Applications of the Electric Dipole Field

From the first antisymmetry law:

$$\underline{E} = -\nabla\phi + \underline{\omega}\,\phi = -\frac{\partial \underline{A}}{\partial t} - \omega_0 \underline{A} \quad - (1)$$

and assuming that

$$\frac{\partial \underline{A}}{\partial t} = 0 \quad - (2)$$

then

$$\underline{E} = -\omega_0 \underline{A} \quad - (3)$$

Assuming that

$$\omega_0 = \frac{mc^2}{2\pi \hbar} \quad - (4)$$

then

$$\underline{E} = -\frac{mc^2}{2\pi \hbar}\underline{A} \quad - (5)$$

where $mc^2/(2\pi\hbar)$ is a universal constant. In general, the electric dipole field is:

$$\underline{E}(\underline{r}) = \frac{3\underline{n}\,(\underline{p}\cdot\underline{n}) - \underline{p}}{4\pi\epsilon_0 |\underline{r} - \underline{r}_0|^3} \quad - (6)$$

where $\underline{n}$ is a unit vector from $\underline{r}$ to $\underline{r}_0$. If the electric dipole moment is in the z axis then:

$$\underline{E} = E_r\,\underline{e}_r + E_\theta\,\underline{e}_\theta \quad - (7)$$

$$= \frac{p}{4\pi\epsilon_0 r^3}\left(2\cos\theta\,\underline{e}_r + \sin\theta\,\underline{e}_\theta\right)$$

spherical polar coordinates.

Therefore:

$$\underline{A} = -\frac{p}{4\pi\epsilon_0\omega_0 r^3}\left(2\cos\theta\,\underline{e}_r + \sin\theta\,\underline{e}_\theta\right) \quad - (8)$$

) In the general case:

$$\underline{A}(\underline{r}) = -\frac{1}{\omega_o}\left(\frac{3\underline{n}(\underline{p}\cdot\underline{n}) - \underline{p}}{4\pi\epsilon_o |\underline{r} - \underline{r_o}|^3}\right) - (9)$$

This is the electric vector potential for the dipole field.
Note that

$$\underline{\nabla}\omega_o = \underline{0} - (10)$$

and

$$\underline{\nabla}\times\underline{A} = \underline{0} - (11)$$

so:

$$\underline{\nabla}\times(\omega_o\underline{A}) = \omega_o\underline{\nabla}\cdot\underline{A} + \underline{A}\cdot\underline{\nabla}\omega_o = \underline{0} - (12)$$

is required for eq. (1) and

$$\underline{\nabla}\times\underline{E} = \underline{0}. - (13)$$

The Faraday law of induction means that:

$$\underline{\nabla}\times\underline{E} + \frac{\partial\underline{B}}{\partial t} = \underline{0} - (14)$$

So

$$\frac{\partial\underline{B}}{\partial t} = \underline{0} - (15)$$

If it is assumed that:

$$\underline{B} = \underline{\nabla}\times\underline{A} - \underline{\omega}\times\underline{A} = \underline{0} - (16)$$

then eq. (15) is true automatically.

The vector antisymmetry law, together with
(16), mean that the spin connections can
calculated by computer algebra as follows:

$$\frac{\partial A_z}{\partial y} = \omega_y A_z, \quad \frac{\partial A_y}{\partial z} = -\omega_z A_y,$$

$$\frac{\partial A_x}{\partial z} = \omega_z A_x, \quad \frac{\partial A_z}{\partial x} = \omega_x A_z, \qquad -(17)$$

$$\frac{\partial A_y}{\partial x} = \omega_x A_y, \quad \frac{\partial A_x}{\partial y} = \omega_y A_x.$$

Therefore eqs. (1) and (8) must be expressed in Cartesian components. To avoid human error this is best done by computer algebra. A preliminary hand calculation gives (Note 385(3)):

$$\underline{A} = -\frac{\rho}{4\pi \epsilon_0 \omega_0 r^5}\left(3xz\,\underline{i} + 3xy\,\underline{j} + (2z^2 - x^2 - y^2)\,\underline{k}\right).$$
$$-(18)$$

where

$$r^5 = (x^2 + y^2 + z^2)^{5/2} \quad -(19)$$

The spin connections calculated in this way conserve antisymmetry. The spin connections have an interesting graphical structure.

The calculation can be repeated for the solution:

$$\omega_0 = -\frac{c}{r} \quad -(20)$$