370(1): Some Remarks on UFT369 and the Usual Lagrangian for the Motion of an Asymmetric Top.

1) It is possible to extend the lagrangian of UFT369 to:

$$\mathcal{L} = \frac{1}{2} m \dot{\underline{r}} \cdot \dot{\underline{r}} + \frac{1}{2} m_1 \dot{\underline{R}} \cdot \dot{\underline{R}} - \frac{mMG}{r} - \frac{m_1 MG}{R} \quad -(1)$$

where the quantities are defined in Fig (1):

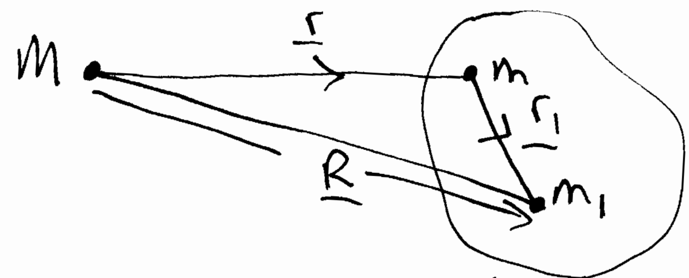

Fig (1)

It is recommended to use the rotation generator defined by Marion and Thornton:

$$\lambda_\theta = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix} \quad -(1)$$

2) The usual method uses the lagrangian:

$$\mathcal{L} = \frac{1}{2} m \dot{\underline{r}} \cdot \dot{\underline{r}} + \frac{1}{2}\left(I_1 \omega_1^2 + I_2 \omega_2^2 + I_3 \omega_3^2 \right) - \frac{mMG}{r} \quad -(2)$$

in which the translational kinetic energy is:

$$T_{trans} = \frac{1}{2} m \dot{\underline{r}} \cdot \dot{\underline{r}} \quad -(3)$$

and the rotational kinetic energy is:

$$T_{rot} = \frac{1}{2} \sum_{i=1 \text{ to } 3} I_i \omega_i^2 \quad -(4)$$

2) Here I_1, I_2 and I_3 are the three principal moments of inertia and

$$\omega_1 = \dot{\phi}_1 + \dot{\theta}_1 + \dot{\psi}_1 = \dot{\phi} \sin\theta \sin\psi + \dot{\theta} \cos\psi \quad -(5)$$

$$\omega_2 = \dot{\phi}_2 + \dot{\theta}_2 + \dot{\psi}_2 = \dot{\phi} \sin\theta \cos\psi - \dot{\theta} \sin\psi \quad -(6)$$

$$\omega_3 = \dot{\phi}_3 + \dot{\theta}_3 + \dot{\psi}_3 = \dot{\phi} \cos\theta + \dot{\psi} \quad -(7)$$

Here θ, ϕ and ψ are the three Euler angles. The rotational motion is solved with the three simultaneous differential equations:

$$\frac{\partial \mathcal{L}}{\partial \theta} = \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}}\right) \quad -(8)$$

$$\frac{\partial \mathcal{L}}{\partial \phi} = \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\phi}}\right) \quad -(9)$$

$$\frac{\partial \mathcal{L}}{\partial \psi} = \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\psi}}\right) \quad -(10)$$

to give $\theta(t)$, $\phi(t)$, $\psi(t)$, $\dot{\theta}(t)$, $\dot{\phi}(t)$, $\dot{\psi}(t)$, ω_1, ω_2 and ω_3 and other functions. The orbital problem is solved using the plane polar coordinate system (r, θ_1), and the

) Euler Lagrange equation:

$$m \frac{d\dot{\underline{r}}}{dt} = -\underline{\nabla} U(r) \quad -(11)$$

which gives the two simultaneous differential equations:

$$\ddot{r} - r\dot{\theta}_1^2 = -\frac{MG}{r^2} \quad -(12)$$

and

$$r\ddot{\theta}_1 + 2\dot{\theta}_1 \dot{r} = 0 \quad -(13)$$

with:

$$L_{\theta_1} = mr^2\dot{\theta}_1 = \text{constant}, \quad -(14)$$

i.e.

$$\frac{dL_{\theta_1}}{dt} = 0 \quad -(15)$$

In this case the set of equations (8) to (10) is independent of the set of equations (11) to (15). This means that there is no dependence of the nutations and precession of the earth on its orbit. In the case of the Milankowitch cycles the general theory seems to indicate that θ, ϕ and ψ depend on r and θ_1. Such a dependence can be introduced through:

$$U = U(r, \theta_1, \theta, \phi, \psi) \quad -(16)$$

in general. This theory is to be developed in UFT370.