1)

THE ORIGIN OF INTRINSIC SPIN IN PHYSICS: THE TETRAD IN ELECTRODYNAMICS, THE DIRAC EQUATION AND STRONG FORCE THEORY.

In these notes for paper 38 of the unified field theory the tetrad is shown to be the wavefunction for electromagnetism, the Dirac equation, and strong force theory. The origin of intrinsic spin in physics is discussed in terms of the tetrad, and it is shown that there exists a basis set of elements in the tangent spacetime at a point P in the base manifold, a basis set which defines the existence of intrinsic spin. In electrodynamics this basis set defines left and right circular polarization and the intrinsic spin field of generally covariant electrodynamics. In the Dirac equation the intrinsic right and left spin of a fermion is defined by the basis set, and in strong field theory the quark colour (R, W and B) is defined by the basis set. In each case the wavefunction is the tetrad q^a_μ, and the index a is the tangent spacetime label of the elements of the basis set.

These findings illustrate the fact that the tetrad is the fundamental field in all physics.

Electrodynamics

The existence of intrinsic spin in electrodynamics was discovered experimentally by Arago in 1811 and is now known to be left and right circular

2) polarization. The vector potentials for left and right circular polarization are:

$$\underline{A}_R^{(1)} = \frac{A^{(0)}}{\sqrt{2}} (\underline{i} - i\underline{j}) e^{i\phi} \quad -(1)$$

$$\underline{A}_L^{(1)} = \frac{A^{(0)}}{\sqrt{2}} (\underline{i} + i\underline{j}) e^{i\phi} \quad -(2)$$

where ϕ is the electromagnetic phase and where the (1) superscript denotes complex conjugate:

$$\underline{A}_R^{(1)} = \underline{A}_R^{(2)*} \quad -(3)$$

$$\underline{A}_L^{(1)} = \underline{A}_L^{(2)*}. \quad -(4)$$

For our present purpose we may simplify the argument by writing:

$$\underline{A}^R = \frac{A^{(0)}}{\sqrt{2}} (\underline{i} - i\underline{j}) e^{i\phi} \quad -(5)$$

$$\underline{A}^L = \frac{A^{(0)}}{\sqrt{2}} (\underline{i} + i\underline{j}) e^{i\phi}. \quad -(6)$$

The basis vectors of the complex circular basis are defined by:

$$\underline{e}^{(1)} = \frac{1}{\sqrt{2}} (\underline{i} - i\underline{j}) \quad -(7)$$

$$\underline{e}^{(2)} = \frac{1}{\sqrt{2}} (\underline{i} + i\underline{j}) \quad -(8)$$

$$\underline{e}^{(3)} = \underline{k}, \quad -(9)$$

where $\underline{i}$, $\underline{j}$ and $\underline{k}$ are Cartesian unit vectors.

Therefore:

$$\underline{A}^R = A^{(0)} e^{i\phi} \underline{e}^{(1)} \quad -(10)$$

$$\underline{A}^L = A^{(0)} e^{i\phi} \underline{e}^{(2)} . \quad -(11)$$

It follows that the right and left basis vectors may be defined as:

$$\underline{e}^R = e^{i\phi} \underline{e}^{(1)} \quad -(12)$$

$$\underline{e}^L = e^{i\phi} \underline{e}^{(2)} . \quad -(13)$$

Within the phase factor $e^{i\phi}$ these are the $\underline{e}^{(1)}$ and $\underline{e}^{(2)}$ basis vectors of the complex circular basis. The components of the right and left basis vectors define a tetrad matrix:

$$q^a_\mu = \begin{bmatrix} e^R_X & e^R_Y \\ e^L_X & e^L_Y \end{bmatrix} \quad -(14)$$

where:

$$e^R_X = \frac{e^{i\phi}}{\sqrt{2}}, \quad e^R_Y = -\frac{i e^{i\phi}}{\sqrt{2}},$$

$$e^L_X = \frac{e^{i\phi}}{\sqrt{2}}, \quad e^L_Y = \frac{i e^{i\phi}}{\sqrt{2}} .$$

