Note 389(7): Solution for ω_0 and ϕ

The problem is to solve:

$$\frac{1}{c^2}\left(\frac{\partial}{\partial t}+\omega_0\right)\phi = (\underline{\nabla}-\underline{\omega})\cdot\underline{A} \quad -(1)$$

and

$$\underline{E} = -\underline{\nabla}\phi + \underline{\omega}\phi = -\frac{\partial \underline{A}}{\partial t} - \omega_0\underline{A} \quad -(2)$$

simultaneously for ω_0 and ϕ, given that $\underline{\omega}$ and $\underline{A}$ have been determined already. From eq. (2):

$$E = |\underline{E}| = \left(E_X^2+E_Y^2+E_Z^2\right)^{1/2} \quad -(3)$$

$$= \left[\left(-\frac{\partial A_X}{\partial t}-\omega_0 A_X\right)^2+\left(-\frac{\partial A_Y}{\partial t}-\omega_0 A_Y\right)^2+\left(-\frac{\partial A_Z}{\partial t}-\omega_0 A_Z\right)^2\right]^{1/2}$$

from which ω_0 can be found by computer algebra.

From eq. (1):

$$\omega_0 = \frac{1}{\phi}\left[c^2(\underline{\nabla}-\underline{\omega})\cdot\underline{A} - \frac{\partial\phi}{\partial t}\right] \quad -(4)$$

Eqs. (3) and (4) must give the same result for ω_0. The electric field strength magnitude E is observable experimentally, so substituting eq. (4) into eq. (3) gives an expression for ϕ and $\partial\phi/\partial t$. This is

$$\underline{E} = -\underline{\nabla}\phi+\underline{\omega}\phi = -\frac{\partial\underline{A}}{\partial t} - \frac{1}{\phi}\left(c^2(\underline{\nabla}-\underline{\omega})\cdot\underline{A}-\frac{\partial\phi}{\partial t}\right)\underline{A} \quad -(5)$$

It is seen that there is a new relation between ϕ and $\underline{A}$.

From the equation: $\Box A^\mu = \mu_0 J^\mu$ — (6)

it is found that: $\underline{\nabla} \cdot \underline{E} = \Box \phi = \rho / \epsilon_0$ — (7)

and $\underline{\nabla} \times \underline{B} - \frac{1}{c^2} \frac{\partial \underline{E}}{\partial t} = \Box \underline{A} = \mu_0 \underline{J}$ — (8)

By measuring $\underline{E}$ and $\underline{B}$ experimentally, ϕ and $\underline{A}$ can be found experimentally. They can also be found experimentally by measuring ρ and $\underline{J}$.

The spin connection vector is found from the vector antisymmetry equations:

$$\frac{\partial A_Z}{\partial Y} + \frac{\partial A_Y}{\partial Z} = \omega_Y A_Z + \omega_Z A_Y \quad - (9)$$

$$\frac{\partial A_X}{\partial Z} + \frac{\partial A_Z}{\partial X} = \omega_Z A_X + \omega_X A_Z \quad - (10)$$

$$\frac{\partial A_Y}{\partial X} + \frac{\partial A_X}{\partial Y} = \omega_X A_Y + \omega_Y A_X \quad - (11)$$

Therefore ω_0 can be computed from eq. (4), and the experimental $\underline{\omega}$ from eqs. (9) to (11) given ϕ and $\underline{A}$. The complete vacuum map is:

$$\omega^\mu = \left(\frac{\omega_0}{c}, \underline{\omega} \right) \quad - (12)$$

The homogeneous field equations must also be obeyed:

$$\underline{\nabla} \cdot \underline{B} = 0 \quad -(13)$$

$$\underline{\nabla} \times \underline{E} + \frac{\partial \underline{B}}{\partial t} = \underline{0} \quad -(14)$$

where

$$\underline{E} = -\underline{\nabla}\phi + \underline{\omega}\phi = -\frac{\partial \underline{A}}{\partial t} - \omega_0 \underline{A} \quad -(15)$$

and

$$\underline{B} = \underline{\nabla} \times \underline{A} - \underline{\omega} \times \underline{A} \quad -(16)$$

