39(2): Scheme of Computation for ECE2 Gravitation

First note that the homogeneous field equations are:

$$\underline{\nabla} \cdot \underline{\Omega} = 0 \quad - (1)$$

and

$$\underline{\nabla} \times \underline{g} + \frac{\partial \underline{\Omega}}{\partial t} = \underline{0} \quad - (2)$$

where

$$\underline{\Omega} = \underline{\nabla} \times \underline{Q} - \underline{\omega} \times \underline{Q} \quad - (3)$$

Eqs (1) and (3) imply:

$$\underline{\nabla} \cdot \underline{\omega} \times \underline{Q} = 0 \quad - (4)$$

Now define the vector potential $\underline{Q}_1$ by:

$$\underline{\nabla} \times \underline{Q}_1 := - \underline{\omega} \times \underline{Q} \quad - (5)$$

and eq. (4) follows from:

$$\underline{\nabla} \cdot \underline{\nabla} \times \underline{Q}_1 = 0 \quad - (6)$$

Q.E.D. It follows that:

$$\underline{\Omega} = \underline{\nabla} \cdot (\underline{Q} + \underline{Q}_1). \quad - (7)$$

Define:

$$\underline{Q}\,(\text{total}) = \underline{Q} + \underline{Q}_1, \quad - (8)$$

then

$$\underline{\Omega} = \underline{\nabla} \times \underline{Q}\,(\text{total}) \quad - (9)$$

Now define:

$$\underline{g} = - \underline{\nabla}\, \underline{\Phi} + \underline{\omega}\, \underline{\Phi} = - \underline{\nabla}\, \underline{\Phi} - \frac{\partial \underline{Q}\,(\text{total})}{\partial t} \quad - (10)$$

and eqns. (1) and (2) follow from eqs. (9) and (10),

Q.E.D.

) It also follows that:

$$\underline{g} = -\underline{\nabla}\Phi - \frac{\partial \underline{Q}}{\partial t}(\text{total}) = -\frac{\partial \underline{Q}}{\partial t} - \omega_0 \underline{Q} \quad -(11)$$

So

$$-\frac{\partial \underline{Q}}{\partial t}(\text{total}) = \underline{g} + \underline{\nabla}\Phi \quad -(12)$$

where

$$\underline{g} = -\frac{\partial \underline{Q}}{\partial t} - \omega_0 \underline{Q} \quad -(13)$$

In the standard model:

$$\underline{g} = -\underline{\nabla}\Phi \quad -(14)$$

so

$$-\frac{\partial \underline{Q}}{\partial t}(\text{total}) = \underline{0} \quad -(15)$$

in the standard model.

<u>A Suggested Procedure</u>

1) Find $\underline{g}$ by Lagrangian methods as in recent UFT papers. The Newtonian $\underline{g}$ is:

$$\underline{g}\,(\text{Newton}) = -\frac{mG}{r^2}\underline{e}_r \quad (16)$$

$$= -\frac{mG}{r^3}\underline{r}$$

This produces an elliptical orbit, a conic section orbit in general.

2) Find the mass density ρ_m from:

$$\underline{\nabla}\cdot\underline{g} = 4\pi G \rho_m \quad -(17)$$

3) Find the current density from:

$$\frac{\partial \rho_m}{\partial t} + \underline{\nabla} \cdot \underline{J}_m = 0 \quad -(18)$$

4) The current density can also be found from:

$$\underline{\nabla} \times \underline{\Omega} - \frac{1}{c^2} \frac{\partial \underline{g}}{\partial t} = \frac{4\pi G}{c^2} \underline{J}_m \quad -(19)$$

where $\underline{\Omega}$ is found from:

$$\underline{\nabla} \times \underline{g} + \frac{\partial \underline{\Omega}}{\partial t} = \underline{0} \quad -(20)$$

5) Find Φ from:

$$\underline{\nabla} \cdot \underline{g} = \Box \, \Phi = 4\pi G \rho_m \quad -(21)$$

6) Find $\underline{Q}$ from:

$$\Box \, \underline{Q} = (4\pi G / c^2) \underline{J}_m \quad -(22)$$

$$= \underline{\nabla} \times \underline{\Omega} - \frac{1}{c^2} \frac{\partial \underline{g}}{\partial t}$$

Graph the scalar and vector potentials.

