1) Note 359(1): Velocity Field for a Newtonian or Coulombic Potential.

For a Newtonian potential it is required that:

$$\underline{g} = -\frac{mG}{r^2}\,\underline{e}_r = (\underline{v}_F\cdot\underline{\nabla})\,\underline{v}_F. \quad -(1)$$

For a Coulombic potential the same inverse square law applies on the left hand side, so:

$$\underline{E} = -\frac{e}{4\pi\epsilon_0 r^2}\,\underline{e}_r = x(\underline{v}_F\cdot\underline{\nabla})\,\underline{v}_F \quad -(2)$$

where x is a proportionality constant.

Let:

$$\underline{v}_F = A f(r)(-Y\underline{i} + X\underline{j}) \quad -(3)$$

where A is a constant to be determined and

$$f(r) = \frac{1}{r^{3/2}} = \frac{1}{(X^2+Y^2)^{3/4}} \quad -(4)$$

It follows that:

$$\underline{v}_F\cdot\underline{\nabla} = A f(r)\left(-Y\frac{\partial}{\partial X} + X\frac{\partial}{\partial Y}\right) \quad -(5)$$

and:

$$(\underline{v}_F\cdot\underline{\nabla})\,\underline{v}_F = A^2 f(r)\left(-Y\frac{\partial}{\partial X} + X\frac{\partial}{\partial Y}\right)\left(f(r)(-Y\underline{i} + X\underline{j})\right)$$

$$= A^2 f^2(r)\left(-Y\frac{\partial}{\partial X} + X\frac{\partial}{\partial Y}\right)(-Y\underline{i} + X\underline{j})$$

$$+ A^2 f(r)(-Y\underline{i} + X\underline{j})\left(-Y\frac{\partial}{\partial X} + X\frac{\partial}{\partial Y}\right)f(r) \quad -(6)$$

2) by using the Leibnitz Theorem.

Now note that:

$$-Y \frac{\partial f(r)}{\partial X} + X \frac{\partial f(r)}{\partial Y} = 0 \quad -(7)$$

This result follows from the fact that:

$$f = \frac{1}{r^{3/2}} = \frac{1}{(X^2+Y^2)^{3/4}} \quad -(8)$$

So

$$\frac{\partial f}{\partial X} = -\frac{3}{2} \frac{X(X^2+Y^2)^{-1/4}}{(X^2+Y^2)^{3/2}}$$

$$= -\frac{3}{2} X (X^2+Y^2)^{-7/4} \quad -(9)$$

and

$$\frac{\partial f}{\partial Y} = -\frac{3}{2} Y (X^2+Y^2)^{-7/4} \quad -(10)$$

and eq. (7) follows, Q.E.D.

Therefore from eq. (6):

$$\underline{g} = -mG \frac{\underline{r}}{r^3}, \quad -(11)$$

$$\underline{g} = -\frac{mG}{r^2} \underline{e}_r \quad -(12)$$

Q.E.D, where:

$$\underline{r} = r \, \underline{e}_r . \quad -(13)$$

3)

Therefore the spacetime-velocity field needed for Newtonian gravitation is:

$$\boxed{\underline{v}_F = \left(\frac{mG}{r^3}\right)^{1/2} \left(-Y\,\underline{i} + X\,\underline{j}\right)} \quad -(14)$$

This result can be written as:

$$\boxed{\underline{v}_F = (mG)^{1/2} \left(\frac{-Y\,\underline{i} + X\,\underline{j}}{(X^2+Y^2)^{3/4}}\right)} \quad -(15)$$

<u>Units</u> <u>Check</u>

$$g = -\frac{mG}{r^2}, \quad \text{so } mG = m^3 s^{-2}$$

and $$v = \frac{m^{3/2}\, s^{-1}}{m^{3/2}}\, m = m\, s^{-1} \quad \checkmark$$

The velocity field needed for a whirlpool galaxy is

$$\underline{v}_F = \frac{L_{FZ}}{m_r (X^2+Y^2)} \left(-Y\,\underline{i} + X\,\underline{j}\right) \quad -(16)$$

Eq. (15) is also the velocity field needed for a Coulomb potential.

