Note 365(4): Central Potential and Absence of Coriolis Accelerations

Considering as in Note 365(1) the classical hamiltonian

$$H = \frac{1}{2} m \dot{\underline{r}} \cdot \dot{\underline{r}} + U \quad -(1)$$

and the lagrangian:
$$\mathcal{L} = \frac{1}{2} m \dot{\underline{r}} \cdot \dot{\underline{r}} - U \quad -(2)$$

the Euler Lagrange equation:

$$\underline{\nabla} \mathcal{L} = \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{\underline{r}}} \quad -(3)$$

gives the force
$$\underline{F} = m \ddot{\underline{r}} = -\underline{\nabla} U \quad -(4)$$

For an assumed central potential in planar orbital theory:

$$U = -\frac{mMG}{r} \quad -(5)$$

the gradient is:

$$\underline{\nabla} U = \frac{\partial U}{\partial r} \underline{e}_r + \frac{1}{r} \frac{\partial U}{\partial \theta} \underline{e}_\theta \quad -(6)$$

$$= \frac{mMG}{r^2} \underline{e}_r$$

because:

$$\frac{\partial U}{\partial \theta} = 0 \quad -(7)$$

so:
$$\underline{F} = m \ddot{\underline{r}} = -\frac{mMG}{r^2} \underline{e}_r \quad -(8)$$

In plane polar coordinates:

$$\underline{F} = m\left(\left(\ddot{r} - r\dot{\theta}^2\right)\underline{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\underline{e}_\theta\right) \quad -(9)$$

$$= m\ddot{\underline{r}}$$

From eqs. (8) and (9) it follows that:

$$\underline{F} = m\left(\ddot{r} - r\dot{\theta}^2\right)\underline{e}_r = -\frac{mMG}{r^2}\underline{e}_r \quad -(10)$$

and

$$\left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\underline{e}_\theta = \underline{0} \quad -(11)$$

Therefore the result:

$$r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 \quad -(12)$$

for any orbit is due to the assumption (5), QED.

In a central force field between m orbiting M there are no Coriolis accelerations.

This result was found in other ways in previous UFT papers, but this method is more general.

Eq. (10) can be written as:

$$F = m\left(\ddot{r} - r\dot{\theta}^2\right) = -\frac{mMG}{r^2} \quad -(13)$$

$$= -\frac{L^2}{mr^2}\left(\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r}\right)$$

3) where:
$$\dot{\theta} = \frac{L}{mr^2} \quad \text{--- (14)}$$

so:
$$F = m\left(\ddot{r} - \frac{L^2}{mr^3}\right) = -\frac{L^2}{mr^2}\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) - \frac{L^2}{mr^3} \quad \text{--- (15)}$$

and
$$\boxed{m\ddot{r} = -\frac{L^2}{mr^2}\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right)} \quad \text{--- (16)}$$

For example, for the hyperbolic spiral orbit:
$$r = \frac{r_0}{\theta} \quad \text{--- (17)}$$

it follows that:
$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) = 0 \quad \text{--- (18)}$$

so
$$m\ddot{r} = 0 \quad \text{--- (19)}$$

and
$$F = -\frac{L^2}{mr^3} \quad \text{--- (20)}$$

Eq. (20) is a purely centrifugal force that comes from
$$F = m\left(\ddot{r} - r\dot{\theta}^2\right) = -\frac{L^2}{mr^2}\left(\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r}\right) \quad \text{--- (21)}$$

The stars are thrown outward by the force (20). These results are generalized by the spin connection.