394(4) : M2 Theory applied to Electrodynamics

1) The first step is to find the scalar potential ϕ_0 and vector potential $\underline{A}_0$ in the hypothetical absence of the vacuum. These are the Lienard-Wiechert potentials obtained from:

$$\Box A_{0\mu} = \mu_0 J_{0\mu} \quad -(1)$$

as in the standard model. Here $J_{0\mu}$ is the charge current density in the hypothetical absence of the vacuum. The standard model does not consider the vacuum.

2) The trace antisymmetry laws are used to find

$$\omega^\mu = \left(\frac{\omega_0}{c}, \underline{\omega} \right) \quad -(2)$$

as follows:

$$\frac{\partial \phi}{\partial t} + \omega_0 \phi = 0 \quad -(3)$$

$$\frac{\partial A_X}{\partial X} + \omega_X A_X = 0 \quad -(4)$$

$$\frac{\partial A_Y}{\partial Y} + \omega_Y A_Y = 0 \quad -(5)$$

$$\frac{\partial A_Z}{\partial Z} + \omega_Z A_Z = 0 \quad -(6)$$

knowing ϕ, A_X, A_Y and A_Z. Therefore ω_0, ω_X, ω_Y and ω_Z can be found, and a vacuum map constructed.

3) The electric field strength $\underline{E}$ and the magnetic flux density $\underline{B}$ in the presence of the vacuum are:

$$\underline{E} = -\underline{\nabla}\phi + \underline{\omega}\phi = -\frac{\partial \underline{A}}{\partial t} - \omega_0 \underline{A},$$

$$\underline{B} = \underline{\nabla} \times \underline{A} - \underline{\omega} \times \underline{A}. \quad -(7)$$

2) Define:

$$\underline{r}_1 = \underline{r} + \delta \underline{r} \quad - (8)$$

then

$$\underline{E} = \frac{e}{4\pi\epsilon_0 r_1^2} \underline{e}_r \quad - (9)$$

where

$$r_1^2 = |\underline{r}_1|^2 \quad - (10)$$

These are the ECE equations for the Coulomb law in the presence of the vacuum. It is assumed that:

$$\phi = \frac{e}{4\pi\epsilon_0 r} \quad - (11)$$

so $\underline{\omega}$ can be found from:

$$\frac{e}{4\pi\epsilon_0 r_1^2} \underline{e}_r = \frac{e}{4\pi\epsilon_0 r^2} \underline{e}_r + \underline{\omega} \frac{e}{4\pi\epsilon_0 r}.$$

$$- (12)$$

From eq. (11):

$$\frac{\partial \phi}{\partial t} = 0 \quad - (13)$$

So:

$$\omega_0 = 0 \quad - (14)$$

for the Coulomb law of electrostatics. It follows from eq. (7) that:

$$\frac{\partial \underline{A}_E}{\partial t} = - \frac{e}{4\pi\epsilon_0 r_1^2} \underline{e}_r \quad - (15)$$

Note carefully that this is not the magnetic potential:

$$\underline{A}_m = \frac{\mu_0}{4\pi} \int \frac{\underline{J}(\underline{x}')}{|\underline{x} - \underline{x}'|} d^3 x' \quad - (16)$$

3) of the standard model, because in electrostatics,

$$\underline{J}(\underline{x}') = \underline{0}. \quad -(17)$$

The electrostatic $\underline{A}_E$ is distinguished from $\underline{A}_m$ for clarity. From the trace equations (4) to (6) it follows that:

$$\frac{\partial A_{EX}}{\partial X} + \omega_X A_{EX} = 0 \quad -(18)$$

$$\frac{\partial A_{EY}}{\partial Y} + \omega_Y A_{EY} = 0 \quad -(19)$$

$$\frac{\partial A_{EZ}}{\partial Z} + \omega_Z A_{EZ} = 0 \quad -(20)$$

where $\underline{\omega}$ is defined by eq. (12). Therefore:

$$\underline{\nabla} \cdot \underline{A}_E = \frac{\partial A_{EX}}{\partial X} + \frac{\partial A_{EY}}{\partial Y} + \frac{\partial A_{EZ}}{\partial Z} \quad -(21)$$

can be found. From eq. (15), note that:

$$\underline{A}_E = \underline{A}_E(\underline{r} + \delta \underline{r}) \quad -(22)$$

but by definition:

$$\phi = \phi(\underline{r}). \quad -(23)$$

If a potential:

$$\phi_1 = \phi_1(\underline{r} + \delta \underline{r}) \quad -(24)$$

is defined, then:

$$\underline{E} = -\underline{\nabla}_1 \phi_1 \quad -(25)$$

$$= -\underline{\nabla} \phi + \underline{\omega} \phi$$

The gradient operator $\underline{\nabla}_1$ is defined by:

$$\underline{\nabla}_1 := \frac{\partial}{\partial X_1}\underline{i} + \frac{\partial}{\partial Y_1}\underline{j} + \frac{\partial}{\partial Z_1}\underline{k}, \quad -(26)$$

and

$$\underline{r}_1 = \underline{r} + \delta\underline{r} = X_1\underline{i} + Y_1\underline{j} + Z_1\underline{k} \quad -(27)$$

Note carefully that the electric vector potential $\underline{A}_E(\underline{r} + \delta\underline{r})$ does not exist in the standard model.