This is achieved by defining:

$$\underline{\nabla} \times \underline{A}_1 := -\underline{\omega} \times \underline{A} \quad -(17)$$

so

$$\underline{B} = \underline{\nabla} \times \underline{A}(\text{total}) \quad -(18)$$

where

$$\underline{A}(\text{total}) = \underline{A} + \underline{A}_1 \quad -(19)$$

and by defining:

$$\underline{E} := -\underline{\nabla}\phi - \frac{\partial \underline{A}(\text{total})}{\partial t} \quad -(20)$$

so

$$-\frac{\partial \underline{A}(\text{total})}{\partial t} := \underline{\omega}\phi \quad -(21)$$

Therefore:

$$A^\mu(\text{total}) = \left(\frac{\phi}{c}, \underline{A}(\text{total}) \right) \quad -(22)$$

The potentials $\underline{A}_1$ and $-\frac{\partial \underline{A}}{\partial t}(\text{total})$ map the interaction with the vacuum.

In eq. (5), ϕ, $\underline{A}$ and $\underline{\omega}$ are known, these quantities must obey eq. (5). This is a law of electrodynamics derived from the anti symmetry constraints (1) and (2). The complete set of antisymmetry laws are as follows:

the trace antisymmetry law or Lindstrom constraint, eq. (1)

the scalar antisymmetry constraint, eq. (2)

the vector antisymmetry constraints, eqs. (9) to (10).

Together, they make up the law of conservation of antisymmetry, a new law of physics.

They are based on the tetrad postulate of Cartan geometry and the antisymmetry of a two-form, discuss the torsion two-form.

The spin connection four-vector (12) and the potential four vector

$$A^{\mu} = \left(\frac{\phi}{c}, \underline{A} \right) \quad -(23)$$

must obey eqs. (4) and (5). There is some freedom of choice of ϕ and $\underline{A}$ is determined. This is because $\underline{E}$ and $\underline{B}$ are invariant under the gauge transformation:

$$A^{\mu}(\text{total}) \rightarrow A^{\mu}(\text{total}) + \partial^{\mu} \phi. \quad -(24)$$

i.e.

$$\phi \rightarrow \phi + \frac{\partial \phi}{\partial t} \quad -(25)$$

and

$$\underline{A}_{\text{total}} \rightarrow \underline{A}_{\text{total}} - \underline{\nabla} \phi \quad -(26)$$

Therefore a function $\partial \phi / \partial t$ can always be added to the experimentally determined ϕ and a function $\underline{\nabla} \phi$ can always be subtracted from the experimentally determined $\underline{A}_{\text{total}}$. This leaves $\underline{E}$ and $\underline{B}$ unchanged.

Therefore, having determined $\underline{E}$ and $\underline{A}$ experimentally, and having found $\underline{\omega}$ from the vector anti-symmetry laws, Eq. (5), it can be regarded as a differential equation for ϕ:

$$\phi\left(\underline{E} + \frac{\partial \underline{A}}{\partial t}\right) = -\left(c^2\left(\underline{\nabla} - \underline{\omega}\right)\cdot\underline{A} - \frac{\partial \phi}{\partial t}\right)\underline{A} \quad -(27)$$

i.e.

$$\boxed{\underline{A}\,\frac{\partial \phi}{\partial t} + \left(\underline{E} + \frac{\partial \underline{A}}{\partial t}\right)\phi = \left(c^2\left(\underline{\omega} - \underline{\nabla}\right)\cdot\underline{A}\right)\underline{A}} \quad -(28)$$

If the ϕ determined from eq. (28) is different from the experimentally measured ϕ, then it is regarded as the gauge shifted ϕ:

$$\phi = \phi\,(\mathrm{exp}) + \frac{\partial \psi}{\partial t} \quad -(29)$$

Therefore the gauge function $\partial\psi / \partial t$ can be found.