385(4): Complete Solution for the Static Magnetic Field

The electric field strength is zero, so:

$$\underline{E} = -\underline{\nabla}\phi + \underline{\omega}\phi = -\frac{\partial \underline{A}_E}{\partial t} - \omega_0 \underline{A}_E \quad -(1)$$
$$= \underline{0}$$

by the first antisymmetry law. This has the solution:

$$\phi = 0, \quad \underline{A}_E = \underline{0} \quad -(2)$$

and

$$\underline{\omega} \neq \underline{0}. \quad -(3)$$

Here $\underline{A}_E$ is the electric vector potential.

The magnetic flux density is defined by:

$$\underline{B} = \underline{\nabla} \times \underline{A} - \underline{\omega} \times \underline{A} \quad -(4)$$

where $\underline{A}$ denotes the magnetic vector potential.

The vector antisymmetry law is:

$$\left(\frac{\partial}{\partial y} - \omega_y\right) A_Z = -\left(\frac{\partial}{\partial Z} - \omega_Z\right) A_y \quad -(5)$$

$$\left(\frac{\partial}{\partial Z} - \omega_Z\right) A_x = -\left(\frac{\partial}{\partial x} - \omega_x\right) A_Z \quad -(6)$$

$$\left(\frac{\partial}{\partial x} - \omega_x\right) A_y = -\left(\frac{\partial}{\partial y} - \omega_y\right) A_x \quad -(7)$$

The time like spin connection is universal:

$$\omega_0 = \frac{mc^2}{2\pi\hbar} \quad -(8)$$

) The Gauss law in the assumed absence of a magnetic monopole is:

$$\underline{\nabla} \cdot \underline{B} = 0 \quad -(9)$$

From eqs. (4) and (9):

$$\underline{\nabla} \cdot \underline{\omega} \times \underline{A} = 0 \quad -(10)$$

i.e.

$$A_x \left(\frac{\partial \omega_z}{\partial y} - \frac{\partial \omega_y}{\partial z} \right) = \omega_x \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) \quad -(11)$$

$$A_y \left(\frac{\partial \omega_z}{\partial x} - \frac{\partial \omega_x}{\partial z} \right) = \omega_y \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) - (12)$$

$$A_z \left(\frac{\partial \omega_y}{\partial x} - \frac{\partial \omega_x}{\partial y} \right) = \omega_z \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) - (13)$$

Eqs. (5) to (7) and (11) to (13) are six equations in six unknowns, $A_x, A_y, A_z, \omega_x, \omega_y, \omega_z$. So the problem is exactly determined, Q.E.D. A package such as Mathematica could probably solve the problem numerically, giving the general $\underline{A}$ and general $\underline{\omega}$.

From the Faraday law of induction:

$$\underline{\nabla} \times \underline{B} = \underline{0} \quad -(14)$$

if it is assumed that:

$$\underline{E} = \underline{0} \quad -(15)$$

) From the Ampère Law:

$$\underline{\nabla} \times \underline{B} = \mu_0 \underline{J} \quad -(16)$$

it follows that:

$$\underline{J} = \underline{0} \quad -(17)$$

From eqs. (4) and (14):

$$\underline{\nabla} \times (\underline{\nabla} \times \underline{A}) = \underline{\nabla} \times (\underline{\omega} \times \underline{A}) \quad -(18)$$

As in Note 381(3) the magnetic vector potential is:

$$\underline{A} = \frac{B_Z}{2} \left(-Y \underline{i} + X \underline{j} \right) \quad -(19)$$

from which:

$$\frac{\partial A_Y}{\partial X} = -\frac{\partial A_X}{\partial Y} \quad -(20)$$

Eq. (13) has the particular solution:

$$\frac{\partial A_Y}{\partial X} = -\frac{\partial A_X}{\partial Y} = \omega_Y A_X = -\omega_X A_Y \quad -(21)$$

which is therefore obeyed by eq. (19), $\underline{\text{Q.E.D.}}$

From eq. (21):

$$\underline{\omega} = -\left(\frac{1}{X} \underline{i} + \frac{1}{Y} \underline{j} \right) \quad -(22)$$

so

$$\underline{\omega} \times \underline{A} = -B_Z \underline{k} \quad -(23)$$

and

$$\boxed{\underline{B} = 2B_Z \underline{k}} \quad -(24)$$

) This is a static magnetic field in $\underline{k}$, Q.E.D.

For this solution:

$$\omega_X = -\frac{1}{X}, \; \omega_Y = \frac{1}{Y}, \; \omega_Z = 0 \quad -(25)$$

$$A_X = -\frac{B^{(0)}}{2} Y, \; A_Y = \frac{B^{(0)}}{2} X, \; A_Z = 0 \quad -(26)$$

so there is conservation of vector antisymmetry, eqs. (5) to (7), Q.E.D.

It also follows that:

$$\underline{\nabla} \cdot \underline{\omega} \times \underline{A} = 0 \quad -(27)$$

because B_Z is constant. Therefore:

$$\underline{\nabla} \times \underline{B} = \underline{0} \quad -(28)$$

and

$$\frac{\partial \underline{B}}{\partial t} = \underline{0}. \quad -(29)$$

Q.E.D. Finally:

$$\underline{J} = \underline{0}. \quad -(30)$$

Therefore the magnetostatic ECE2 equations are obeyed, and vector antisymmetry is conserved in three dimensions. The static magnetic field is used in applications such as NMR and ESR, the Zeeman effect and so on. The general solution of eqs. (5) to (7) and (11) to (13) should give many new experimental effects.