387(1): Calculation of the Vacuum Charge from the ECE2 Theory

In general, the electric field strength in volts per metre is calculated from:

$$\underline{E} = -\underline{\nabla}\phi + \underline{\omega}\phi = -\frac{\partial \underline{A}}{\partial t} - \omega_0 \underline{A} \quad -(1)$$

and the magnetic flux density is calculated from:

$$\underline{B} = \underline{\nabla} \times \underline{A} - \underline{\omega} \times \underline{A} \quad -(2)$$

Electrostatics is defined by:

$$\underline{\nabla} \times \underline{E} = \underline{0} \quad -(3)$$

so the Faraday law of induction implies that:

$$\frac{\partial \underline{B}}{\partial t} = \underline{0} \quad -(4)$$

and electrostatics does not imply the absence of $\underline{B}$, it implies that the latter must be time independent.

If it is assumed that electrostatics implies:

$$\frac{\partial \underline{A}}{\partial t} = \underline{0} \quad -(5)$$

then

$$\underline{E} = -\omega_0 \underline{A}. \quad -(6)$$

In general the electromagnetic field tensor is:

$$F_{\mu\nu} = D_\mu A_\nu - D_\nu A_\mu \quad -(7)$$

where

$$D_\mu = \partial_\mu + \omega_\mu \quad -(8)$$

is the covariant derivative, and where:

$$F_{\mu\nu} = -F_{\nu\mu} \quad -(9)$$

so the fundamental antisymmetry law is:

2)

$$D_\mu A_\nu = - D_\nu A_\mu \quad - (10)$$

i.e.

$$(\partial_\mu + \omega_\mu) A_\nu = -(\partial_\nu + \omega_\nu) A_\mu \quad - (11)$$

Eq. (11) transfers into eqs. (1) and

$$\frac{\partial A_Z}{\partial Y} + \frac{\partial A_Y}{\partial Z} = \omega_Y A_Z + \omega_Z A_Y \quad - (12)$$

$$\frac{\partial A_X}{\partial Z} + \frac{\partial A_Z}{\partial X} = \omega_Z A_X + \omega_X A_Z \quad - (13)$$

$$\frac{\partial A_Y}{\partial X} + \frac{\partial A_X}{\partial Y} = \omega_X A_Y + \omega_Y A_X \quad - (14)$$

From eqs. (3) and (6)

$$\underline{\nabla} \times (\omega_0 \underline{A}) = \underline{0} \quad - (15)$$

From the Coulomb laws:

$$\underline{\nabla} \cdot \underline{E} = \rho / \epsilon_0 \quad - (16)$$

$$\underline{\nabla} \cdot (\omega_0 \underline{A}) = - \rho / \epsilon_0 \quad - (17)$$

Now define the vacuum scalar potential by:

$$- \underline{\nabla} \phi(\text{vac}) = \underline{\omega} \phi. \quad - (18)$$

This is analogous to:

$$\underline{\nabla} \times \underline{\alpha}(\text{vac}) = \underline{\omega} \times \underline{A} \quad - (19)$$

in UFT386.

So the vacuum electric field strength is:

$$\underline{E}(\text{vac}) = - \underline{\nabla} \phi(\text{vac}) = \underline{\omega} \phi \quad - (20)$$

so the spin connection $\underline{\omega}$ is the intermediary between the material scalar potential ϕ and the vacuum electric field strength $\underline{E}$ (vac). Eq. (20) is analogous to

$$\underline{B}\,(\text{vac}) = \underline{\omega} \times \underline{A} \quad - (21)$$

in UFT386.

We have:

$$\phi = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\underline{x}')}{|\underline{x}-\underline{x}'|}\, d^3x' \quad - (22)$$

and

$$\phi\,(\text{vac}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\text{vac})(\underline{x}')}{|\underline{x}-\underline{x}'|}\, d^3x' \quad - (23)$$

where ρ (vac) is the vacuum charge density.

The total electric field strength is:

$$\underline{E}\,(\text{total}) = \underline{E} + \underline{E}\,(\text{vac}) \quad - (24)$$

where

$$\underline{\nabla} \cdot \underline{E}(\text{vac}) = \frac{\rho(\text{vac})}{\epsilon_0} \quad - (25)$$

$$- (26)$$

The Coulomb law is:

$$\underline{\nabla} \cdot (\underline{E} + \underline{E}\,(\text{vac})) = \frac{1}{\epsilon_0}(\rho + \rho\,(\text{vac}))$$

where

$$\underline{E}\,(\text{vac}) := -\underline{\nabla}\,\phi\,(\text{vac}) = \underline{\omega}\,\phi \quad - (27)$$

So the vacuum charge density is defined by:

$$\boxed{\underline{\nabla} \cdot (\underline{\omega}\,\phi) = \frac{\rho(\text{vac})}{\epsilon_0}} \quad - (28)$$

) which is analogous to:

$$\boxed{\underline{\nabla} \times (\underline{\omega} \times \underline{A}) = \mu_0 \, \underline{J}(\text{vac})} \quad -(29)$$

The vacuum charge current density is:

$$J^\mu(\text{vac}) = \left(c\rho(\text{vac}), \, \underline{J}(\text{vac}) \right) \quad -(30)$$

$$= \left(c\epsilon_0 \, \underline{\nabla} \cdot (\underline{\omega} \phi), \, \frac{1}{\mu_0} \underline{\nabla} \times (\underline{\omega} \times \underline{A}) \right) \quad -(31)$$

here:

$$\epsilon_0 \mu_0 = \frac{1}{c^2} \quad -(32)$$

so

$$\boxed{J^\mu(\text{vac}) = \frac{1}{\mu_0} \left(\frac{1}{c} \underline{\nabla} \cdot (\underline{\omega} \phi), \, \underline{\nabla} \times (\underline{\omega} \times \underline{A}) \right)} \quad -(33)$$

Units

$$\begin{aligned}
\phi &= J C^{-1} \\
\underline{\omega} &= m^{-1} \\
\rho &= C m^{-3} \\
\underline{J} &= C m^{-2} s^{-1} \\
\underline{A} &= J s C^{-1} m^{-1} \\
\epsilon_0 &= J^{-1} C^{2} m^{-1} \\
\mu_0 &= J s^2 C^{-2} m^{-1}
\end{aligned} \quad -(34)$$

Therefore the vacuum four-current density $J^\mu(\text{vac})$ depends only on ϕ, $\underline{A}$ and $\underline{\omega}$.