Note 393(7): Checking Note 393(5), Eq. (5)

Consider:

$$\left\langle (\underline{r} + \delta\underline{r})(2\underline{r}\cdot\delta\underline{r} + \delta\underline{r}\cdot\delta\underline{r}) \right\rangle$$

$$= \Big\langle \underline{i}\,(x+\delta x)(2x\,\delta x + 2y\,\delta y + 2z\,\delta z + \delta x^2 + \delta y^2 + \delta z^2)$$

$$+ \underline{j}\,(y+\delta y)(2x\,\delta x + 2y\,\delta y + 2z\,\delta z + \delta x^2 + \delta y^2 + \delta z^2) \qquad -(1)$$

$$+ \underline{k}\,(z+\delta z)(2x\,\delta x + 2y\,\delta y + 2z\,\delta z + \delta x^2 + \delta y^2 + \delta z^2) \Big\rangle .$$

$$= \Big\langle (x\underline{i} + y\underline{j} + z\underline{k})(\delta x^2 + \delta y^2 + \delta z^2)$$

$$+ (\delta x\,\underline{i} + \delta y\,\underline{j} + \delta z\,\underline{k})(\delta x^2 + \delta y^2 + \delta z^2)$$

$$+ 2\underline{i}\,\langle x^2\delta x + xy\,\delta y + xz\,\delta z + (\delta x)^2\delta x + \delta x(\delta y)^2 + \delta x(\delta z)^2 \rangle$$

$$+ 2\underline{j}\,\langle yx\,\delta x + x^2\delta y + yz\,\delta z + \delta y(\delta x)^2 + (\delta y)^2\delta y + \delta y(\delta z)^2 \rangle$$

$$+ 2\underline{k}\,\langle zx\,\delta x + zy\,\delta y + z^2\delta z + \delta z(\delta x)^2 + \delta z(\delta y)^2 + (\delta z)^2\delta z \rangle$$

2) By isotropy:

$$\langle \delta \underline{r} \rangle = \underline{0} \quad -(2)$$

So:

$$\langle \delta x \rangle = \langle \delta y \rangle = \langle \delta z \rangle = 0 \quad -(3)$$

Therefore:

$$\langle (\delta x \underline{i} + \delta y \underline{j} + \delta z \underline{k})(\delta x^2 + \delta y^2 + \delta z^2) \rangle = 0$$

$$-(4)$$

Also:

$$\langle x^2 \delta x + xy \delta y + xz \delta z \rangle$$
$$= \langle yx \delta x + y^2 \delta y + \delta z \delta z \rangle \quad -(5)$$
$$= \langle z x \delta x + zy \delta y + z^2 \delta z \rangle = 0$$

and

$$\langle (\delta x)^2 \delta x + \delta x (\delta y)^2 + \delta x (\delta z)^2 \rangle$$
$$= \langle \delta y (\delta x)^2 + (\delta y)^2 \delta y + \delta y (\delta z)^2 \rangle \quad -(6)$$
$$= \langle \delta z (\delta x)^2 + \delta z (\delta y)^2 + (\delta z)^2 \delta z \rangle = 0$$

So:

$$\langle (\underline{r} + \delta \underline{r})(2 \underline{r} \cdot \delta \underline{r} + \delta \underline{r} \cdot \delta \underline{r}) \rangle$$
$$= \langle (x \underline{i} + y \underline{j} + z \underline{k})(\delta x^2 + \delta y^2 + \delta z^2) \rangle$$
$$= \underline{r} \langle \delta \underline{r} \cdot \delta \underline{r} \rangle \quad -(7)$$

Q.E.D.