5(1): Spin Connections of the Magnetic Dipole Field

It follows by defining the Hodge dual of the electromagnetic field tensor that:

$$c\underline{B} = -\underline{\nabla}\phi + \underline{\omega}\phi = -\frac{\partial \underline{A}}{\partial t} - \omega_0 \underline{A}, \quad -(1)$$

The Hodge dual field tensor is defined as:

$$\tilde{F}_{\mu\nu} := \partial_\mu A_\nu - \partial_\nu A_\mu + \omega_\mu A_\nu - \omega_\nu A_\mu \quad -(2)$$

The antisymmetry laws are:

$$\partial_\mu A_\nu + \omega_\mu A_\nu = -(\partial_\nu A_\mu + \omega_\nu A_\mu) \quad -(3)$$

The Hodge dual is defined as:

$$\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma} \quad -(4)$$

where $\epsilon^{\mu\nu\rho\sigma}$ is the totally antisymmetric unit tensor in four dimensions. In ECE2:

$$F_{\rho\sigma} = D_\rho A_\sigma - D_\sigma A_\rho \quad -(5)$$

where

$$D_\rho = \partial_\rho + \omega_\mu \quad -(6)$$

where ω_μ is the spin connection four vector. Therefore:

$$\tilde{F}^{\mu\nu} = (D^\mu A^\nu)_{HD} - (D^\nu A^\mu)_{HD} \quad -(7)$$

where $(D^\mu A^\nu)_{HD} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} D_\rho A_\sigma . \quad -(8)$

For example:

$$(D^0 A^1)_{HD} = \frac{1}{2}\left(\epsilon^{0123} D_2 A_3 + \epsilon^{0132} D_3 A_2\right) \quad -(9)$$

By antisymmetry:

$$\epsilon^{0123} = -\epsilon^{0132} = 1 \quad -(10)$$

nd

$$D_2 A_3 = -D_3 A_2. \quad -(11)$$

So:

$$(D^0 A^1)_{HD} = D_2 A_3, \quad -(12)$$

and

$$\tilde{F}^{01} = F_{23} \quad -(13)$$

By definition:

$$F_{\rho\sigma} = \begin{bmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & -cB_3 & cB_2 \\ -E_2 & cB_3 & 0 & -cB_1 \\ -E_3 & -cB_2 & cB_1 & 0 \end{bmatrix} \quad -(14)$$

So

$$\tilde{F}^{\mu\nu} = \begin{bmatrix} 0 & -cB^1 & -cB^2 & -cB^3 \\ cB^1 & 0 & E^3 & -E^2 \\ cB^2 & -E^3 & 0 & E^1 \\ cB^3 & E^2 & -E^1 & 0 \end{bmatrix} \quad -(15)$$

in which

$$B^1 = -B_1 = B_x \quad -(16)$$

nd so on.

It is seen that:

$$\underline{E} \rightarrow c\underline{B} \quad -(17)$$

$$c\underline{B} \rightarrow -\underline{E} \quad -(18)$$

when eq. (14) is transformed to eq. (15)

It follows that:

3)

$$\underline{E} = -\underline{\nabla}\phi + \underline{\omega}\phi = -\frac{\partial \underline{A}}{\partial t} - \omega_0 \underline{A} \quad -(19)$$

is transformed to Eq. (1), Q.E.D.

Furthermore,

$$\underline{B} = \underline{\nabla} \times \underline{A} - \underline{\omega} \times \underline{A} \quad -(20)$$

is transformed to:

$$\underline{E} = c\left(\underline{\omega} \times \underline{A} - \underline{\nabla} \times \underline{A}\right) \quad -(21)$$

If it is assumed that:

$$\frac{\partial \underline{A}}{\partial t} = \underline{0} \quad -(22)$$

then:

$$c\underline{B} = -\omega_0 \underline{A} \quad -(23)$$

If it is assumed that:

$$\underline{E} = \underline{0} \quad -(24)$$

then

$$\underline{\nabla} \times \underline{A} = \underline{\omega} \times \underline{A} \quad -(25)$$

The vector antisymmetry law is:

$$\left(\frac{\partial}{\partial Y} - \omega_Y\right) A_Z = -\left(\frac{\partial}{\partial Z} - \omega_Z\right) A_Y \quad -(26)$$

$$\left(\frac{\partial}{\partial Z} - \omega_Z\right) A_X = -\left(\frac{\partial}{\partial X} - \omega_X\right) A_Z \quad -(27)$$

$$\left(\frac{\partial}{\partial X} - \omega_X\right) A_Y = -\left(\frac{\partial}{\partial Y} - \omega_Y\right) A_X \quad -(28)$$

