Note 359(1): Velocity Field for a Newtonian or Coulombic Potential.

For a Newtonian potential it is required that:

$$\underline{g} = -\frac{mG}{r^2}\underline{e}_r = (\underline{v}_F \cdot \underline{\nabla})\underline{v}_F. \quad -(1)$$

For a Coulombic potential the same inverse square law applies on the left hand side, so:

$$\underline{E} = -\frac{e}{4\pi\epsilon_0 r^2}\underline{e}_r = x(\underline{v}_F \cdot \underline{\nabla})\underline{v}_F \quad -(2)$$

where x is a proportionality constant.

Let:

$$\underline{v}_F = A f(r)(-Y\underline{i} + X\underline{j}) \quad -(3)$$

where A is a constant to be determined and

$$f(r) = \frac{1}{r^{3/2}} = \frac{1}{(X^2+Y^2)^{3/2}} \quad -(4)$$

It follows that:

$$\underline{v}_F \cdot \underline{\nabla} = A f(r)\left(-Y\frac{\partial}{\partial X} + X\frac{\partial}{\partial Y}\right) \quad -(5)$$

and:

$$(\underline{v}_F \cdot \underline{\nabla})\underline{v}_F = A^2 f(r)\left(-Y\frac{\partial}{\partial X} + X\frac{\partial}{\partial Y}\right)\left(f(r)(-Y\underline{i} + X\underline{j})\right)$$

$$= A^2 f^2(r)\left(-Y\frac{\partial}{\partial X} + X\frac{\partial}{\partial Y}\right)(-Y\underline{i} + X\underline{j})$$

$$+ A^2 f(r)(-Y\underline{i} + X\underline{j})\left(-Y\frac{\partial}{\partial X} + X\frac{\partial}{\partial Y}\right)f(r) \quad -(6)$$

2) by using the Leibnitz Theorem.

Now note that:

$$-Y\frac{\partial f(r)}{\partial X} + X\frac{\partial f(r)}{\partial Y} = 0. \quad -(7)$$

This result follows from the fact that:

$$f = \frac{1}{r^{3/2}} = \frac{1}{(X^2+Y^2)^{3/2}} \quad -(8)$$

so

$$\frac{\partial f}{\partial X} = -2X\frac{(X^2+Y^2)^{1/2}}{(X^2+Y^2)^3} \quad -(9)$$

and

$$\frac{\partial f}{\partial Y} = -2Y\frac{(X^2+Y^2)^{1/2}}{(X^2+Y^2)^3} \quad -(10)$$

and eq. (7) follows, Q. E. D.

Therefore from eq. (6):

$$\underline{g} = -A^2\frac{\underline{r}}{r^3} = -\frac{A^2}{r^2}\underline{e}_r \quad -(11)$$

$$= -\frac{mG}{r^2}\underline{e}_r$$

if

$$A = (mG)^{1/2} \quad -(12)$$

<u>Q. E. D.</u>, or

$$A^2 = mG \quad -(13)$$

3)

Therefore the spacetime velocity field needed for Newtonian gravitation is:

$$\boxed{\underline{v}_F = \left(\frac{mG}{r^3}\right)^{1/2}\left(-Y\,\underline{i} + X\,\underline{j}\right)} \quad -(14)$$

This result can be written as:

$$\boxed{\underline{v}_F = (mG)^{1/2}\left(\frac{-Y\,\underline{i} + X\,\underline{j}}{(X^2+Y^2)^{3/2}}\right)} \quad -(15)$$

<u>Units</u> <u>Check</u>

$$g = -\frac{mG}{r^2}, \quad \text{so } mG = m^3 s^{-2}$$

and

$$v = \frac{m^{3/2}\, s^{-1}\, m}{m^{3/2}} = m\, s^{-1} \quad ✓$$

The velocity field needed for a whirlpool galaxy is

$$\underline{v}_F = \frac{L_{Fz}}{m_r(X^2+Y^2)}\left(-Y\,\underline{i} + X\,\underline{j}\right) \quad -(16)$$

Eq. (15) is also the velocity field needed for a Coulomb potential.