394(6): Vacuum Effects in the Magnetic Dipole Potential and Field

The magnetic dipole potential is:

$$\underline{A} = \frac{\mu_0}{4\pi} \frac{\underline{m} \times \underline{r}}{r^3} \quad -(1)$$

as in previous notation, and the magnetic field is:

$$\underline{B} = -\frac{\mu_0}{4\pi} \underline{m} \nabla^2 \left(\frac{1}{r} \right) + \frac{\mu_0}{4\pi r^3} \left(3\underline{m} \cdot \frac{\underline{r}\,\underline{r}}{r^2} - \underline{m} \right) \quad -(2)$$

$$:= \underline{B}_c + \underline{B}_d$$

in which the contact field is:

$$\underline{B}_c = -\frac{\mu_0}{4\pi} \underline{m} \nabla^2 \left(\frac{1}{r} \right)$$

$$= \mu_0 \underline{m} \, \delta_D(\underline{r}) \quad -(3)$$

where $\delta_D(\underline{r})$ is the Dirac delta function.

The magnetic dipole field is:

$$\underline{B}_d = \frac{\mu_0}{4\pi r^3} \left(3\underline{m} \cdot \frac{\underline{r}\,\underline{r}}{r^2} - \underline{m} \right) \quad -(4)$$

For each function in the presence of the vacuum we propose the postulate:

$$\underline{r} \longrightarrow \underline{r} + \delta \underline{r} \quad -(5)$$

and

$$r = |\underline{r}| \rightarrow |\underline{r} + \delta \underline{r}| \quad -(6)$$

1) This postulate is applied to all equations of physics, meaning that the $\underline{r}$ coordinate shivers in the presence of the vacuum.

As in previous work:

$$|\underline{r}+\delta\underline{r}| = \left(r^2 + 2\underline{r}\cdot\delta\underline{r} + \delta\underline{r}\cdot\delta\underline{r}\right)^{1/2}$$

$$= r\left(1 + \frac{2\underline{r}\cdot\delta\underline{r} + \delta\underline{r}\cdot\delta\underline{r}}{r^2}\right)^{1/2}$$

$$:= r(1+x)^{1/2} \quad -(7)$$

where

$$x = \left(2\underline{r}\cdot\delta\underline{r} + \delta\underline{r}\cdot\delta\underline{r}\right)/r^2 \quad -(8)$$

So the magnetic dipole potential in the presence of the vacuum is:

$$\underline{A} = \frac{\mu_0}{4\pi}\frac{\underline{m}\times(\underline{r}+\delta\underline{r})}{|\underline{r}+\delta\underline{r}|^3}$$

$$= \frac{\mu_0}{4\pi r^3}\frac{\underline{m}\times(\underline{r}+\delta\underline{r})}{(1+x)^{3/2}} \quad -(9)$$

$$= \frac{\mu_0\,\underline{m}\times(\underline{r}+\delta\underline{r})}{4\pi r^3}(1+x)^{-3/2}$$

Using the binomial expansion as developed by Newton:

$$(1+x)^{-3/2} = 1 - \frac{3x}{2} + \frac{15}{8}x^2 + \ldots \quad -(10)$$

So in the presence of the vacuum:

$$\underline{A} = \frac{\mu_0}{4\pi r^3}\,\underline{m}\times(\underline{r}+\delta\underline{r})\left(1 - \frac{3x}{2} + \frac{15}{8}x^2 + \ldots\right) \quad -(11)$$

The average value $\langle \underline{A} \rangle$ is finally evaluated using many terms as needed. The final result is graphed.

In note 393(4) it was shown that the dipole electric field in the presence of the vacuum is:

$$\underline{E} = \frac{1}{4\pi\epsilon_0}\left(\frac{(\underline{r}+\delta\underline{r})(\underline{p}\cdot(\underline{r}+\delta\underline{r}))}{r^5}\left(1-\frac{5x}{2}+\frac{35}{8}x^2+\ldots\right)\right.$$
$$\left.-\frac{\underline{p}}{r^3}\left(1-\frac{3x}{2}+\frac{15x^2}{8}+\ldots\right)\right) \quad -(12)$$

The dipole magnetic field in the presence of the vacuum has the same structure:

$$\underline{B} = \frac{\mu_0}{4\pi}\left(\frac{(\underline{r}+\delta\underline{r})(\underline{p}\cdot(\underline{r}+\delta\underline{r}))}{r^5}\left(1-\frac{5x}{2}+\frac{35}{8}x^2+\ldots\right)\right.$$
$$\left.-\frac{\underline{p}}{r^3}\left(1-\frac{3x}{2}+\frac{15x^2}{8}+\ldots\right)\right) \quad -(13)$$

Therefore after averaging it will display the same new features as the electric dipole field. The contact field will be worked out in the next note.