

359(2): Newtonian Velocity Field in Three Dimensions

This is:

$$\underline{g} = \frac{1}{2(x^2 + y^2 + z^2)^{3/2}} \left((x^2 + y^2)^{3/2} \underline{g}_1 + (x^2 + z^2)^{3/2} \underline{g}_2 + (y^2 + z^2)^{3/2} \underline{g}_3 \right) \quad - (1)$$

where:

$$\underline{g}_1 = -\frac{mG}{(x^2 + y^2)^{3/2}} (x \underline{i} + y \underline{j}) \quad - (2)$$

$$\underline{g}_2 = -\frac{mG}{(y^2 + z^2)^{3/2}} (y \underline{j} + z \underline{k}) \quad - (3)$$

$$\underline{g}_3 = -\frac{mG}{(x^2 + z^2)^{3/2}} (x \underline{i} + z \underline{k}) \quad - (4)$$

and

$$\underline{g}_1 = (\underline{v}_{F1} \cdot \underline{\nabla}) \underline{v}_{F1} \quad - (5)$$

$$\underline{g}_2 = (\underline{v}_{F2} \cdot \underline{\nabla}) \underline{v}_{F2} \quad - (6)$$

$$\underline{g}_3 = (\underline{v}_{F3} \cdot \underline{\nabla}) \underline{v}_{F3} \quad - (7)$$

Here:

$$\underline{v}_{F1} = \frac{(mG)^{1/2} (-y \underline{i} + x \underline{j})}{(x^2 + y^2)^{3/2}} \quad (a)$$

2)

$$\underline{V}_{F2} = (mG)^{1/3} \left(\frac{-Z\underline{i} + X\underline{j}}{(X^2 + Z^2)^{3/2}} \right) \quad - (9)$$

$$\underline{V}_{F3} = (mG)^{1/3} \left(\frac{-Z\underline{i} + Y\underline{j}}{(Y^2 + Z^2)^{3/2}} \right) \quad - (10)$$

Therefore:

$$\underline{g} = -\frac{mG}{r^3} \underline{e}_r \quad - (11)$$

$$\text{also } r^2 = X^2 + Y^2 + Z^2 \quad - (12)$$

QED.