

358(6): Spacetime Current of the Whirlpool Galaxy.

This is defined as:

$$\underline{J}_F = a_0^2 \underline{\nabla} \times (\underline{\nabla} \times \underline{v}_F) - \frac{\partial}{\partial t} \left((\underline{v}_F \cdot \underline{\nabla}) \underline{v}_F \right)$$

where $\underline{v}_F = \frac{L_F Z}{m r_F^2} (-Y \underline{i} + X \underline{j}) = \begin{pmatrix} -Y \\ X \\ 0 \end{pmatrix}$

is the velocity field of spacetime associated with a whirlpool galaxy.

As in previous notes:

$$\begin{aligned} \underline{g}(\text{matter}) &= (\underline{v}_F \cdot \underline{\nabla}) \underline{v}_F - (3) \\ &= -\frac{L_F Z}{m^2 r_F^3} \underline{e}_r \end{aligned}$$

So if r_F is independent of time:

$$\frac{\partial}{\partial t} \left((\underline{v}_F \cdot \underline{\nabla}) \underline{v}_F \right) = \underline{0} - (4)$$

As in previous work: - (5)

$$\underline{w} = \underline{\nabla} \times \underline{v}_F = \frac{2L_F Z}{m r_F^2} \underline{k} = \frac{2L_F Z}{m(x^2 + y^2)} \underline{k}$$

Therefore:

$$\underline{\nabla} \times (\underline{\nabla} \times \underline{v}_F) = \frac{2L_F Z}{m} \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & \frac{1}{(x^2 + y^2)} \end{vmatrix} - (6)$$

2) So:

$$I_F = \frac{4a_0^2 L_F z}{m r_F^4} (-\underline{y}_i + \underline{x}_j) - (7)$$

$$= \frac{4a_0^2}{r_F^2} \underline{v}_F$$

where

$$r_F^2 = x^2 + y^2 - (8)$$

This is the spacetime current that gives
rise to the whirlpool galaxy.