359(2): Newtonian Velocity Field in Three Dimensions

This is:

$$\underline{g} = \frac{1}{2(x^2+y^2+z^2)^{3/2}} \Big((X^2+Y^2)^{3/2} \underline{g}_1 + (X^2+Z^2)^{3/2} \underline{g}_2 + (Y^2+Z^2)^{3/2} \underline{g}_3 \Big) \quad -(1)$$

where:

$$\underline{g}_1 = -\frac{mG}{(x^2+y^2)^{3/2}} \left(X \underline{i} + Y \underline{j} \right) \quad -(2)$$

$$\underline{g}_2 = -\frac{mG}{(y^2+z^2)^{3/2}} \left(Y \underline{j} + Z \underline{k} \right) \quad -(3)$$

$$\underline{g}_3 = -\frac{mG}{(x^2+z^2)^{3/2}} \left(X \underline{i} + Z \underline{k} \right) \quad -(4)$$

and

$$\underline{g}_1 = \left(\underline{v}_{F1} \cdot \underline{\nabla} \right) \underline{v}_{F1} \quad -(5)$$

$$\underline{g}_2 = \left(\underline{v}_{F2} \cdot \underline{\nabla} \right) \underline{v}_{F2} \quad -(6)$$

$$\underline{g}_3 = \left(\underline{v}_{F3} \cdot \underline{\nabla} \right) \underline{v}_{F3} . \quad -(7)$$

Here:

$$\underline{v}_{F1} = (mG)^{1/2} \frac{\left(-Y \underline{i} + X \underline{j} \right)}{(x^2+y^2)^{3/4}}$$

2)

$$\underline{V}_{F2} = (mG)^{1/2} \frac{\left(-Z\,\underline{k} + X\,\underline{i}\right)}{\left(X^2 + Z^2\right)^{3/4}} \quad - (9)$$

$$\underline{V}_{F3} = (mG)^{1/2} \left(\frac{-Z\,\underline{k} + Y\,\underline{j}}{\left(Y^2 + Z^2\right)^{3/4}} \right) \quad - (10)$$

Therefore:

$$\underline{g} = -\frac{mG}{r^2}\,\underline{e}_r \quad - (11)$$

where

$$r^2 = X^2 + Y^2 + Z^2 \quad - (12)$$

QED.

59(3): Details of the Three Dimensional Calculation

Consider the velocity fields:

$$\underline{v}_{F1} = \frac{A}{(X^2+Y^2)^{3/4}}\left(-Y\underline{i} + X\underline{j}\right) \quad -(1)$$

$$\underline{v}_{F2} = \frac{A}{(X^2+Z^2)^{3/4}}\left(-Z\underline{i} + X\underline{k}\right) \quad -(2)$$

$$\underline{v}_{F3} = \frac{A}{(Y^2+Z^2)^{3/4}}\left(-Z\underline{j} + Y\underline{k}\right) \quad -(3)$$

In three dimensions:

$$\underline{\nabla} = \underline{i}\frac{\partial}{\partial X} + \underline{j}\frac{\partial}{\partial Y} + \underline{k}\frac{\partial}{\partial Z} \quad -(4)$$

It follows that:

$$\underline{v}_{F1}\cdot\underline{\nabla} = \frac{A}{(X^2+Y^2)^{3/4}}\left(-Y\frac{\partial}{\partial X} + X\frac{\partial}{\partial Y}\right) \quad -(5)$$

$$\underline{v}_{F2}\cdot\underline{\nabla} = \frac{A}{(X^2+Z^2)^{3/4}}\left(-Z\frac{\partial}{\partial X} + X\frac{\partial}{\partial Z}\right) \quad -(6)$$

$$\underline{v}_{F3}\cdot\underline{\nabla} = \frac{A}{(Y^2+Z^2)^{3/4}}\left(-Z\frac{\partial}{\partial Y} + Y\frac{\partial}{\partial Z}\right) \quad -(7)$$

Now define:

$$\underline{g}_1 = \left(\underline{v}_{F1}\cdot\underline{\nabla}\right)\underline{v}_{F1} \quad -(8)$$

$$\underline{g}_2 = \left(\underline{v}_{F2} \cdot \underline{\nabla}\right) \underline{v}_{F2} \quad - (9)$$

$$\underline{g}_3 = \left(\underline{v}_{F3} \cdot \underline{\nabla}\right) \underline{v}_{F3} \quad - (10)$$

