10(1): The Newtonian and Coulombic Limits of ECE2 Physics

The Newtonian limit is defined by:

$$\Phi = -\frac{MG}{r} \quad -(1)$$

which is the gravitational potential. Here M is the mass of the attracting object, G is Newton's constant and r the distance between M and an object m in orbit. Now apply the standard procedure of UFT 389.

The spin connection is found from:

$$\frac{\partial^2 \Phi}{\partial t^2} = \underline{\nabla} \cdot (\underline{\omega}\, \Phi) = 0 \quad -(2)$$

i.e.

$$\Phi\, \underline{\nabla} \cdot \underline{\omega} + \underline{\omega} \cdot \underline{\nabla} \Phi = 0. \quad -(3)$$

In the Newtonian limit:

$$\underline{g} = -\underline{\nabla} \Phi = -\frac{MG}{r^2} \underline{e}_r \quad -(4)$$

so

$$\underline{\omega} \cdot \underline{g} = \Phi\, \underline{\nabla} \cdot \underline{\omega} \quad -(5)$$

and

$$\underline{\nabla} \cdot \underline{\omega} = \frac{\underline{\omega} \cdot \underline{r}}{r^2} \quad -(6)$$

From eqs. (4) and (6):

$$\underline{\omega} = \underline{0} \quad -(7)$$

and $\underline{g}$ is given by eq. (4).

For a planar orbit the vector antisymmetry law reduces to:

$$\frac{\partial Q_y}{\partial X} + \frac{\partial Q_x}{\partial Y} = 0, \quad -(8)$$

where we have used eq. (7). A solution of Eq. (8) is

$$\underline{Q} = \frac{\Omega^{(0)}}{2}\left(-X\underline{i} + Y\underline{j}\right) \quad -(9)$$

) The gravitomagnetic field is:

$$\underline{\Omega} = \underline{\nabla} \times \underline{Q} = \Omega^{(0)}\underline{k} \quad -(10)$$

4) The total potential is:

$$\underline{Q}\,(\text{total}) = \underline{Q} \quad -(11)$$

5) The scalar spin connection is found from:

$$\underline{g} = -\frac{\partial \underline{Q}}{\partial t} - \omega_0 \underline{Q} = -\omega_0 \underline{Q} \quad -(12)$$

In Cartesian coordinates:

$$-\frac{mG\left(X\underline{i} + Y\underline{j}\right)}{\left(X^2+Y^2\right)^{3/2}} = -\frac{\omega_0 \Omega^{(0)}}{2}\left(-Y\underline{i} + X\underline{j}\right) \quad -(13)$$

Therefore

$$\omega_0 \Omega^{(0)} = \frac{2mG}{r^3} \quad -(14)$$

In this equation, the units are as follows:

$$r = m, \quad \omega_0 = s^{-1}, \quad \Omega^{(0)} = s^{-1}, \quad mG = m^3 s^{-2} \quad -(15)$$

) The Lindstrom constraint is:

$$\frac{1}{c^2}\left(\frac{\partial}{\partial t} + \omega_0\right)\underline{\Phi} = \left(\underline{\nabla} - \underline{\omega}\right)\cdot\underline{Q} \quad -(16)$$

i.e. $$\omega_0 \underline{\Phi} = c^2\, \underline{\nabla}\cdot\underline{Q} \quad -(17)$$

) because:
$$\frac{\partial \underline{\Phi}}{\partial t} = 0, \quad \underline{\omega} = \underline{0} \quad - (18)$$

From eq. (9):
$$\underline{\nabla} \cdot \underline{Q} = 0 \quad - (19)$$

but
$$\omega_0 \underline{\Phi} \neq 0. \quad - (20)$$

Therefore in order to satisfy eqn. (17), regauge:
$$\underline{\Phi} \rightarrow \underline{\Phi} + \frac{\partial \phi}{\partial t} \quad - (21)$$
$$\underline{Q} \rightarrow \underline{Q} - \underline{\nabla} \phi \quad - (22)$$

This leaves $\underline{g}$ and $\underline{\Omega}$ unchanged. So:
$$\omega_0 \left(\underline{\Phi} + \frac{\partial \phi}{\partial t} \right) = c^2 \underline{\nabla} \cdot \left(\underline{Q} - \underline{\nabla} \phi \right) = - c^2 \nabla^2 \phi \quad - (23)$$

If it is assumed that ϕ is independent of time:
$$\nabla^2 \phi = - \frac{\omega_0 \underline{\Phi}}{c^2} \quad - (24)$$

and ϕ can be found.

