365(6): Iterative Scheme for Calculating the Orbit from the Fluid Dynamic Binet Equation

This equation is:

$$\left(1+\Omega^1{}_{01r}\right)\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right)+\frac{1}{r}=-\frac{m r^2}{L^2}F(r) \quad -(1)$$

Step 1

In note 365(2) it was shown that the assumption of an inverse square law for $F(r)$ leads to the orbit:

$$r=\frac{\alpha}{1+\epsilon\cos\left(\frac{\theta}{\left(1+\Omega^1{}_{01r}\right)^{1/2}}\right)} \quad -(2)$$

where

$$\Omega^1{}_{01r}=\frac{\partial R}{\partial r}. \quad -(3)$$

Here

$$R=R\left(r(t),\theta(t),t\right) \quad -(4)$$

is the position of an element of fluid spacetime. In this approximation it was assumed that:

$$\frac{\partial R}{\partial \theta}\sim 0 \quad -(5)$$

The above can be regarded as the first step in an iterative scheme.

Step 2

This is to use eq. (2) in Eq. (1) to find the force law for the orbit (2). This can be

2) done by computer algebra. The refined force law is a function of $\partial R / \partial r$, and can be compared with the inverse square force law:

$$F(r) = -\frac{mMG}{r^2} \quad - (6)$$

It will be interesting to find the properties of $F(r)$, and compare it with the Einstein theory, whose force law is a combination of terms inverse square in r and inverse fourth power in r.

The true precessing ellipse is known from previous work to be the solution of two simultaneous equations, the ECE2 hamiltonian and lagrangian. The numerical true solution can be compared with the analytical eq. (2). The true orbit is not the Einstein orbit and is not:

$$r = \frac{\alpha}{1 + \epsilon \cos(x\theta)} \quad - (7)$$

where x is a constant. Note carefully that from eq. (2):

$$\frac{1}{r} = \frac{1}{\alpha}\left(1 + \epsilon \cos\left(\frac{\theta}{\left(1 + \frac{\partial R}{\partial r}\right)^{1/2}}\right)\right) \quad - (8)$$

and $\partial R / \partial r$ is a function of $r(t)$.