401(3): Simple ECE2 Theory of Orbital Precession

Consider the fundamental definition of force in ECE2 gravitational theory:

$$\underline{F} = - \underline{\nabla} \, \underline{\Phi} + \underline{\omega} \, \underline{\Phi} \quad -(1)$$

where $\underline{\Phi}$ is the gravitational potential and $\underline{\omega}$ the spin connection. The vacuum force is:

$$\underline{F}(\text{vac}) = \underline{\omega} \, \underline{\Phi} \quad -(2)$$

where $\underline{\Phi}$ is the gravitational potential in the absence of the vacuum:

$$\underline{\Phi} = - \frac{mG}{r} \quad -(3)$$

In Cartesian coordinates eq. (1) is:

$$\ddot{X} = - \frac{mG}{(X^2+Y^2)^{1/2}} \left(\frac{X}{X^2+Y^2} + \omega_X \right) \quad -(4)$$

$$\ddot{Y} = - \frac{mG}{(X^2+Y^2)^{1/2}} \left(\frac{Y}{X^2+Y^2} + \omega_Y \right) \quad -(5)$$

From the tensorial Taylor expansion of the previous note:

$$\omega_X \, \underline{\Phi} = \langle F_X(\text{vac}) \rangle^{(2)} + \langle F_X(\text{vac}) \rangle^{(4)} + \dots \quad -(6)$$

$$\omega_Y \, \underline{\Phi} = \langle F_Y(\text{vac}) \rangle^{(2)} + \langle F_Y(\text{vac}) \rangle^{(4)} + \dots \quad -(7)$$

in which $\underline{\Phi}$ is given by eq. (3).

In ECE2 theory the gravitational force (1) is equivalent to the lagrangian:

2)

$$\mathcal{L} = -\frac{mc^2}{\gamma} + \frac{mMG}{r} \quad -(8)$$

and the Hamiltonian:

$$H = \gamma mc^2 - \frac{mMG}{r} \quad -(9)$$

and the field equations of ECE2 gravitation, written in a space with finite torsion and curvature.

Forward precession is produced by the Euler Lagrange equations:

$$\frac{\partial \mathcal{L}}{\partial X} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{X}} \quad -(10)$$

$$\frac{\partial \mathcal{L}}{\partial Y} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{Y}} \quad -(11)$$

and retrograde precession is produced by the relativistic Newton equation:

$$\gamma^3 m \ddot{\underline{r}} = -mMG\frac{\underline{r}}{r^3} \quad -(12)$$

which is obtained from the Euler Lagrange equation:

$$\frac{\partial \mathcal{L}}{\partial \underline{r}} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\underline{r}}} \quad -(13)$$

and Lagrangian (8).

As in Note 401(1), forward precession is given from eqs. (10) and (11) by:

$$\ddot{X} = \frac{mG}{\gamma(x^2+y^2)^{3/2}}\left(\frac{\dot{x}\dot{y}y+x\dot{x}^2}{c^2}-x\right) \quad -(14)$$

$$\ddot{y} = \frac{mG}{\gamma(x^2+y^2)^{3/2}}\left(\frac{\dot{y}\dot{x}x+y\dot{y}^2}{c^2}-y\right) \quad -(15)$$

The spin connections for forward precession are found by a comparison of eqns. (4) and (5) and (14) and (15). In these equations:

$$\frac{1}{\gamma} = \left(1-\frac{\dot{x}^2+\dot{y}^2}{c^2}\right)^{1/2} \quad -(16)$$

For retrograde precession:

$$\ddot{X} = -\frac{mG}{\gamma^3}\frac{X}{(x^2+y^2)^{3/2}} \quad -(17)$$

$$\ddot{y} = -\frac{mG}{\gamma^3}\frac{y}{(x^2+y^2)^{3/2}} \quad -(18)$$

and the spin connections for retrograde precession are found by a comparison of eqs. (4) and (5) and (17) and (18).

These spin connection components can now be used in eqs. (6) and (7). In previous work it was shown that

$$\langle F_x(\mathrm{vac})\rangle^{(2)} = \langle F_y(\mathrm{vac})\rangle^{(2)} = 0$$

$$-(19)$$

4) Assuming that the tensorial Taylor series converge rapidly, then:

$$\omega_x \Phi = \langle F_x(\mathrm{vac}) \rangle^{(4)} \quad -(20)$$

$$\omega_y \Phi = \langle F_y(\mathrm{vac}) \rangle^{(4)} \quad -(21)$$

in which Φ, ω_x and ω_y are known. It is therefore possible to find $\langle (\delta \underline{r} \cdot \delta \underline{r})^2 \rangle$ because:

$$\langle F_x(\mathrm{vac}) \rangle^{(4)} = \frac{1}{216} \langle (\delta \underline{r} \cdot \delta \underline{r})^2 \rangle \left(\frac{\partial^4 F_x}{\partial x^4} + \frac{\partial^4 F_x}{\partial y^4} + 6 \frac{\partial^4 F_x}{\partial x^2 \partial y^2} \right) \quad -(22)$$

and

$$\langle F_y(\mathrm{vac}) \rangle^{(4)} = \frac{1}{216} \langle (\delta \underline{r} \cdot \delta \underline{r})^2 \rangle \left(\frac{\partial^4 F_y}{\partial x^4} + \frac{\partial^4 F_y}{\partial y^4} + 6 \frac{\partial^4 F_y}{\partial x^2 \partial y^2} \right) \quad -(23)$$

in an orbit in the XY plane.

It would be very interesting to graph the vacuum fluctuation $\langle (\delta \underline{r} \cdot \delta \underline{r})^2 \rangle$ for forward and retrograde precession, showing that the origin of precession is vacuum fluctuation on the classical ECE2 relativity level. The overall aim of the calculation is to reproduce observed orbital precession with precision.