As in Note 359(1):

$$\underline{g}_1 = -\frac{mG}{(X^2+Y^2)^{3/2}} \left(X\underline{i} + Y\underline{j}\right) - (11)$$

using

$$A^2 = mG. \quad - (12)$$

Now consider:

$$\left(\underline{v}_{F2} \cdot \underline{\nabla}\right) \underline{v}_{F2} = A^2 f_2 \left(-Z\frac{\partial}{\partial X} + X\frac{\partial}{\partial Z}\right)\left(f_2\left(-Z\underline{i} + X\underline{k}\right)\right).$$

$$= A^2 f_2 \left(-Z\frac{\partial}{\partial X} + X\frac{\partial}{\partial Z}\right)\left(-Z\underline{i} + X\underline{k}\right)$$

$$+ A^2 f_2 \left(-Z\underline{i} + X\underline{k}\right)\left(-Z\frac{\partial}{\partial X} + X\frac{\partial}{\partial Z}\right) f_2 . \quad - (13)$$

Note that:

$$-Z\frac{\partial f_2}{\partial X} + X\frac{\partial f_2}{\partial Z} = 0 \quad - (14)$$

because:

$$f_2 = (X^2+Z^2)^{-3/4} - (15)$$

so

$$\frac{\partial f_2}{\partial X} = -\frac{3}{2}\frac{X(X^2+Z^2)^{-1/4}}{(X^2+Z^2)^{3/2}} - (16)$$

and: $$\frac{\partial f_2}{\partial Z} = -\frac{3Z}{2}\frac{(X^2+Z^2)^{-1/4}}{(X^2+Z^2)^{3/2}} \quad -(17)$$

Q.E.D.

So:

$$\underline{g}_2 = (\underline{V}_{F2}\cdot\underline{\nabla})\underline{V}_{F2}$$
$$= \frac{A^2}{(X^2+Z^2)^{3/2}}\left(-Z\frac{\partial}{\partial X}+X\frac{\partial}{\partial Z}\right)(-Z\underline{i}+X\underline{k})$$
$$= -\frac{mG}{(X^2+Z^2)^{3/2}}(X\underline{i}+Z\underline{k}) \quad -(18).$$

Finally consider:

$$(\underline{V}_{F3}\cdot\underline{\nabla})\underline{V}_{F3} = A^2 f_3\left(-Z\frac{\partial}{\partial Y}+Y\frac{\partial}{\partial Z}\right)\left(f_3(-Z\underline{j}+Y\underline{k})\right)$$
$$= A^2 f_3\left(-Z\frac{\partial}{\partial Y}+Y\frac{\partial}{\partial Z}\right)(-Z\underline{j}+Y\underline{k})$$
$$+A^2 f_3(-Z\underline{j}+Y\underline{k})\left(-Z\frac{\partial}{\partial Y}+Y\frac{\partial}{\partial Z}\right)f_3$$
$$= -\frac{mG}{(Y^2+Z^2)^{3/2}}(Y\underline{j}+Z\underline{k}) \quad -(19)$$

Therefore:

4)

Therefore:

$$\underline{g}_1 + \underline{g}_2 + \underline{g}_3$$

$$= -mG\left(\frac{X\underline{i}+Y\underline{j}}{(X^2+Y^2)^{3/2}} + \frac{X\underline{i}+Z\underline{k}}{(X^2+Z^2)^{3/2}} + \frac{Y\underline{j}+Z\underline{k}}{(Y^2+Z^2)^{3/2}}\right)$$

$$= (\underline{v}_{F1}\cdot\underline{\nabla})\underline{v}_{F1} + (\underline{v}_{F2}\cdot\underline{\nabla})\underline{v}_{F2} + (\underline{v}_{F3}\cdot\underline{\nabla})\underline{v}_{F3} \quad -(20)$$

