16(b): Comparison of Precession Equations

Consider the lagrangian of ECE2 theory:

$$\mathcal{L} = -\frac{mc^2}{\gamma} - U \quad -(1)$$

The relativistic momentum is defined by:

$$\underline{p} = \frac{\partial \mathcal{L}}{\partial \dot{\underline{r}}} = m \frac{d\underline{r}}{d\tau} \quad -(2)$$

(Marion and Thornton chapter 15). It follows that:

$$\underline{F} = \frac{\partial \mathcal{L}}{\partial \underline{r}} = \frac{d}{d\tau} \frac{\partial \mathcal{L}}{\partial \dot{\underline{r}}} = m \frac{d}{d\tau}\left(\frac{d\underline{r}}{d\tau}\right) = m\gamma^4 \frac{d^2\underline{r}}{dt^2} \quad -(3)$$

where $\underline{F}$ is the Minkowski force, the relativistic version of the Newtonian force:

$$\underline{F}(\text{Newton}) = m \frac{d^2\underline{r}}{dt^2} \quad -(4)$$

In the limit:

$$\gamma \rightarrow 1 \quad -(5)$$

the Minkowski force reduces to the Newtonian force.

If

$$U = -\frac{mMG}{r} \quad -(6)$$

then:

$$\frac{\partial \mathcal{L}}{\partial \underline{r}} = -mMG\frac{\underline{r}}{r^3} \quad -(7)$$

It follows that the Minkowski force equation

2) is:

$$\boxed{\underline{F} = m\gamma^4 \frac{d^2 \underline{r}}{dt^2} = -\frac{mMG\,\underline{r}}{r^3}} \quad -(8)$$

This gives a precessing orbit when solved.

Eq. (8) is a new and original discovery.

It is obtained from the relativistic Euler Lagrange equation:

$$\boxed{\frac{\partial \mathcal{L}}{\partial \underline{r}} = \frac{d}{d\tau} \frac{\partial \mathcal{L}}{\partial \dot{\underline{r}}}} \quad -(9)$$

with the ECE2 Lagrangian (1).

Eq. (9) is also a new and original discovery.

Note carefully that the equation:

$$\ddot{\underline{r}} = \frac{\gamma MG}{r^3} \left(\frac{\underline{v}\,(\underline{v} \cdot \underline{r})}{c^2} - \underline{r} \right) \quad -(10)$$

is obtained from:

$$\frac{\partial \mathcal{L}}{\partial \underline{r}} = \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{\underline{r}}} \quad -(11)$$

Eq. (10) gives precession but the newly discovered and rigorously self consistent equation is equation (9).

Eq. (8) must be solved simultaneously with the field equations of gravitostatics:

$$\underline{\nabla} \times \underline{g} = \underline{0} \quad - (12)$$

$$\underline{\nabla} \cdot \underline{g} = \underline{\kappa} \cdot \underline{g} = 4\pi G \rho_m \quad - (13)$$

$$\frac{\partial \underline{g}}{\partial t} = \underline{0} \quad - (14)$$

Eq. (12) has been checked by computer algebra. We have:

$$g_x = \frac{\ddot{X}}{\left(1 + \frac{\dot{X}^2 + \dot{Y}^2}{c^2}\right)^2} = -mG \frac{X}{(X^2+Y^2)^{3/2}} \quad - (15)$$

$$g_Y = \frac{\ddot{Y}}{\left(1 + \frac{\dot{X}^2 + \dot{Y}^2}{c^2}\right)^2} = -mG \frac{Y}{(X^2+Y^2)^{3/2}} \quad - (16)$$

Eq. (13) is:

$$\frac{\partial g_x}{\partial X} + \frac{\partial g_Y}{\partial Y} = \underline{\kappa} \cdot \underline{g} = 4\pi G \rho_m \quad - (17)$$

We have:

$$\frac{\partial g_x}{\partial X} = -mG \left(\frac{(X^2+Y^2)^{3/2} - X \frac{\partial}{\partial X}\left((X^2+Y^2)^{3/2}\right)}{(X^2+Y^2)^3} \right) \quad - (18)$$

$$= -\frac{mG}{(X^2+Y^2)^3} \left((X^2+Y^2)^{3/2} - 3X^2 (X^2+Y^2)^{1/2} \right)$$

$$= -mG\left(\frac{1}{(x^2+y^2)^{3/2}} - \frac{3x^2}{(x^2+y^2)^{5/2}}\right)$$

Similarly:

$$\frac{\partial g_y}{\partial y} = -mG\left(\frac{1}{(x^2+y^2)^{3/2}} - \frac{3y^2}{(x^2+y^2)^{5/2}}\right) \quad -(19)$$

So:

$$\frac{\partial g_x}{\partial x} + \frac{\partial g_y}{\partial y} = \frac{2mG}{(x^2+y^2)^{3/2}} = 4\pi G \rho_m \quad -(20)$$

So:

$$\boxed{\rho_m = \frac{m}{2\pi(x^2+y^2)^{3/2}}} \quad -(21)$$

Therefore eq. (8) is:

$$\ddot{x} = -2\pi \rho_m G \left(1 - \frac{\dot{x}^2+\dot{y}^2}{c^2}\right)^2 x \quad -(22)$$

$$\ddot{y} = -2\pi \rho_m G \left(1 - \frac{\dot{x}^2+\dot{y}^2}{c^2}\right)^2 y \quad -(23)$$

Eqs. (21) to (23) must be solved simultaneously to give ~~a~~ precessing orbit.