392(8): ECE2 Zitterbewegung Theory

In this theory the total scalar potential is written as:

$$\phi_t = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\underline{x}')\, d^3x'}{|\underline{x} - \underline{x}' - \delta(\underline{x} - \underline{x}')|} \quad -(1)$$

and the total vector potential is:

$$\underline{A}_t = \frac{\mu_0}{4\pi} \int \frac{\underline{J}(\underline{x}')\, d^3x'}{|\underline{x} - \underline{x}' - \delta(\underline{x} - \underline{x}')|} \quad -(2)$$

where $\delta(\underline{x} - \underline{x}')$ is the vacuum fluctuation used to calculate the anomalous g factor of the electron and the Lamb shift. This was first carried out by H. A. Bethe as is well known. It is known as zitterbewegung or jitterbugging, and the potentials defined by eqs. (1) and (2) automatically define interaction with the vacuum.

The Bethe theory and later improvements give an accurate description of the radiative corrections, and can be used as in Eqs. (1) and (2).

Therefore the electric field strength of electrostatics is:

$$\underline{E}_t(\underline{x}) = -\frac{1}{4\pi\epsilon_0} \underline{\nabla} \int \frac{\rho(\underline{x}')\, d^3x'}{|\underline{x} - \underline{x}' - \delta(\underline{x} - \underline{x}')|} \quad -(3)$$

$$= -\underline{\nabla}\phi + \underline{\omega}\phi = -\frac{\partial \underline{A}_E}{\partial t} - \omega_0 \underline{A}_E \quad -(4)$$

where $\underline{A}_E$ is the electric vector potential of Note 392(7).

The magnetic flux density of magnetostatics is:

$$\underline{B}_t = \frac{\mu_0}{4\pi} \underline{\nabla} \times \int \frac{\underline{J}(\underline{x}') \, d^3x'}{|\underline{x} - \underline{x}' - \delta(\underline{x} - \underline{x}')|}$$

$$= \underline{\nabla} \times \underline{A} - \underline{\omega} \times \underline{A} \qquad -(5)$$

In eq. (4):

$$\phi = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\underline{x}') \, d^3x'}{|\underline{x} - \underline{x}'|}, \qquad -(6)$$

In eq. (5):

$$\underline{A} = \frac{\mu_0}{4\pi} \int \frac{\underline{J}(\underline{x}') \, d^3x'}{|\underline{x} - \underline{x}'|}, \qquad -(7)$$

these are the scalar and vector potentials in the hypothetical absence of the vacuum.

The existence of the radiative corrections means that the vacuum is always present in any situation in physics.

The fields in the hypothetical absence of the vacuum are, in electrostatics:

$$\underline{E} = -\underline{\nabla} \phi \qquad -(8)$$

and

$$\underline{B} = \underline{\nabla} \times \underline{A} \qquad -(9)$$

It follows that:

$$\boxed{\begin{aligned} \underline{E}_t - \underline{E} &= \underline{\omega} \phi \qquad -(10) \\ \text{and} \quad \underline{B}_t - \underline{B} &= -\underline{\omega} \times \underline{A} \qquad -(11) \end{aligned}}$$

3) where:

$$\underline{E} = -\frac{1}{4\pi\epsilon_0}\underline{\nabla}\int\frac{\rho(\underline{x}')}{|\underline{x}-\underline{x}'|}d^3x' \quad -(12)$$

and

$$\underline{B} = \frac{\mu_0}{4\pi}\underline{\nabla}\times\int\frac{\underline{J}(\underline{x}')\,d^3x'}{|\underline{x}-\underline{x}'|} \quad -(13)$$

It follows that:

$$\underline{\nabla}\left(\int\frac{\rho(\underline{x}')\,d^3x'}{|\underline{x}-\underline{x}'-\delta(\underline{x}-\underline{x}')|}-\int\frac{\rho(\underline{x}')\,d^3x'}{|\underline{x}-\underline{x}'|}\right)$$

$$= \underline{\omega}\int\frac{\rho(\underline{x}')\,d^3x'}{|\underline{x}-\underline{x}'|} \quad -(14)$$

i.e.:

$$\underline{\nabla}\int\rho(\underline{x}')\left(\frac{1}{|\underline{x}-\underline{x}'-\delta(\underline{x}-\underline{x}')|}-\frac{1}{|\underline{x}-\underline{x}'|}\right)d^3x'$$

$$= \underline{\omega}\int\frac{\rho(\underline{x}')}{|\underline{x}-\underline{x}'|}d^3x' \quad -(15)$$

