371(5): The Planar Orbit of ECE2 Relativity

The orbit can be found from the lagrangian:

$$\mathcal{L} = -mc^2\left(1 - \frac{v^2}{c^2}\right)^{1/2} - U \quad -(1)$$

where

$$U = -\frac{mMG}{r} \quad -(2)$$

and

$$v^2 = \dot{r}^2 + r^2\dot{\theta}^2 \quad -(3)$$

The proper Lagrange variables are r and θ and the Euler Lagrange equations are:

$$\frac{\partial \mathcal{L}}{\partial r} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{r}} \quad -(4)$$

and

$$\frac{\partial \mathcal{L}}{\partial \theta} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\theta}} \quad -(5)$$

These can be solved simultaneously with Maxima to give the orbit $r(\theta)$. The latter can be found from

$$\frac{dr}{d\theta} = \frac{dr}{dt}\frac{dt}{d\theta} = \frac{\dot{r}}{\dot{\theta}} \quad -(6)$$

We have

$$r = \int \frac{dr}{d\theta}\, d\theta \quad -(7)$$

For the Sommerfeld atom:

$$U = -\frac{e^2}{4\pi\epsilon_0 r} \quad -(8)$$

) and the orbits of the Sommerfeld atom can be obtained in the same way.

The integration in eq. (7) can be carried out numerically to any degree of precision. The result should be a precessing ellipse.

The three dimensional relativistic orbit is given by using

$$v^2 = \dot{r}^2 + r^2 \dot{\beta}^2 \qquad -(9)$$

$$= \dot{r}^2 + \dot{\theta}^2 + \dot{\phi}^2 \sin^2\theta$$

and the proper Lagrange variables r and β or the proper Lagrange variables r, θ and ϕ, as outlined in Note 371(5).

Quantum effects can also be worked out with a Lagrangian theory.
