (6) and (16):

$$\Omega_0^2 \langle \delta \underline{r} \cdot \delta \underline{r}^* \rangle^{1/2} = \frac{2}{3} MG \frac{\langle \delta \underline{r} \cdot \delta \underline{r} \rangle}{r^4} \quad - (17)$$

Assume that $\langle \delta \underline{r} \cdot \delta \underline{r} \rangle$ on the right hand side of eq. (17) is generalized to $\langle \delta \underline{r} \cdot \delta \underline{r}^* \rangle$ in a complex valued Taylor series. Equivalently assume that the physical part of the function on the left hand side of eq. (17) is its real part:

$$\text{Real} \langle \delta \underline{r} \cdot \delta \underline{r}^* \rangle^{1/2} = \langle \delta \underline{r} \cdot \text{Real}\, \delta \underline{r}^* \rangle^{1/2}$$

$$= \langle \delta \underline{r} \cdot \delta \underline{r} \rangle^{1/2} \quad - (18)$$

It follows that

$$\boxed{\Omega_0^2 = \frac{2}{3} MG \frac{\langle \delta \underline{r} \cdot \delta \underline{r} \rangle^{1/2}}{r^4}} \quad - (19)$$

From eqs. (8) and (16):

$$|\underline{\omega}|\, \phi = \frac{2}{3}\, mMG \frac{\langle \delta \underline{r} \cdot \delta \underline{r} \rangle}{r^4} \quad - (20)$$

) where

$$\phi = -\frac{mMG}{r} \quad -(21)$$

so

$$\boxed{\langle \delta \underline{r} \cdot \delta \underline{r} \rangle = -\frac{3r^3}{2} |\underline{\omega}|} \quad -(22)$$

Therefore $\langle \delta \underline{r} \cdot \delta \underline{r} \rangle$ can be found from:

$$|\underline{\omega}| = \left(\omega_x^2 + \omega_y^2 \right)^{1/2} \quad -(23)$$

and Ω_0^2 found from eq. (15). The spin connection components are found from the force equation of ECE2:

$$\underline{F} = -\underline{\nabla}\phi + \underline{\omega}\phi \quad -(24)$$

and a lagrangian analysis as in Note 401(3), and previous UFT papers.

From eq. (16) the magnitude of the total force is:

$$F = \frac{mMG}{r^2} \left(1 + \frac{2}{3} \langle \delta \underline{r} \cdot \delta \underline{r} \rangle \right) \quad -(25)$$

This is the vacuum correction of the magnitude of the Newtonian force. The use of the Newtonian force magnitude in eq. (13) means that a second order theory can be used as in eq. (16), thus greatly simplifying the calculation.