Similarly:

$$\underline{\nabla}\times\int\underline{J}(\underline{x}')\left(\frac{1}{|\underline{x}-\underline{x}'-\delta(\underline{x}-\underline{x}')|}-\frac{1}{|\underline{x}-\underline{x}'|}\right)d^3x'$$

$$= -\underline{\omega}\times\int\frac{\underline{J}(\underline{x}')}{|\underline{x}-\underline{x}'|}d^3x' \quad -(16)$$

4)

On the one electron level....

$$\underline{E}_t = -\frac{e}{4\pi\epsilon_0 (r - \delta r)^2} \quad (17)$$

and

$$\underline{E} = -\frac{e}{4\pi\epsilon_0 r^2}\, \underline{e}_r$$

So

$$\left(\frac{1}{(r-\delta r)^2} - \frac{1}{r^2}\right) \underline{e}_r = \frac{\omega}{r^2} \quad -(18)$$

i.e.

$$\boxed{\underline{\omega} = \left(\frac{r^2}{(r-\delta r)^2} - 1\right) \underline{e}_r} \quad -(19)$$

where

$$\underline{r} = r\, \underline{e}_r \quad -(20)$$

so

$$\boxed{\underline{\omega} = \left(\frac{r}{(r-\delta r)^2} - \frac{1}{r}\right) \underline{r}} \quad -(21)$$

Therefore $\underline{\omega}$ can be worked out from the Bethe theory of the Lamb shift for H. This is a very accurate theory.

Alternatively, δr can be regarded as empirical. The vacuum electric field strength in volts per metre is defined by the scalar antisymmetry law:

5) $$\underline{E}_A = -\underline{\nabla}\phi + \underline{\omega}\phi = -\frac{\partial \underline{A}_E}{\partial t} - \omega_0 \underline{A}_E \quad -(22)$$

so

$$\underline{E}(vac) = \underline{\omega}\phi \quad -(23)$$

i.e.

$$\boxed{\begin{aligned}\underline{E}(vac) &= -\frac{e}{4\pi\epsilon_0 r}\left(\frac{r}{(r-\delta r)^2} - \frac{1}{r}\right)\underline{r} \\ &= -\frac{e}{4\pi\epsilon_0}\left(\frac{1}{(r-\delta r)^2} - \frac{1}{r^2}\right)\underline{r}\end{aligned}} \quad -(24)$$

This is the well known jitterbugging field of the vacuum, observed in the radiative corrections.

ECE shows that its origin is the spin connection (21).

The vacuum magnetic flux density is:

$$\underline{B}(vac) = -\underline{\omega} \times \underline{A} \quad -(25)$$

where

$$\underline{\omega} = \left(\frac{r}{(r-\delta r)^2} - \frac{1}{r}\right)\underline{r} \quad -(26)$$

and

$$\underline{A} = \frac{\mu_0}{4\pi}\int \frac{\underline{J}(\underline{x}')}{|\underline{x}-\underline{x}'|}\, d^3x' \quad -(27)$$

The vector potential $\underline{A}$ can be calculated for a current loop as in Maxwell Heaviside theory.

Alternatively, $\underline{A}$ can be calculated from

6) the vector antisymmetry law:

$$\frac{\partial A_Z}{\partial Y} + \frac{\partial A_Y}{\partial Z} = \omega_Y A_Z + \omega_Z A_Y \quad -(27)$$

$$\frac{\partial A_X}{\partial Z} + \frac{\partial A_Z}{\partial X} = \omega_Z A_X + \omega_X A_Z \quad -(28)$$

$$\frac{\partial A_Y}{\partial X} + \frac{\partial A_X}{\partial Y} = \omega_X A_Y + \omega_Y A_X \quad -(29)$$

This is a general method and gives the general vector potential $\underline{A}$ for the jitterbugging spin connection (26).