The Newtonian gravitational field is defined as:

$$\underline{g} = \frac{1}{2(X^2+Y^2+Z^2)^{3/2}}\left((X^2+Y^2)^{3/2}\underline{g}_1 + (X^2+Z^2)^{3/2}\underline{g}_2 + (Y^2+Z^2)^{3/2}\underline{g}_3\right)$$

$$= -\frac{mG}{r^3}\underline{r} \quad -(21)$$

where

$$\underline{r} = X\underline{i} + Y\underline{j} + Z\underline{k}. \quad -(22)$$

i.e.

$$\underline{g} = -\frac{mG}{r^2}\underline{e}_r, \quad -(23)$$

where

$$\underline{r} = r\,\underline{e}_r \quad -(24)$$

59(4): Calculation of Charges and Vorticity

Consider the velocity fields:

$$\underline{v}_{F1} = \frac{(mG)^{1/2}}{(X^2+Y^2)^{3/4}}(-Y\underline{i}+X\underline{j}) \quad -(1)$$

$$\underline{v}_{F2} = \frac{(mG)^{1/2}}{(X^2+Z^2)^{3/4}}(-Z\underline{i}+X\underline{k}) \quad -(2)$$

$$\underline{v}_{F3} = \frac{(mG)^{1/2}}{(Y^2+Z^2)^{3/4}}(-Z\underline{j}+Y\underline{k}) \quad -(3)$$

These produce the three Kambe charges:

$$q_{F1} = \underline{\nabla}\cdot\underline{g}_{F1} = \frac{mG}{(X^2+Y^2)^{3/2}} \quad -(4)$$

$$q_{F2} = \underline{\nabla}\cdot\underline{g}_{F2} = \frac{mG}{(X^2+Z^2)^{3/2}} \quad -(5)$$

$$q_{F3} = \underline{\nabla}\cdot\underline{g}_{F3} = \frac{mG}{(Y^2+Z^2)^{3/2}} \quad -(6)$$

where

$$\underline{g}_i = (\underline{v}_{Fi}\cdot\underline{\nabla})\underline{v}_{Fi} \quad -(6a)$$

$$i = 1, 2, 3$$

These spacetime or vacuum frequencies are ~~generated by~~ a Newtonian or Coulombic field and are related to mass density.

The three velocity fields (1) to (3) also

generate the three spacetime vorticities or Cartesian
magnetic fields:

$$\underline{W}_{F1} = \underline{\nabla} \times \underline{V}_{F1} = -\frac{3}{2} \frac{(X+Y)(MG)^{1/2}}{(X^2+Y^2)^{7/2}} \underline{k} \quad -(7)$$

$$\underline{W}_{F2} = \underline{\nabla} \times \underline{V}_{F2} = -\frac{3}{2} \frac{(X+Z)(MG)^{1/2}}{(X^2+Y^2)^{7/2}} \underline{j} \quad -(8)$$

$$\underline{W}_{F3} = \underline{\nabla} \times \underline{V}_{F3} = -\frac{3}{2} \frac{(Y+Z)(MG)^{1/2}}{(Y^2+Z^2)^{7/2}} \underline{i} \quad -(9)$$

These spacetime or vacuum or aether vortices are generated by a Newtonian or Coulombic field. These properties of the vacuum can be graphed in Cartesian or spherical polar coordinates. These graphs would show a richly structured vacuum or aether or spacetime. The Newtonian acceleration itself is defined as in Note 359(3). The gravitomagnetic fields have the property

$$\underline{\nabla} \cdot \underline{W}_{F1} = \underline{\nabla} \cdot \underline{W}_{F2} = \underline{\nabla} \cdot \underline{W}_{F3} = 0 \quad -(10)$$

as can be verified from eqs. (7) to (9) using:

$$\underline{\nabla} = \underline{i}\frac{\partial}{\partial X} + \underline{j}\frac{\partial}{\partial Y} + \underline{k}\frac{\partial}{\partial Z} \quad -(11)$$

It follows that

$$\underline{\nabla} \cdot \underline{W}_F = 0 \quad -(12)$$

where

$$\underline{W}_F = \underline{W}_{F1} + \underline{W}_{F2} + \underline{W}_{F3} \quad -(13)$$

is the total vorticity of the vacuum. The total vorticity can be graphed in Cartesian and spherical polar coordinates.