369(5): Checking the Spin Connection

We have:

$$\frac{D}{Dt}\begin{bmatrix}\dot{r}\\ r\dot{\theta}\\ r\sin\theta\,\dot{\phi}\end{bmatrix}=\frac{d}{dt}\begin{bmatrix}\dot{r}\\ r\dot{\theta}\\ r\sin\theta\,\dot{\phi}\end{bmatrix}+\begin{bmatrix}0 & -\dot{\theta} & -\sin\theta\,\dot{\phi}\\ \dot{\theta} & 0 & -\cos\theta\,\dot{\phi}\\ \sin\theta\,\dot{\phi} & \dot{\phi}\cos\theta & 0\end{bmatrix}\begin{bmatrix}\dot{r}\\ r\dot{\theta}\\ r\sin\theta\,\dot{\phi}\end{bmatrix} \quad -(1)$$

This gives:

$$a_r=\ddot{r}-r\dot{\theta}^2-r\dot{\phi}^2\sin^2\theta \quad -(2)$$ ✓

(checked by computer algebra)

$$a_\theta=\frac{d}{dt}\left(r\dot{\theta}\right)+\dot{\theta}\dot{r}-r\cos\theta\sin\theta\,\dot{\phi}^2$$

$$=\dot{r}\dot{\theta}+r\ddot{\theta}+\dot{r}\dot{\theta}-r\cos\theta\sin\theta\,\dot{\phi}^2$$ ✓

$$=2\dot{r}\dot{\theta}+r\ddot{\theta}-r\sin\theta\cos\theta\,\dot{\phi}^2$$

(checked by computer algebra)

and

$$a_\phi=\frac{d}{dt}\left(r\sin\theta\,\dot{\phi}\right)+\dot{\phi}\dot{r}\sin\theta+r\dot{\theta}\dot{\phi}\cos\theta$$

$$=\dot{r}\sin\theta\,\dot{\phi}+r\frac{d}{dt}\left(\dot{\phi}\sin\theta\right)+\dot{\phi}\dot{r}\sin\theta+r\dot{\theta}\dot{\phi}\cos\theta$$

$$=\dot{r}\sin\theta\,\dot{\phi}+r\ddot{\phi}\sin\theta+r\dot{\phi}\dot{\theta}\cos\theta+\dot{\phi}\dot{r}\sin\theta+r\dot{\theta}\dot{\phi}\cos\theta$$

$$=2\dot{\phi}\dot{r}\sin\theta+2r\dot{\phi}\dot{\theta}\cos\theta+r\ddot{\phi}\sin\theta$$ ✓

(Double checked by hand).