Definition of ECE2 Electrostatics

The electrostatic field in the presence of the vacuum is:

$$\underline{E} = \underline{E}_0 + \underline{\omega}\,\phi \quad -(28)$$

$$= -\frac{\partial \underline{A}_E}{\partial t}$$

where

$$\underline{E}_0 = -\underline{\nabla}\phi \quad -(29)$$

It follows that:

$$\underline{\nabla} \times \underline{E}_0 = \underline{0} \quad -(30)$$

Eq. (28) is true for any electrostatic field. Therefore:

$$\underline{\nabla} \times \underline{E} = \underline{\nabla} \times (\underline{\omega}\,\phi) \quad -(31)$$

The new trace law (3) means that for electrostatics in general:

$$\omega_0 = 0 \quad -(32)$$

but

$$\underline{\omega} \neq \underline{0}. \quad -(33)$$

In electrostatics:

$$\underline{B} = \underline{\nabla} \times \underline{A}_m - \underline{\omega} \times \underline{A}_m = \underline{0} \quad -(34)$$

because

$$\underline{A}_m = \underline{0} \quad -(35)$$

) In magnetostatics:

$$\underline{B}(\underline{r}+\delta\underline{r}) = \underline{\nabla} \times \underline{A}_m(\underline{r}) - \underline{\omega} \times \underline{A}_m(\underline{r}) \quad -(36)$$

where

$$\underline{A}_m = \frac{\mu_0}{4\pi} \int \frac{\underline{J}(\underline{x}')}{|\underline{x}-\underline{x}'|} d^3x' \quad -(37)$$

Given the magnetic vector potential $\underline{A}_m$ of eq. (37), the magnetostatic spin connection is defined by eqs. (4) to (6). The vector antisymmetry law is:

$$(\underline{B}(\underline{r}+\delta\underline{r}))_{ij} = -(\underline{B}(\underline{r}+\delta\underline{r}))_{ji} \quad -(37)$$

and given $\underline{\omega}$ and $\underline{A}_m$ is defined automatically by eq. (36).

The scalar spin connection ω_0 does not enter into magnetostatics, because ϕ vanishes for magnetostatics.

<u>Electrodynamics</u>

Defining:

$$\underline{r}_1 := \underline{r} + \delta\underline{r} \quad -(38)$$

the MZ field equations of electrodynamics are:

$$\underline{\nabla}_1 \cdot \underline{B}(\underline{r}_1) = 0 \quad -(39)$$

$$\underline{\nabla}_1 \times \underline{E}(\underline{r}_1) + \frac{\partial \underline{B}(\underline{r}_1)}{\partial t} = \underline{0} \quad -(40)$$

$$\underline{\nabla}_1 \cdot \underline{E}(\underline{r}_1) = \rho_1 / \epsilon_0 \quad -(41)$$

$$\underline{\nabla}_1 \times \underline{B}(\underline{r}_1) + \frac{1}{c^2} \frac{\partial \underline{E}_1(\underline{r}_1)}{\partial t} = \mu_0 \underline{J}_1 \quad -(42)$$

i) in vector:

$$\underline{\nabla}_1 \times \underline{E}(\underline{r}_1) = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \frac{\partial}{\partial x_1} & \frac{\partial}{\partial y_1} & \frac{\partial}{\partial z_1} \\ E_x & E_y & E_z \end{vmatrix} \quad -(43)$$

The homogeneous field equations are obeyed by defining:

$$\underline{B}(\underline{r}_1) = \underline{\nabla}_1 \times \underline{A}(\underline{r}_1) \quad -(44)$$

and

$$\underline{E}(\underline{r}_1) = -\underline{\nabla}_1 \phi(\underline{r}_1) - \frac{\partial \underline{A}(\underline{r}_1)}{\partial t} \quad -(45)$$

In electrodynamics, no distinction is made between $\underline{A}_E$ and $\underline{A}_m$. Similarly, in standard model electrodynamics:

$$\underline{E} = -\underline{\nabla}\phi - \frac{\partial \underline{A}}{\partial t} \quad -(46)$$

$$\underline{B} = \underline{\nabla} \times \underline{A} \quad -(47)$$

but in standard model electrostatics:

$$\underline{E} = -\underline{\nabla}\phi \quad -(48)$$

$$\underline{B} = \underline{0} \quad -(49).$$

These equations can be developed in many different ways.