7) Find $\underline{\omega}$ from eqs. (15) to (17) of Note 389(1). Graph the vector spin connection $\underline{\omega}$. This is a map of the vacuum.

8) Find $\underline{\Omega}$ from:

$$\underline{\Omega} = \underline{\nabla} \times \underline{Q} - \underline{\omega} \times \underline{Q} \quad -(23)$$

Graph the magnetogravitational field $\underline{\Omega}$.

9) Graph:

$$\underline{\Omega}\,(\text{interaction with vacuum}) = -\underline{\omega}\times\underline{Q} \quad -(24)$$

10) Find ω_0 from the Lindstrom constraint:

$$\frac{1}{c^2}\left(\frac{\partial}{\partial t}+\omega_0\right)\Phi = (\underline{\nabla}-\underline{\omega})\cdot\underline{Q} \quad -(25)$$

and graph ω_0.

11) Find $\partial\underline{Q}/\partial t$ from:

$$-\frac{\partial\underline{Q}}{\partial t} = \underline{g} + \omega_0\underline{Q} \quad -(26)$$

and graph $\partial\underline{Q}/\partial t$.

12) Find $\partial\underline{Q}(\text{total})/\partial t$ from:

$$-\frac{\partial\underline{Q}(\text{total})}{\partial t} = \underline{g} + \underline{\nabla}\Phi \quad -(27)$$

and graph $\partial\underline{Q}(\text{total})/\partial t$.

Special Case of Newtonian $\underline{g}$

In this case:

$$\underline{\nabla}\times\underline{g} = \underline{0} \quad -(28)$$

so

$$\frac{\partial\underline{\Omega}}{\partial t} = \underline{0}. \quad -(29)$$

Also in Newtonian gravitation, the central mass M is static so

$$\underline{J}_m = \underline{0} \quad -(30)$$

Therefore:

$$\Box \underline{Q} = 0 \quad -(31)$$

At the ECE2 level there is a vector potential and a spin connection $\underline{\omega}$, in the Newtonian level neither exists. Eq. (31) is a d'Alembert equation with well known solutions for $\underline{Q}$. Knowing these solutions, the spin connection vector $\underline{\omega}$ is found from eqs. (15) to (17) of Note 389(1).

Graph the "Newtonian" $\underline{Q}$ and $\underline{\omega}$.

Find $\underline{\Omega}$ in the Newtonian limit from:

$$\underline{\Omega} = \underline{\nabla} \times \underline{Q} - \underline{\omega} \times \underline{Q} \quad -(32)$$

From eq. (20):

$$\frac{\partial \underline{\Omega}}{\partial t} = \underline{0} \quad -(33)$$

if

$$\underline{\nabla} \times \underline{g} = \underline{0} \quad -(34)$$

so $\underline{Q}$ and $\underline{\omega}$ in the Newtonian limit must be independent of time, and:

$$\Box = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 \longrightarrow -\nabla^2 \quad -(35)$$

so eq. (31) reduces to:

$$\nabla^2 \underline{Q} = \underline{0}, \quad -(36)$$

whose solutions are well known.

In the Newtonian limit, eq. (21) reduces to the Poisson equation:

$$\nabla^2 \underline{\Phi} = -4\pi G \rho_m \quad -(37)$$

. the Lindstrom constraint equation (25) reduces to:

$$\omega_0 \underline{\Phi} = (\underline{\nabla} - \underline{\omega}) \cdot \underline{Q} \quad -(38)$$

with:

$$\underline{g} = -\omega_0 \underline{Q} \quad -(39)$$

and

$$\nabla^2 \underline{Q} = \underline{0} \quad -(40)$$

Therefore $\underline{Q}$ is found from eq. (40), $\underline{g}$ is known experimentally, and ω_0 can be found from eq. (39). This can be graphed and is another map of the vacuum.

Finally the covariant derivative of $\underline{Q}$ is found from eq. (38):

$$(\underline{\nabla} - \underline{\omega}) \cdot \underline{Q} = \omega_0 \underline{\Phi} \quad -(41)$$

so

$$\underline{\nabla} \cdot \underline{Q} = \omega_0 \underline{\Phi} + \underline{\omega} \cdot \underline{Q} \quad -(42)$$