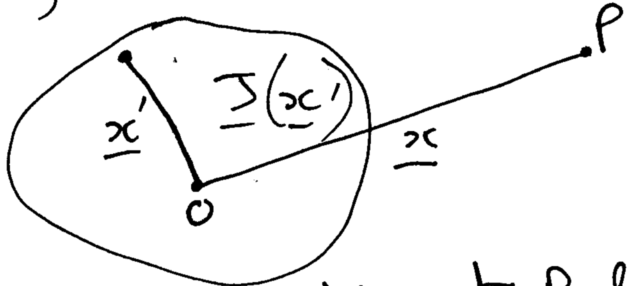
94(5): Magnetic Dipole Field, Applications to NMR.

With reference to Fig (1)



The dipole magnetic potential at point P due to a local charge distribution and current density $\underline{J}(\underline{x}')$ is:

$$\underline{A}(\underline{x}) = \frac{\mu_0}{4\pi} \frac{\underline{m} \times \underline{x}}{|\underline{x}|^3} \quad -(1)$$

where $\underline{m}$ is the magnetic dipole moment and μ_0 the vacuum permeability.

In applications to NMR and similar, the notation of eq. (1) is changed to:

$$\underline{A} = \frac{\mu_0}{4\pi} \frac{\underline{m} \times \underline{r}}{r^3} \quad -(2)$$

where

$$r = |\underline{r}| \quad -(3)$$

The nuclear magnetic dipole moment is used to produce:

$$\underline{A} = \frac{\gamma_N \mu_0}{4\pi r^3} \underline{I} \times \underline{r} \quad -(4)$$

where

$$\underline{m} = \gamma_N \underline{I} \quad -(5)$$

is the nuclear magnetic dipole moment.

Using:

$$\underline{\nabla}\left(\frac{1}{r}\right) = -\frac{\underline{r}}{r^3} \quad -(6)$$

then

$$\underline{A} = -\frac{\mu_0}{4\pi} \underline{m} \times \underline{\nabla}\left(\frac{1}{r}\right) \quad -(7)$$

The curl of $\underline{A}$ is evaluated with:

$$\nabla \times (\underline{F} \times \underline{G}) = \underline{F}(\underline{\nabla} \cdot \underline{G}) - (\underline{\nabla} \cdot \underline{F})\underline{G} + (\underline{G} \cdot \underline{\nabla})\underline{F} - (\underline{F} \cdot \underline{\nabla})\underline{G} \quad -(8)$$

...ng

$$\underline{F} = \underline{m}, \quad \underline{G} = \underline{\nabla} \frac{1}{r} \quad -(9).$$

...t follows that:

$$\underline{\nabla} \times \underline{A} = -\frac{\mu_0}{4\pi} \left(\underline{m} \left(\underline{\nabla} \cdot \underline{\nabla} \frac{1}{r} \right) - (\underline{\nabla} \cdot \underline{m}) \underline{\nabla} \left(\frac{1}{r} \right) + \left(\underline{\nabla} \left(\frac{1}{r} \right) \cdot \underline{\nabla} \right) \underline{m} - (\underline{m} \cdot \underline{\nabla}) \underline{\nabla} \frac{1}{r} \right) \quad -(10)$$

$$= -\frac{\mu_0}{4\pi} \left(\underline{m} \nabla^2 \frac{1}{r} - (\underline{m} \cdot \underline{\nabla}) \left(\underline{\nabla} \frac{1}{r} \right) \right)$$

because m is a constant nuclear quantity and has no dependence on $\underline{r}$.

Use:

$$(\underline{m} \cdot \underline{\nabla})(\underline{\nabla} r^{-1}) = -\underline{m} \cdot \underline{\nabla} (\underline{r} / r^3) \quad -(11)$$

$$= -\left(m_x \frac{\partial}{\partial x} + m_y \frac{\partial}{\partial y} + m_z \frac{\partial}{\partial z} \right) \left(\frac{x\underline{i} + y\underline{j} + z\underline{k}}{r^3} \right)$$

$$= -\frac{\underline{m}}{r^3} - \underline{r}\, \underline{m} \cdot \underline{\nabla} \frac{1}{r^3}$$

$$= -\frac{\underline{m}}{r^3} + \frac{3\underline{r}(\underline{m} \cdot \underline{r})}{r^5}$$

$$= \frac{1}{r^3} \left(3\underline{m} \cdot \frac{\underline{r}\,\underline{r}}{r^2} - \underline{m} \right)$$

The total magnetic flux density is:

$$\underline{B} = -\frac{\mu_0}{4\pi}\,\underline{m}\,\nabla^2\left(\frac{1}{r}\right) + \frac{\mu_0}{4\pi}\left(\frac{3\,\underline{m}\cdot\underline{r}\,\underline{r}}{r^5} - \frac{\underline{m}}{r^3}\right) \quad -(12)$$

In NMR theory the spherical or spatial average is:

$$\langle \underline{B} \rangle = -\frac{2}{3}\,\frac{\mu_0}{4\pi}\,\underline{m}\,\nabla^2\frac{1}{r} \quad -(13)$$

producing the contact term and spin spin splitting. This is experimentally observable in NMR theory and is affected by the vacuum.

In the macroscopic zitterbewegung theory:

$$\underline{r} \rightarrow \underline{r} + \delta\underline{r} \quad -(14)$$

so the average magnetic field (13) of spin spin theory is changed to:

$$\langle \underline{B} \rangle = -\frac{2}{3}\,\frac{\mu_0}{4\pi}\,\underline{m}\,\nabla^2\frac{1}{|\underline{r}+\delta\underline{r}|} \quad -(15)$$

Making the change of variable:

$$\underline{r}_1 = \underline{r} + \delta\underline{r} \quad -(16)$$

$$= X_1\,\underline{i} + Y_1\,\underline{j} + Z_1\,\underline{k}$$

the magnetic dipole potential in the presence of the vacuum is:

$$\underline{A}(\underline{r}+\delta\underline{r}) = \frac{\mu_0}{4\pi}\,\underline{m}\times\frac{(\underline{r}+\delta\underline{r})}{|\underline{r}+\delta\underline{r}|^3} \quad -(17)$$

$$= \frac{\mu_0}{4\pi}\,\underline{m}\times\frac{\underline{r}_1}{r_1^3}$$