359(7): A New and General Dynamics for Planar Orbits.

Consider the orbital velocity in plane polar coordinates:

$$\underline{v} = \dot{r}\,\underline{e}_r + r\dot{\theta}\,\underline{e}_\theta \quad -(1)$$

where

$$\underline{e}_r = \underline{i}\cos\theta + \underline{j}\sin\theta \quad -(2)$$

and

$$\underline{e}_\theta = -\underline{i}\sin\theta + \underline{j}\cos\theta \quad -(3)$$

In fluid gravitation there is a new and general relation between the orbital velocity $\underline{v}$ and the Newtonian acceleration $\underline{g}$:

$$\boxed{\underline{g} = (\underline{v}\cdot\underline{\nabla})\,\underline{v}} \quad -(4)$$

This is a new type of general dynamics. The Newtonian acceleration is the convective derivative of the orbital velocity.

For a planar orbit in the XY plane:

$$\underline{v} = (MG)^{1/2}\,\frac{(-Y\,\underline{i} + X\,\underline{j})}{(X^2+Y^2)^{3/4}} \quad -(5)$$

It follows that:

$$(MG)^{1/2}\,\frac{X}{(X^2+Y^2)^{3/4}} = \dot{r}\sin\theta + r\dot{\theta}\cos\theta \quad -(6)$$

$$(MG)^{1/2}\,\frac{Y}{(X^2+Y^2)^{3/4}} = r\dot{\theta}\sin\theta - \dot{r}\cos\theta \quad -(7)$$

2)

and:

$$v^2 = \dot{r}^2 + r\dot{\theta}^2 = \frac{mG}{(X^2+Y^2)^{1/2}} \quad -(8)$$

This is a general result valid for all planar orbits, for example the conic sections and the whirlpool galaxy.

It follows that:

$$X = \frac{(mG)^{1/2}}{v^3}\left(\dot{r}\sin\theta + r\dot{\theta}\cos\theta\right) \quad -(9)$$

$$= (mG)^{1/2}\frac{(\dot{r}\sin\theta + r\dot{\theta}\cos\theta)}{(\dot{r}^2 + r\dot{\theta}^2)^{3/2}}$$

and

$$Y = (mG)^{1/2}\frac{(r\dot{\theta}\sin\theta - \dot{r}\cos\theta)}{(\dot{r}^2 + r\dot{\theta}^2)^{3/2}} \quad -(10)$$

These are also general results valid for all planar orbits.

Therefore eq. (4) is a new general equation of all planar orbits, and also three dimensional orbits.