Therefore in the Newtonian limit, $\underline{g}$ is accompanied by a gravitomagnetic field perpendicular to the plane of the orbit. As shown in UFT 119 this is observed in equinoctial precession.

The overall results for the Newtonian limit are:

4)

$$\underline{\Phi} = -\frac{mG}{r} \qquad -(25)$$

$$\underline{g} = -\frac{mG}{r^2}\,\underline{e}_r = -\frac{mG}{r^3}\,\underline{r} \qquad -(26)$$

$$\underline{\omega} = \underline{0} \qquad -(27)$$

$$\underline{q} = \frac{\Omega^{(0)}}{2}\left(-X\,\underline{i} + Y\,\underline{j}\right) \qquad -(28)$$

$$\underline{\Omega} = \Omega^{(0)}\,\underline{k} \qquad -(29)$$

$$\omega_0\,\Omega^{(0)} = \frac{2mG}{r^3} \qquad -(30)$$

The gauge function ϕ is defined by the Lindstrom constraint:

$$\omega_0\left(\underline{\Phi} + \frac{\partial\phi}{\partial t}\right) = -c^2\nabla^2\phi \qquad -(31)$$

As shown in UFT119, $\underline{\Omega}$ is observed in the phenomena of equinoctial precession, with:

$$\underline{\nabla}\times\underline{\Omega} = \frac{4\pi G}{c^2}\,\underline{J} \qquad -(32)$$

From eq. (29):

$$\underline{J} = \underline{0} \qquad -(33)$$

So there is no current of mass density in the Newtonian limit because the source (m) is static. The mass density is defined as usual in Newtonian theory:

$$\square\phi = \underline{\nabla}\cdot\underline{g} = 4\pi G\rho \qquad -(34)$$

The vacuum map is

$$\omega^\mu = \left(\frac{\omega_0}{c}, \underline{0}\right) \qquad -(35)$$

Coulombic Limit

$$\phi = -\frac{e^2}{4\pi\epsilon_0 r} \quad -(36)$$

$$\underline{E} = -\frac{e^2}{4\pi\epsilon_0 r^2}\,\underline{e}_r \quad -(37)$$

$$\underline{\omega} = \underline{0} \quad -(38)$$

$$\underline{A} = \frac{B^{(0)}}{2}\left(-X\underline{i} + Y\underline{j}\right) \quad -(39)$$

$$\underline{B} = B^{(0)}\,\underline{k} \quad -(40)$$

$$\omega_0 B^{(0)} = \frac{2}{r^3}\left(\frac{e^2}{4\pi\epsilon_0}\right) \quad -(41)$$

and the gauge function is defined by

$$\omega_0\left(\phi + \frac{\partial\phi}{\partial t}\right) = -c^2\nabla^2\phi \quad -(42)$$

where

$$\phi \rightarrow \phi + \frac{\partial\phi}{\partial t} \quad -(43)$$

$$A \rightarrow A - \nabla\phi \quad -(44)$$

Assumption of Zero Vector Potential

If it is assumed that $\underline{A}$ and $\underline{Q}$ are zero, then $\underline{g}$ and $\underline{E}$ vanish, because:

$$\underline{g} = -\underline{\nabla}\,\underline{\Phi} + \underline{\omega}\,\underline{\Phi} = -\frac{\partial\underline{Q}}{\partial t} - \omega_0\underline{Q} \quad -(45)$$

$$\underline{E} = -\underline{\nabla}\phi + \underline{\omega}\phi = -\frac{\partial\underline{A}}{\partial t} - \omega_0\underline{A} \quad -(46)$$

However, in electrostatics for example:

$$\underline{E} = -\underline{\nabla}\phi = -\omega_0\underline{A} \quad -(47)$$

6) and $$\omega_0 B^{(0)} = \frac{2}{r^3}\left(\frac{e^2}{4\pi\epsilon_0}\right) \quad -(48)$$

so $\underline{E}$ is determined by the product of ω_0 and $\underline{A}$. The existence of $\underline{B}$ in eq. (40) must be measured experimentally.

Therefore ECE2 physics in the Newtonian and Coulombic limits is a much richer subject than the standard model. ECE2 physics is generally covariant in the Coulombic or Newtonian limit, gives a vector potential, a vacuum, a spin connection, and a gauge function.