Note 365(5): Motion of Stars in a Whirlpool Galaxy

In conventional classical dynamics the motion is governed by the Binet equation:

$$F = -\frac{L^2}{mr^2}\left(\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r}\right) \quad -(1)$$

If:

$$r = \frac{r_0}{\theta} \quad -(2)$$

then:

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) = 0 \quad -(3)$$

so

$$F = -\frac{L^2}{mr^3} \quad -(4)$$

and

$$m\ddot{r} = 0 \quad -(5)$$

In this case:

$$\frac{dr}{dt} = \frac{dr}{d\theta}\frac{d\theta}{dt} = \frac{L}{mr^2}\frac{dr}{d\theta}$$

$$= -\frac{L}{mr^2}\frac{r_0}{\theta^2} = -\frac{L}{mr_0}. \quad -(6)$$

so

$$r = -\frac{L}{mr_0}t. \quad -(7)$$

There is a contradiction between eqs. (4) and (7) because eq. (4) is an outward, centrifugal force and eq. (7) indicates that r becomes more negative with increasing t.

This contradiction is resolved by using:

$$\omega = \frac{d\theta}{dt} = -\frac{L}{mr^2} \quad -(8)$$

2) i.e. by reversing the sign of the angular momentum:

$$\underline{L} = \underline{r} \times \underline{p} \quad -(9)$$

to

$$\underline{L} = -\underline{r} \times \underline{p} \quad -(10)$$

This is equivalent to reversing the motion of $\underline{p}$ or using motion reversal:

$$t \rightarrow -t \quad -(11)$$

The magnitude of $\underline{L}$, denoted L, is not changed by motion reversal, so:

$$\hat{T}(L) = L \quad -(12)$$

where $\hat{T}$ is the motion reversal operator. The angular velocity is

$$\underline{\omega} = \omega \underline{k} \quad -(13)$$

and

$$\hat{T}(\underline{\omega}) = -\underline{\omega} \quad -(14)$$

so eq. (8) results, QED.

Motion reversal changes eq. (7) to

$$r = \frac{L}{m r_0} t \quad -(15)$$

if

$$\frac{d\theta}{dt} = -\frac{L}{mr^2} \quad -(16)$$

The centrifugal force (4) has the property:

$$\hat{T}(F) = F \quad -(17)$$

because:

$$\hat{T}\left(\frac{L^2}{mr^3}\right) = \frac{L^2}{mr^3} \quad -(18)$$

Eqs. (4) and (15) mean that the stars

3) move outwards.

However, if we use:

$$r = -\frac{r_0}{\theta} \quad -(19)$$

and

$$\omega = -\frac{L}{mr^2} \quad -(20)$$

it follows that:

$$r = -\frac{L}{mr_0}t. \quad -(21)$$

The centrifugal force from eqs. (4) and (19) is:

$$F = \frac{L^2\theta^3}{mr_0^3} \quad -(22)$$

and is positive valued. Eqs. (21) and (22) show that the stars move inwards.