The tetrad in eq. (14) obeys the Evans wave equation in the limit of zero photon mass:

$$kT = \left(\frac{mc}{\hbar}\right)^2 \rightarrow 0, \quad -(15)$$

4) so that:

$$\square q^a_\mu = 0. \quad -(16)$$

With the Evans Ansatz:

$$A^a_\mu = A^{(0)} q^a_\mu \quad -(17)$$

eqn. (16) is the d'Alembert wave equation in free space:

$$\square A^a_\mu = 0. \quad -(18)$$

The tetrad q^a_μ is always defined by:

$$V^a = q^a_\mu V^\mu \quad -(19)$$

where V^a is a vector in the tangent spacetime and V^μ is a vector in the base manifold.

Define:

$$V^\mu = \begin{bmatrix} e_X \\ e_Y \end{bmatrix} = \begin{bmatrix} e^{-i\phi} \\ e^{-i\phi} \end{bmatrix} \quad -(20)$$

and

$$V^a = \begin{bmatrix} e^R \\ e^L \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1-i \\ 1+i \end{bmatrix} \quad -(21)$$

and it follows from eqns. (14) and (19) – (21) that:

5)

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1-i \\ 1+i \end{bmatrix} = \frac{e^{i\phi}}{\sqrt{2}}\begin{bmatrix} 1 & -i \\ 1 & i \end{bmatrix}\begin{bmatrix} e^{-i\phi} \\ e^{-i\phi} \end{bmatrix} \quad -(18)$$

i.e. $$V^a = q^a_\mu V^\mu \quad -(19)$$

<u>Q.E.D.</u>

<u>The Intrinsic Spin Basis Set</u>

From eqn (16) it is seen that the basis set for the intrinsic spin of electromagnetism is:

$$\left.\begin{aligned} \underline{e}^{(1)} \times \underline{e}^{(2)} &= i\underline{e}^{(3)*} \\ \underline{e}^{(2)} \times \underline{e}^{(3)} &= i\underline{e}^{(1)*} \\ \underline{e}^{(3)} \times \underline{e}^{(1)} &= i\underline{e}^{(2)*} \end{aligned}\right\} \quad -(20)$$

i.e. the basis set made up of the complex circular unit vectors. Eqn. (20) has O(3) symmetry.

This reasoning may now be extended to the whole of physics (i.e. to all radiated and matter fields).

6)

FERMIONIC MATTER FIELDS AND DIRAC EQUATION.

The tetrad field for the Dirac equation is:

$$q^a_\mu = \begin{bmatrix} q^R_1 & q^R_2 \\ q^L_1 & q^L_2 \end{bmatrix} \quad -(21)$$

where the Pauli spinors are defined by:

$$\xi^R = \begin{bmatrix} q^R_1 \\ q^R_2 \end{bmatrix}, \quad \xi^L = \begin{bmatrix} q^L_1 \\ q^L_2 \end{bmatrix}. \quad -(22)$$

the tetrad field is defined by:

$$V^a = q^a_\mu V^\mu \quad -(23)$$

where:

$$V^a = \begin{bmatrix} e^R \\ e^L \end{bmatrix}, \quad V^\mu = \begin{bmatrix} e^1 \\ e^2 \end{bmatrix}. \quad -(24)$$

The column vector V^μ is a two-dimensional column vector in the base manifold and transforms under SU(2) symmetry. Similarly the column vector V^a is a two-dimensional column vector in the tangent spacetime.

