390(4): Simultaneous Solution of the ECE2 Wave Equation and Scalar Antisymmetry Law.

This procedure produces eq. (12) of Note 390(3):

$$\frac{1}{c^2}\frac{\partial^2 \underline{\Phi}}{\partial t^2} = \underline{\nabla} \cdot \left(\underline{\omega}\, \underline{\Phi} \right) \quad -(1)$$

This is satisfied in the Newtonian limit:

$$\left.\begin{array}{l} c \rightarrow \infty \\ \underline{\omega} \rightarrow \underline{0} \end{array}\right\} -(2)$$

So

$$\underline{\nabla} \cdot \left(\underline{\omega}\, \underline{\Phi} \right) \rightarrow 0 \quad -(3)$$

This has been verified numerically by co author Horst Eckardt. For retrograde precession in three dimensions:

$$\ddot{X} = - \frac{mG}{\gamma^3 (X^2+Y^2+Z^2)^{3/2}} X \quad -(4)$$

and

$$\ddot{Y} = \frac{-mG}{\gamma^3 (X^2+Y^2+Z^2)^{3/2}} Y \quad -(5)$$

where

$$\gamma = \left(1 - \frac{v^2}{c^2} \right)^{-1/2} \quad -(6)$$

in which

$$\underline{v} = \dot{X}\, \underline{i} + \dot{Y}\, \underline{j} + \dot{Z}\, \underline{k} \quad -(7)$$

The spin connections in 3-D for retrograde precession are:

$$\omega_X = \left(\frac{X}{X^2+Y^2+Z^2} \right) \left(\frac{1}{\gamma^3} - 1 \right) \quad -(8)$$

$$\omega_Y = \left(\frac{Y}{X^2+Y^2+Z^2} \right) \left(\frac{1}{\gamma^3} - 1 \right) \quad -(9)$$

2)
$$\omega_Z = \frac{Z}{(X^2+Y^2+Z^2)}\left(\frac{1}{\gamma^3}-1\right) \quad -(10)$$

In this case, computation shows that:

$$\underline{\nabla}\cdot(\underline{\omega}\,\underline{\Phi}) = 0 \quad -(11)$$

in which:

$$\underline{\Phi} = -\frac{mG}{r} \quad -(12)$$

and

$$r = (X^2+Y^2+Z^2)^{1/2} \quad -(13)$$

It follows from eqs. (1) and (11) that:

$$\frac{1}{c^2}\frac{\partial^2\underline{\Phi}}{\partial t^2} = 0 \quad -(14)$$

for retrograde precession, where

$$\underline{\Phi} = -\frac{mG}{r(t)} \quad -(15)$$

so

$$\frac{\partial\underline{\Phi}}{\partial t} = \frac{\partial\underline{\Phi}}{\partial r}\frac{\partial r}{\partial t} = -\frac{mG}{r^2}\frac{\partial r}{\partial t} \quad -(16)$$

and

$$\frac{\partial^2\underline{\Phi}}{\partial t^2} = -mG\frac{\partial}{\partial t}\left(\frac{1}{r^2}\frac{\partial r}{\partial t}\right) = 0 \quad -(17)$$

The dependence of r on t is determined by:

$$\frac{\partial}{\partial t}\left(\frac{1}{r^2}\frac{\partial r}{\partial t}\right) = 0 \quad -(18)$$

in retrograde precession.

In a static ellipse:

3)
$$\frac{dr}{dt} = \left(\frac{2}{m} \left(E - U \right) - \frac{L^2}{m^2 r^2} \right)^{1/2} \quad -(19)$$

because the hamiltonian is defined by:

$$H = E = \frac{1}{2} m \dot{r}^2 + \frac{1}{2} \frac{L^2}{mr^2} + U(r) \quad -(20)$$

in which

$$U(r) = m\Phi \quad -(21)$$

Here E and L are constants of motion. For retrograde precession, dr/dt is no longer given by eq. (19). However in both cases:

$$\frac{1}{c^2} \frac{\partial^2 \Phi}{\partial t^2} = 0 \quad -(22)$$

In the classical case, this can be expressed with:

$$c \to \infty \quad -(23)$$

for any $\partial^2 \Phi / \partial t^2$. However for retrograde precession:

$$\frac{\partial^2 \Phi}{\partial t^2} = 0 \quad -(24)$$

which leads to eq. (18).

Eq. (18) can be expressed as:

$$\left(\frac{\partial}{\partial t} \left(\frac{1}{r^2} \right) \right) \left(\frac{\partial r}{\partial t} \right) + \frac{1}{r^2} \frac{\partial^2 r}{\partial t^2} = 0 \quad -(25)$$

and integrated numerically to give dr/dt for retrograde precession. Finally this can be compared with Eq. (19) for the static ellipse.

i) Propagating gravitational waves exist at the Newtonian level. This can be shown as follows by considering the Newtonian:

$$\underline{g} = -\underline{\nabla}\,\underline{\Phi} = -\frac{mG}{r^3}\,\underline{r} = -\frac{mG}{r^2}\,\underline{e}_r \quad -(26)$$

Here

$$\underline{r} = r\,\underline{e}_r \quad -(27)$$

It follows that:

$$-\nabla^2 \underline{\Phi} = -mG\,\underline{\nabla}\cdot\left(\frac{1}{r^2}\underline{e}_r\right)$$

$$= -mG\frac{\partial}{\partial r}\left(\frac{1}{r^2}\right) = \frac{2mG}{r^3} \quad -(28)$$

In Newtonian theory:

$$\frac{dr}{dt} = \frac{dr}{d\phi}\frac{d\phi}{dt} = \frac{L}{mr^2}\frac{dr}{d\phi} \quad -(29)$$

The orbit is:

$$r = \frac{\alpha}{1+\epsilon\cos\phi} \quad -(30)$$

so:

$$\frac{d\phi}{dt} = \frac{L}{mr^2} \quad -(31)$$

and

$$\frac{dt}{d\phi} = \frac{m}{L\alpha^2}\left(1+\epsilon\cos\phi\right)^2 \quad -(32)$$

This can be integrated to give $\phi(t)$ and

$$r(t) = \frac{\alpha}{1+\epsilon\cos\phi(t)} \quad -(33)$$

so

$$\underline{\Phi} = -\frac{mG}{r(t)} \quad -(34)$$

and $\dfrac{\partial^2 \underline{\Phi}}{\partial t^2}$ can be found

) The Newtonian wave equation is:

$$\Box \Phi = 4\pi G \rho_m \quad -(35)$$

where the d'Alembertian is:

$$\Box = \frac{1}{v^2}\frac{\partial^2}{\partial t^2} - \nabla^2 \quad -(36)$$

in which the orbital v is:

$$v^2 = MG\left(\frac{2}{r} - \frac{1}{a}\right) \quad -(37)$$

So in Newtonian orbital theory a particle m in orbit around M emits propagating gravitational radiation from eq. (35).