This vector potential can be used in the design of circuits.

Trace Antisymmetry Law of Lindstrom

This is derived from:

$$\mathrm{Trace}\,(\Gamma_{\mu\nu}) = \Gamma_{00} + \Gamma_{11} + \Gamma_{22} + \Gamma_{33}$$

$$= 0 \quad -(30)$$

The electrostatic field tensor is:

$$F_{\mu\nu E} = \begin{bmatrix} 0 & F_{01} & F_{02} & F_{03} \\ F_{10} & 0 & 0 & 0 \\ F_{20} & 0 & 0 & 0 \\ F_{30} & 0 & 0 & 0 \end{bmatrix} \quad -(31)$$

The magnetostatic field tensor is:

$$F_{\mu\nu B} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & F_{12} & F_{13} \\ 0 & F_{21} & 0 & F_{23} \\ 0 & F_{31} & F_{32} & 0 \end{bmatrix} \quad -(32)$$

So in electrostatics only the scalar antisymmetry law applies:

$$F_{0i} = -F_{i0} \qquad -(33)$$

$$i = 1, 2, 3$$

In magnetostatics only the vector antisymmetry law applies:

$$\left.\begin{aligned} F_{12} &= -F_{21} \\ F_{13} &= -F_{31} \\ F_{23} &= -F_{32} \end{aligned}\right\} \qquad -(34)$$

The field tensor of magnetostatics can be considered to be:

$$F_{ij} = \begin{bmatrix} 0 & F_{12} & F_{13} \\ F_{21} & 0 & F_{23} \\ F_{31} & F_{32} & 0 \end{bmatrix} \qquad -(35)$$

and the relevant trace for magnetostatics is:

$$\text{Trace } \Gamma_{ij} = \Gamma_{11} + \Gamma_{22} + \Gamma_{33} \qquad -(36)$$

i.e.

$$-\underline{\nabla} \cdot \underline{A} + \underline{\omega} \cdot \underline{A} = 0 \qquad -(37)$$

which gives the constraint:

$$\boxed{\underline{\nabla} \cdot \underline{A} = \underline{\omega} \cdot \underline{A}} \qquad -(38)$$

Knowing $\underline{\omega}$ and $\underline{A}$, the divergence $\underline{\nabla} \cdot \underline{A}$ can be found. In the standard model, various gauge assumptions are used to find $\underline{\nabla} \cdot \underline{A}$, an arbitrary procedure.

8) As in Note 358(4), the trace antisymmetry law is developed from the tetrad postulate:

$$\Gamma^a_{\mu\nu} = \partial_\mu q^a_\nu + \omega^a_{\mu b} q^b_\nu . \quad -(39)$$

By antisymmetry: $$\Gamma^a_{\mu\nu} = - \Gamma^a_{\nu\mu} . \quad -(40)$$

The antisymmetry implies that:

$$\text{Trace } \Gamma_{\mu\nu} = \Gamma_{00} + \Gamma_{11} + \Gamma_{22} + \Gamma_{33} = 0 \quad -(41)$$

i.e. $$\frac{1}{c^2}\left(\frac{\partial \phi}{\partial t} + \omega_0 \phi\right) - \underline{\nabla} \cdot \underline{A} + \underline{\omega} \cdot \underline{A} = 0 \quad -(41)$$

in electromagnetic theory. In magnetostatics:

$$\underline{\nabla} \cdot \underline{A} = \underline{\omega} \cdot \underline{A} \quad -(42)$$

and this result is equivalent to the absence of ϕ in magnetostatics.

Therefore in electrostatics:

$$\frac{\partial \phi}{\partial t} + \omega_0 \phi = 0 \quad -(43)$$

It follows that: $$\frac{\partial \phi_t}{\partial t} + \omega_0 \phi_t = 0 \quad -(44)$$

and ω_0 can be found. Note that the jitterbugging scalar potential of Eq. (1) is time dependent because $\delta(\underline{x} - \underline{x}')$ <u>fluctuates</u>. Similarly:

$$\underline{\nabla} \cdot \underline{A}_t = \underline{\omega} \cdot \underline{A}_t \quad -(45)$$

and $\underline{A}_t$ fluctuates in space.