The tetrad field q^a_μ defined by eqn. (21) obeys the Evans wave equation:

$$(\Box + kT)\, q^a_\mu = 0 \quad -(25)$$

7) The <u>Dirac equation</u> is recovered in the limit

$$kT \to (mc/\hbar)^2, \quad -(26)$$

$$T \to \frac{m}{V}. \quad -(26a)$$

Here m is the mass of the fermion, $\hbar$ is the reduced Planck constant, c is the velocity of light, V is <u>the rest volume of the fermion</u>:

$$V = \frac{\hbar^2 k}{mc^2}. \quad -(27)$$

In the limit (26) <u>the Dirac spinor</u> is defined by:

$$\psi = \begin{bmatrix} q_1^R \\ q_2^R \\ q_1^L \\ q_2^L \end{bmatrix}, \quad -(28)$$

and the Dirac equation is:

$$\left(\Box + \left(\frac{mc}{\hbar}\right)^2\right)\psi = 0. \quad -(29)$$

This is a free-particle equation, in that no gravitational attraction exists between fermions in eqn. (29). To describe ~~this~~ gravitational attraction we need the Evans wave equation (25).

STRONG FIELD THEORY

In strong field theory there are thought to exist six quark flavours and three quark colours. If we accept this view uncritically the Evans unified field theory can be applied to the n-quark model, where $n = 2, \ldots, 6$. These models have $SU(n)$ symmetry.

Two-Quark Model ($n = 2$, $SU(2)$).

In this case:

$$V^{\mu} = \begin{bmatrix} u \\ d \end{bmatrix} = \begin{bmatrix} e^1 \\ e^2 \end{bmatrix} \quad - (30)$$

and

$$V^{a} = \begin{bmatrix} e^R \\ e^W \\ e^B \end{bmatrix} . \quad - (31)$$

The V^a colour spinor is a 3-spinor, and the colours R, W and B play a role analagous to intrinsic spin. In Dirac theory the intrinsic spin is half-integral spin, and eqn (23) defines the Pauli exclusion principle in general relativity. From eqns (30) and (31):

$$\begin{bmatrix} e^R \\ e^W \\ e^B \end{bmatrix} = \begin{bmatrix} q_1^R & q_2^R \\ q_1^W & q_2^W \\ q_1^B & q_2^B \end{bmatrix} \begin{bmatrix} e^1 \\ e^2 \end{bmatrix} \quad -(32)$$

which is an example of:

$$V^a = q^a_\mu V^\mu. \quad -(33)$$

Therefore <u>the flavour-colour tetrad</u> for the two-quark model is:

$$q^a_\mu = \begin{bmatrix} q_1^R & q_2^R \\ q_1^W & q_2^W \\ q_1^B & q_2^B \end{bmatrix} \quad -(34)$$

and obeys the Evans wave equation:

$$(\square + kT)\, q^a_\mu = 0 \quad -(35)$$

where q^a_μ is the <u>strong matter field</u>

<u>Three-Quark Model ($n = 3$, $SU(3)$)</u>

In this case:

$$V^\mu = \begin{bmatrix} u \\ d \\ s \end{bmatrix} = \begin{bmatrix} e^1 \\ e^2 \\ e^3 \end{bmatrix} \quad -(36)$$

and:

and :

$$\begin{bmatrix} e^R \\ e^W \\ e^B \end{bmatrix} = \begin{bmatrix} q_1^R & q_2^R & q_3^R \\ q_1^W & q_2^W & q_3^W \\ q_1^B & q_2^B & q_3^B \end{bmatrix} \begin{bmatrix} e^1 \\ e^2 \\ e^3 \end{bmatrix} . \quad -(37)$$

The strong matter field for ~~the three~~-quark model is therefore :

$$q_\mu^a = \begin{bmatrix} q_1^R & q_2^R & q_3^R \\ q_1^W & q_2^W & q_3^W \\ q_1^B & q_2^B & q_3^B \end{bmatrix} \quad -(38)$$

and is governed by eqns (35) and (38).

<u>Four-Quark Model (n = 4, su(4))</u>

In this case

$$\begin{bmatrix} e^R \\ e^W \\ e^B \end{bmatrix} = \begin{bmatrix} q_1^R & q_2^R & q_3^R & q_4^R \\ q_1^W & q_2^W & q_3^W & q_4^W \\ q_1^B & q_2^B & q_3^B & q_4^B \end{bmatrix} \begin{bmatrix} e^1 \\ e^2 \\ e^3 \\ e^4 \end{bmatrix} \quad -(39)$$

and so on up to the six-quark model (n = 6, su(6)).