398(1): Computation of the Vector Spin Corrections

In general, it has been shown by computer algebra in UFT397 that the 'volumically averaged' change in a scalar function f due to the vacuum fluctuation δr is:

$$\langle \Delta f \rangle = \langle \Delta f \rangle^{(2)} + \langle \Delta f \rangle^{(4)} + \langle \Delta f \rangle^{(6)} + \dots \quad -(1)$$

In Cartesian coordinates:

$$\langle \Delta f \rangle^{(2)} = \frac{1}{6} \langle \underline{\delta r} \cdot \underline{\delta r} \rangle \left(\frac{\partial^2 f}{\partial X^2} + \frac{\partial^2 f}{\partial Y^2} + \frac{\partial^2 f}{\partial Z^2} \right) \quad -(2)$$

$$\langle \Delta f \rangle^{(4)} = \frac{1}{216} \langle (\underline{\delta r} \cdot \underline{\delta r})^2 \rangle \left(\frac{\partial^4 f}{\partial X^4} + \frac{\partial^4 f}{\partial Y^4} + \frac{\partial^4 f}{\partial Z^4} + 6 \left(\frac{\partial^4 f}{\partial Y^2 \partial Z^2} + \frac{\partial^4 f}{\partial X^2 \partial Z^2} + \frac{\partial^4 f}{\partial X^2 \partial Y^2} \right) \right) \quad -(3)$$

$$\langle \Delta f \rangle^{(6)} = \frac{\langle (\underline{\delta r} \cdot \underline{\delta r})^3 \rangle}{19440} \left(\frac{\partial^6 f}{\partial X^6} + \frac{\partial^6 f}{\partial Y^6} + \frac{\partial^6 f}{\partial Z^6} + 15 \left(\frac{\partial^6 f}{\partial Y^4 \partial Z^2} + \frac{\partial^6 f}{\partial Y^2 \partial Z^4} + \frac{\partial^6 f}{\partial X^4 \partial Z^2} + \frac{\partial^6 f}{\partial X^4 \partial Y^2} + \frac{\partial^6 f}{\partial X^2 \partial Z^4} + \frac{\partial^6 f}{\partial X^2 \partial Y^4} \right) + 90 \frac{\partial^6 f}{\partial X^2 \partial Y^2 \partial Z^2} \right) \quad -(4)$$

When f is the Coulomb potential between the electron and proton in the H atom:

$$f = -\frac{e^2}{4\pi\epsilon_0 r} \quad -(5)$$

Eq. (1) produces higher order corrections to the Lamb shift.

2) Eq. (1) can be used to compute the ECE2 spin connection for the electric field strength $\underline{E}$ in volts m^{-1}, and the magnetic flux density $\underline{B}$ in tesla.

For the electric field strength:

$$\langle \Delta \underline{E}_0 \rangle = \underline{E}(\text{vac}) \quad -(6)$$

$$= \langle \Delta E_{X0} \rangle \underline{i} + \langle \Delta E_{Y0} \rangle \underline{j} + \langle \Delta E_{Z0} \rangle \underline{k}$$

$$= \underline{\omega} \, \phi_0$$

where $\underline{\omega}$ is the vector spin connection or "vacuum map", and where

$$\phi_0 = - \frac{e^2}{4 \pi \epsilon_0 r} \quad -(7)$$

with

$$r = (X^2 + Y^2 + Z^2)^{1/2} \quad -(8)$$

From eq. (1):

$$\langle \Delta E_{X0} \rangle = \langle \Delta E_{X0} \rangle^{(2)} + \langle \Delta E_{X0} \rangle^{(4)} + \langle \Delta E_{X0} \rangle^{(6)} + \dots$$

$$\langle \Delta E_{Y0} \rangle = \langle \Delta E_{Y0} \rangle^{(2)} + \langle \Delta E_{Y0} \rangle^{(4)} + \langle \Delta E_{Y0} \rangle^{(6)} + \dots$$

$$\langle \Delta E_{Z0} \rangle = \langle \Delta E_{Z0} \rangle^{(2)} + \langle \Delta E_{Z0} \rangle^{(4)} + \langle \Delta E_{Z0} \rangle^{(6)} + \dots$$

$$-(9)$$

Here:

$$\underline{E}_0 = - \frac{e^2}{4 \pi \epsilon_0} \frac{\underline{r}}{r^3} \quad -(10)$$

is the Coulombic electric field strength in the hypothetical absence of the vacuum:

$$\underline{E}_0 = - \underline{\nabla} \phi_0 \quad -(11)$$

3) From eq. (10):

$$E_{0x} = -\frac{e^2}{4\pi\epsilon_0} \cdot \frac{X}{(X^2+Y^2+Z^2)^{3/2}} \quad -(12)$$

and similarly for E_{0Y} and E_{0Z}.

Therefore the components of the electric spin connection vector can be computed from eqs. (6) - (10).

Similarly, the magnetic spin connection vector can be computed from:

$$\langle \Delta \underline{B}_0 \rangle = \underline{B}(vac) = -\underline{\omega} \times \underline{A}_0 \quad -(13)$$

where $\underline{A}_0$ is a vector potential. For example if $\underline{A}_0$ is the dipole potential:

$$\underline{A}_0 = \frac{\mu_0}{4\pi} \frac{\underline{m} \times \underline{r}}{r^3} \quad -(14)$$

then:

$$\underline{B}_0 = \underline{\nabla} \times \underline{A}_0 = \frac{\mu_0}{4\pi}\left[\frac{1}{r^3} 3\underline{m} \cdot \frac{\underline{r}\,\underline{r}}{r^2} - \frac{\underline{m}}{r^3}\left(1 + r^3 \nabla^2\left(\frac{1}{r}\right)\right)\right] \quad -(15)$$

This expression simplifies if it is assumed that:

$$\nabla^2\left(\frac{1}{r}\right) = 0, \quad -(16)$$

which is the result of vector analysis.

However, on the quantum level:

$$\nabla^2\left(\frac{1}{r}\right) = -4\pi\,\delta(\underline{r}) \quad -(17)$$

where $\delta(\underline{r})$ is the Dirac delta function.

The averages $\langle \delta\underline{r} \cdot \delta\underline{r} \rangle$, $\langle (\delta\underline{r} \cdot \delta\underline{r})(\delta\underline{r} \cdot \delta\underline{r}) \rangle$ and $\langle (\delta\underline{r} \cdot \delta\underline{r})^3 \rangle$ must be evaluated with mode theory, this is developed in the next note.