376(4): Comparison of Precession Theories

1) Precession Based on ECE2 Covariance

Orbital precession energy from the fundamental equation of ECE2 covariance:

$$\mathcal{L} = -\frac{mc^2}{\gamma} - U \quad -(1)$$

$$\underline{p} = \gamma m \underline{v} = \frac{\partial \mathcal{L}}{\partial \underline{\dot{r}}} = m \frac{d\underline{r}}{d\tau} \quad -(2)$$

and the relativistic Euler Lagrange equations:

$$\frac{\partial \mathcal{L}}{\partial r} = \frac{d}{d\tau} \frac{\partial \mathcal{L}}{\partial \dot{r}} \quad -(3)$$

giving the relativistic force equation:

$$\underline{F} = \gamma^4 m \frac{d^2 \underline{r}}{dt^2} = -mMG\frac{\underline{r}}{r^3} \quad -(4)$$

The relevant field potential equations are:

$$\underline{\nabla} \times \underline{g} = \underline{0} \quad -(5)$$

$$\underline{\nabla} \cdot \underline{g} = \underline{\kappa} \cdot \underline{g} = 4\pi G \rho_m \quad -(6)$$

$$\frac{\partial \underline{g}}{\partial t} = \underline{0} \quad -(7)$$

in which the gravitomagnetic field $\underline{\Omega}$ is neglected. These are the equations of gravitostatics.

2) Precession due to Fluid Spacetime

The force equation is:

$$\underline{F} = m\underline{g} = m\left(\frac{\partial \underline{v}}{\partial t} + (\underline{v} \cdot \underline{\nabla})\underline{v}\right) = -mMG\frac{\underline{r}}{r^3} \quad -(8)$$

2) where:

$$\underline{v} = \frac{\partial \underline{R}}{\partial t} + (\underline{v} \cdot \underline{\nabla})\underline{R} \quad -(9)$$

in which

$$\underline{R} = R(x(t), y(t), t) \quad -(10)$$

is the position element of fluid spacetime.

From eqs. (4) and (8):

$$\gamma^4 \frac{d^2 \underline{r}}{dt^2} = \frac{\partial \underline{v}}{\partial t} + (\underline{v} \cdot \underline{\nabla})\underline{v} \quad -(11)$$

where

$$\underline{v} = \underline{v}(\dot{x}(t), \dot{y}(t), t) \quad -(12)$$

In the Newtonian limit:

$$\underline{v}(\dot{x}(t), \dot{y}(t), t) \rightarrow \underline{v}(t) \quad -(13)$$

and

$$\gamma \rightarrow 1 \quad -(14)$$

so Eq. (11) reduces to:

$$\underline{v} = \dot{\underline{r}} \quad -(15)$$

From eq. (11), the effect of a fluid spacetime is to change the Newton equation in the Minkowski equation.

The equations of the fluid spacetime are the Kambe equations:

$$\underline{\nabla} \cdot \underline{E}_F = q_F \quad -(16)$$

$$\underline{\nabla} \cdot \underline{B}_F = 0 \quad -(17)$$

$$\underline{\nabla} \times \underline{E}_F + \frac{\partial \underline{B}_F}{\partial t} = \underline{0} \quad -(18)$$

$$\underline{\nabla} \times \underline{B}_F - \frac{1}{a_0^2} \frac{\partial \underline{E}_F}{\partial t} = \frac{1}{a_0^2} \underline{J}_F \quad -(19)$$

3) These are ECE2 covariant and relativistic, and have the same overall structure as the ECE2 covariant equations of gravitation and electrodynamics. In these equations:

$$\underline{E}_F = \left(\underline{v}_F \cdot \underline{\nabla} \right) \underline{v}_F = -\underline{\nabla} \Phi_F - \frac{\partial \underline{v}_F}{\partial t} \quad -(20)$$

so:

$$\frac{\partial \underline{v}_F}{\partial t} + \left(\underline{v}_F \cdot \underline{\nabla} \right) \underline{v}_F = -\underline{\nabla} \Phi_F . \quad -(21)$$

If

$$\Phi_F = -\frac{MG}{r}, \quad -(22)$$

the gravitational potential, then:

$$\underline{g} = -\underline{\nabla} \Phi_F = \frac{\partial \underline{v}}{\partial t} + \left(\underline{v} \cdot \underline{\nabla} \right) \underline{v} \quad -(23)$$

which is eq. (8), Q.E.D., with:

$$\underline{v} = \underline{v}_F . \quad -(24)$$

If the vorticity is neglected, then eqs. (16) to (19) reduce to:

$$\underline{\nabla} \cdot \underline{E}_F = q_F \quad -(25)$$

$$\underline{\nabla} \times \underline{E}_F = \underline{0} \quad -(26)$$

$$\frac{\partial \underline{E}_F}{\partial t} = \underline{0} \quad -(27)$$

therefore:

$$\boxed{\underline{g} = \frac{\partial \underline{v}}{\partial t} + \underline{E}_F} \quad -(28)$$

4) Therefore $\underline{E}_F$ is a relativistic correction to $\underline{g}$. Eqs. (25) to (27) are hydrodynamical equations of a fluid with no vorticity, more generally they are fluid spacetime equations with no vorticity. They correspond to gravitostatics and electrostatics.

It follows that eq. (11) is:

$$\boxed{\gamma^4 \frac{d^2 \underline{r}}{dt^2} = \frac{\partial \underline{v}}{\partial t} + \underline{E}_F} \quad -(29)$$

Eq. (29) means that the fluid property of spacetime which produces the Minkowski force equation, results in a precessing orbit.

From eqs. (4) and (6):

$$\underline{\nabla} \cdot \left(\gamma^4 \frac{d^2 \underline{r}}{dt^2} \right) = \underline{\kappa} \cdot \left(\gamma^4 \frac{d^2 \underline{r}}{dt^2} \right)$$

$$= 4\pi G \rho_m \quad -(30)$$

i.e.

$$\underline{\nabla} \cdot \left(\frac{\partial \underline{v}}{\partial t} + \underline{E}_F \right) = \underline{\kappa} \cdot \left(\frac{\partial \underline{v}}{\partial t} + \underline{E}_F \right)$$

$$= 4\pi G \rho_m \quad -(31)$$