(2): Development of the Covariant Derivative in General Dynamics.

The acceleration is considered to be:

$$\underline{a} = \frac{\partial \underline{v}}{\partial t} + \left(\underline{v} \cdot \underline{\nabla}\right) \underline{v} \quad -(1)$$

and not

$$\underline{a} = \frac{d \underline{v}}{dt} \quad -(2)$$

In Cartesian coordinates in three dimensions:

$$\underline{a} = \frac{\partial \underline{v}}{\partial t} + \left(v_X \frac{\partial}{\partial X} + v_Y \frac{\partial}{\partial Y} + v_Z \frac{\partial}{\partial Z} \right) \underline{v} \quad -(3)$$

So:

$$\begin{bmatrix} a_X \\ a_Y \\ a_Z \end{bmatrix} = \frac{\partial}{\partial t} \begin{bmatrix} v_X \\ v_Y \\ v_Z \end{bmatrix} + \begin{bmatrix} \partial v_X / \partial X & \partial v_X / \partial Y & \partial v_X / \partial Z \\ \partial v_Y / \partial X & \partial v_Y / \partial Y & \partial v_Y / \partial Z \\ \partial v_Z / \partial X & \partial v_Z / \partial Y & \partial v_Z / \partial Z \end{bmatrix} \begin{bmatrix} v_X \\ v_Y \\ v_Z \end{bmatrix} \quad -(4)$$

In note 361(1) it was shown that eq. (1) is an example of the Cartan covariant derivative of $\underline{v}$ provided that the Jacobian matrix is the spin connection matrix:

$$\begin{bmatrix} \omega^1{}_{01} & \omega^1{}_{02} & \omega^1{}_{03} \\ \omega^2{}_{01} & \omega^2{}_{02} & \omega^2{}_{03} \\ \omega^3{}_{01} & \omega^3{}_{02} & \omega^3{}_{03} \end{bmatrix} = \begin{bmatrix} \partial v_X / \partial X & \partial v_X / \partial Y & \partial v_X / \partial Z \\ \partial v_Y / \partial X & \partial v_Y / \partial Y & \partial v_Y / \partial Z \\ \partial v_Z / \partial X & \partial v_Z / \partial Y & \partial v_Z / \partial Z \end{bmatrix} \quad -(5)$$

Therefore eq. (1) reduces to eq. (2)

if the spin connection and Jacobian matrices vanish. this means that the Cartan derivative becomes the ordinary derivative:

$$\frac{DV^a}{\partial x^\mu} = \frac{\partial V^a}{\partial x^\mu} \quad -(6)$$

Therefore eq. (1) is an equation of general relativity with a moving frame of reference. Eq. (2) is written in a static frame of reference.

The frame moves if and only if the spin connection and Jacobian matrices are non-zero. If they are zero we obtain the non relativistic limit, eq. (2). The latter equation means that:

$$\frac{\partial v_x}{\partial X} = \frac{\partial v_x}{\partial Y} = \frac{\partial v_x}{\partial Z} = \frac{\partial v_Y}{\partial X} = \frac{\partial v_Y}{\partial Y} = \frac{\partial v_Y}{\partial Z} = \frac{\partial v_Z}{\partial X} = \frac{\partial v_Z}{\partial Y} = \frac{\partial v_Z}{\partial Z}$$

$$= 0 \quad -(7)$$

Therefore the usual dynamics, the non-relativistic dynamics of eq. (2), mean that $\underline{v}$ is not a function of X, Y and Z. So:

$$\underline{v} = \underline{v}(t) \quad -(8)$$

and

$$\underline{a} = \frac{\partial \underline{v}}{\partial t} = \frac{d\underline{v}}{dt} \quad -(9)$$

i.e the partial derivative is the total derivative.

) Eq. (1) is well known in fluid dynamics and is known as the Euler equation. Within the context of ECE2 unified field theory and fluid gravitation, it can be applied to gravitational theory. Therefore the acceleration due to gravity is:

$$\underline{g} = \frac{\partial \underline{v}}{\partial t} + \left(\underline{v} \cdot \underline{\nabla} \right) \underline{v} \quad -(10)$$

In the Newtonian theory:

$$\underline{g} = \frac{d \underline{v}}{dt} = \frac{d^2 \underline{r}}{dt^2} = -\frac{mMG}{r^2} \underline{e}_r \quad -(11)$$

and in the Newtonian or inertial frame:

$$\left(\underline{v} \cdot \underline{\nabla} \right) \underline{v} = 0. \quad -(12)$$

and the mass m is attracted to M and eventually collides with it.

Without the presence of:

$$\left(\underline{v} \cdot \underline{\nabla} \right) \underline{v} \neq \underline{0} \quad -(13)$$

the inverse square law of Hooke, developed by Newton from 1665 to 1687, does not produce an orbit.

The reason is that:

$$\underline{g} = \frac{\partial \underline{v}}{\partial t} + \left(\underline{v} \cdot \underline{\nabla} \right) \underline{v} \quad -(14)$$

$$= \frac{\partial \underline{v}}{\partial t} - r\omega^2 \underline{e}_r \quad -(15)$$

for a two dimensional orbit. Eq. (15) is derived from the expression for acceleration in plane polar coordinates (r, θ):

$$\underline{a} = (\ddot{r} - r\dot{\theta}^2)\underline{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\underline{e}_\theta \quad -(16)$$

As shown in previous UFT papers:

$$r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 \quad -(17)$$

for a planar orbit.

Therefore in a planar orbit:

$$\underline{g}_1 = (\underline{v}\cdot\underline{\nabla})\underline{v} = -\omega^2 r \underline{e}_r \quad -(18)$$

is <u>the centrifugal acceleration</u>. In general:

$$\boxed{\underline{g}_1 = (\underline{v}\cdot\underline{\nabla})\underline{v} = -\underline{\omega}\times(\underline{\omega}\times\underline{r})} \quad -(19)$$

The minus sign means that the centrifugal force is directed outward from the centre of rotation.

Therefore centrifugal acceleration is a consequence of Cartan geometry, and the orbital equation is an equation of general relativity:

$$\underline{g} = \frac{d^2 \underline{r}}{dt^2} - \omega^2 r \, \underline{e}_r = -\frac{mMG}{r^2}\underline{e}_r \quad (20)$$

$$= \frac{d^2 \underline{r}}{dt^2} + (\underline{v} \cdot \underline{\nabla}) \underline{v}$$

This is the Leibnitz equation of 1689. It was unknown to Newton.

Therefore:

$$\frac{d^2 \underline{r}}{dt^2} = \omega^2 r \, \underline{e}_r - \frac{mMG}{r^2} \underline{e}_r$$

$$= \left(\omega^2 r - \frac{mMG}{r^2} \right) \underline{e}_r \quad - (21)$$

$$= -(\underline{v} \cdot \underline{\nabla}) \underline{v} - \frac{mMG}{r^2} \underline{e}_r$$

In general:

$$\underline{v} = \underline{v}(t, X, Y) \quad - (22)$$

for an orbit, but in Newtonian dynamics:

$$\underline{v} = \underline{v}(t) \quad - (23)$$

$$= \frac{d\underline{r}}{dt}$$