4) Eqs. (25) to (28) give:

$$\frac{\partial A_Z}{\partial Y} = \omega_Y A_Z, \quad \frac{\partial A_Y}{\partial Z} = \omega_Z A_Y, \quad -(29)$$

$$\frac{\partial A_X}{\partial Z} = \omega_Z A_X, \quad \frac{\partial A_Z}{\partial X} = \omega_X A_Z, \quad -(30)$$

$$\frac{\partial A_Y}{\partial X} = \omega_X A_Y, \quad \frac{\partial A_X}{\partial Y} = \omega_Y A_X \quad -(31)$$

Assume that:

$$\omega_0 = \frac{mc^2}{2\pi \hbar} \quad -(32)$$

Consider various experimentally well known magnetic flux densities $\underline{B}$, calculate the potential $\underline{A}$ from

$$\underline{A} = -\frac{c}{\omega_0}\underline{B} = -\frac{2\pi\hbar}{mc}\underline{B}, \quad -(33)$$

and finally calculate the spin connections from eqs. (29) to (31). Here ω_0 is the rest frequency in hertz of the vacuum particle of mass m.

This procedure conserves the ECE2 antisymmetry laws for any static magnetic flux density $\underline{B}$,

Q. E. D.

The magnetic field far from a current loop of radius a and current I is given by Jackson, eq. (5.41) in spherical polar coordinates:

$$B_r = \frac{\mu_0}{4\pi}\left(I\pi a^2\right)\frac{\cos\theta}{r^3} \quad -(34)$$

$$B_\theta = \frac{\mu_0}{4\pi}\left(I\pi a^2\right)\frac{\sin\theta}{r^3} \quad -(35)$$

So it is a dipole magnetic field. The magnetic dipole moment of the loop is:

$$m = \pi I a^2 \quad -(36)$$

Eqs. (34) and (35) have the same structure as the electrostatic dipole field:

$$E_r = \frac{2p\cos\theta}{4\pi\epsilon_0 r^3} \quad -(37)$$

and

$$E_\theta = \frac{p\sin\theta}{4\pi\epsilon_0 r^3} \quad -(38)$$

The ECE2 magnetostatic equations are:

$$\underline{\nabla}\cdot\underline{B} = 0, \quad -(39)$$

$$\underline{\nabla}\times\underline{B} = \mu_0 \underline{J} \quad -(40)$$

It is well known that eqs. (39) and (40) are solved in the standard model using the magnetic scalar potential, used in regions where $\underline{J} = \underline{0}$:

$$\underline{B} = -\underline{\nabla}\phi \quad -(41)$$

(Jackson, third edition, page 180). In ECE2, eq. (41) of the standard model is replaced by the Hodge dual scalar antisymmetry law (1) for the dual field tensor.

From eqs. (1) and (39):

$$\underline{\nabla} \cdot (\omega_0 \underline{A}) = 0 \quad - (42)$$

i.e.

$$\underline{A} \cdot \underline{\nabla} \omega_0 + \omega_0 \underline{\nabla} \cdot \underline{A} = 0. \quad - (43)$$

If it is assumed that ω_0 is a constant then

$$\underline{\nabla} \cdot \underline{A} = 0 \quad - (44)$$

i.e

$$\underline{\nabla} \cdot \underline{B} = 0 \quad - (45)$$

So $\underline{A}$ from the dipole field automatically obeys eq. (44) because $\underline{B}$ is a solution of eq. (45).

If it is assumed that ω_0 varies, then it must be worked out from eq. (43)

Given the spin connection vector from eqs. (29) to (31) then from eqs (1) and (39):

$$\nabla^2 \phi - \underline{\nabla} \cdot (\underline{\omega} \phi) = 0 \quad - (46)$$

ϕ can be worked out, Q.E.D.

Finally, for the magnetic dipole field:

$$\underline{\nabla} \times \underline{B} = \underline{0} \quad - (47)$$

far from the loop:

$$\underline{J} \rightarrow \underline{0}. \quad - (48)$$

It follows from eq. (1) that:

$$\underline{\nabla} \times (\underline{\omega} \phi) = \underline{0}, \quad - (49)$$

using eq. (47).

) Note carefully that:

$$\underline{A} = -\frac{c}{\omega_0}\underline{B}$$

$$= -\frac{c}{\omega_0}\frac{\mu_0}{4\pi}\cdot I\pi a^2\cdot\left(\frac{\cos\theta}{r^3}\underline{e}_r + \frac{\sin\theta}{r^3}\underline{e}_\theta\right)$$

$$= -\frac{c}{\omega_0}\cdot\frac{\mu_0 I a^2}{4r^3}\left(\cos\theta\,\underline{e}_r + \sin\theta\,\underline{e}_\theta\right)$$

, an entirely new type of vector potential that does not exist in the standard model.