5(3) : Complete Solution for the Electric Dipole Field

The first antisymmetry law:

$$\underline{E} = -\underline{\nabla}\phi + \underline{\omega}\phi = -\frac{\partial \underline{A}}{\partial t} - \omega_0 \underline{A} \quad -(1)$$

s considered with:

$$\underline{\nabla} \times \underline{E} = \underline{0} \quad -(2)$$

to give:

$$\underline{\nabla} \times \frac{\partial \underline{A}}{\partial t} + \underline{\nabla} \times (\omega_0 \underline{A}) = \underline{0}. \quad -(3)$$

In electrostatics:

$$\frac{\partial \underline{A}}{\partial t} = \underline{0} \quad -(4)$$

so

$$\underline{\nabla} \times (\omega_0 \underline{A}) = \underline{0}. \quad -(5)$$

This means that:

$$\underline{\nabla} \times \underline{E} = \underline{0} \quad -(6)$$

so the electric dipole field must be irrotational. Using the Faraday law:

$$\underline{\nabla} \times \underline{E} + \frac{\partial \underline{B}}{\partial t} = \underline{0} \quad -(7)$$

eq. (6) implies:

$$\frac{\partial \underline{B}}{\partial t} = \underline{0}, \quad -(8)$$

possible solution of which is:

$$\underline{B} = \underline{0}. \quad -(9)$$

There is also the Coulomb law:

$$\underline{\nabla} \cdot \underline{E} = \rho / \epsilon_0 \quad -(10)$$

From eqs. (1) and (4):

$$\underline{E} = -\omega_0 \underline{A} \quad -(11)$$

) Eq. (5) implies that:

$$\omega_0 \underline{\nabla} \times \underline{A} + \underline{A} \times \underline{\nabla} \omega_0 = \underline{0} \quad -(12)$$

Eq. (10) implies that

$$\omega_0 \underline{\nabla} \cdot \underline{A} + \underline{A} \cdot \underline{\nabla} \omega_0 = -\frac{\rho}{\epsilon_0} \quad -(13)$$

There are four equations in four unknowns, ω_0, A_x, A_y and A_z. So this system can be solved for the unknowns given an experimentally determined ρ.

The package Mathematica can probably solve the eqs. (12) and (13). It may be possible to solve the system with Maxima. Once A_x, A_y and A_z are known the antisymmetry laws are obeyed automatically by choice of spin connection vector.

It must first be checked by computer algebra that the electric dipole field:

$$\underline{E}(\underline{r}) = \frac{3\underline{n}(\underline{p} \cdot \underline{n}) - \underline{p}}{4\pi \epsilon_0 |\underline{r} - \underline{r}_0|^3} \quad -(14)$$

is irrotational, i.e.

$$\underline{\nabla} \times \underline{E} = \underline{0} \quad -(15)$$

The Maxima package can be used for this check.

Here $\underline{n}$ is a unit vector from $\underline{r}$ to $\underline{r}_0$:

$$\underline{n} = \frac{\underline{r} - \underline{r}_0}{\left((\underline{r} - \underline{r}_0) \cdot (\underline{r} - \underline{r}_0)\right)^{1/2}} \quad -(16)$$

If the dipole moment $\underline{p}$ is in the Z axis:

$$\underline{E} = E_r \underline{e}_r + E_\theta \underline{e}_\theta \quad -(17)$$

here

$$E_r = \frac{2p\cos\theta}{4\pi\epsilon_0 r^3} \quad -(18)$$

$$E_\theta = \frac{p\sin\theta}{4\pi\epsilon_0 r^3} \quad -(19)$$

in spherical polar coordinates.

Using eqs. (17) to (19), it must be checked by computer algebra:

$$\underline{\nabla} \times \underline{E} = \underline{0} \quad -(20)$$

where:

$$\underline{\nabla}\times\underline{E} = \frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}(E_\phi \sin\theta) - \frac{\partial E_\theta}{\partial\phi}\right)\underline{e}_r + \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial E_r}{\partial\phi} - \frac{\partial}{\partial r}(rE_\phi)\right)\underline{e}_\theta + \frac{1}{r}\left(\frac{\partial}{\partial r}(rE_\theta) - \frac{\partial E_r}{\partial\theta}\right)\underline{e}_\phi \quad -(21)$$

From eqs. (17) to (21):

$$\frac{\partial}{\partial r}(rE_\theta) - \frac{\partial E_r}{\partial\theta} = 0 \quad -(22)$$

i.e.

$$\frac{\partial}{\partial r}\left(\frac{p\sin\theta}{4\pi\epsilon_0 r^2}\right) - \frac{\partial E_r}{\partial\theta} = 0 \quad -(23)$$

or

$$\frac{p}{4\pi\epsilon_0}\left(\frac{\partial}{\partial r}\left(\frac{\sin\theta}{r^2}\right) - \frac{\partial}{\partial\theta}\left(\frac{2\cos\theta}{r^3}\right)\right) = 0 \quad -(24)$$

i.e.

$$\frac{p}{4\pi\epsilon_0}\left(-\frac{2}{r^3}\sin\theta+\frac{2}{r^3}\sin\theta\right)=0 \quad -(25)$$

<u>Q.E.D.</u>

So the <u>electric</u> dipole <u>field (17) is indeed irrotational</u>, <u>and p</u> <u>is a constant</u>.

The computer can be used to check that eq. (14) is also irrotational, so $\underline{E}(\underline{r})$ is a static electric field.

A units analysis shows that

$$E = \text{volt } m^{-1} = J C^{-1} m^{-1}$$

and

$$A = J s C^{-1} m^{-1}$$

so

$$\underline{E} = -\omega_0 \underline{A} \quad -(26)$$

is correct. The vector potential $\underline{A}$ is irrotational if

$$\underline{A} = -\frac{1}{\omega_0}\left(\frac{2p\cos\theta}{4\pi\epsilon_0 r^3}\underline{e}_r+\frac{p\sin\theta\,\underline{e}_\theta}{4\pi\epsilon_0 r^3}\right)$$

$$= -\frac{p}{4\pi\epsilon_0\,\omega_0 r^3}\left(2\cos\theta\,\underline{e}_r+\sin\theta\,\underline{e}_\theta\right) \quad -(27)$$

if ω_0 is a constant.

It follows that

$$\underline{\nabla}\times\underline{A}=0 \quad -(28)$$

) and $\underline{\nabla}\omega_0 = \underline{0}$ — (29)

o eq. (12) is obeyed, Q.E.D.

From eq. (13):

$$\underline{\nabla} \cdot \underline{A} = -\frac{\rho}{\omega_0 \epsilon_0} \quad -(30)$$

It is proposed that ω_0 be the frequency in ertz associated with the fundamental vacuum article. Defining:

$$\Omega_0 = 2\pi\omega_0 \quad -(31)$$

hen the relativistic energy of the vacuum particle is:

$$E = \gamma mc^2 = \hbar\Omega_0 \quad -(32)$$

For a vacuum particle at rest:

$$E = mc^2 = \hbar\Omega_0 \quad -(33)$$

hich is the de Broglie equation. So:

$$\boxed{\omega_0 = \frac{mc^2}{2\pi\hbar}} \quad -(34)$$

where m is the mass of the vacuum particle (see ECE2, UFT 366). This can be measured experimentally from the missing mass of the universe.