358(2): The Origin of Mass Density in Spacetime

In fluid gravitation, the acceleration due to gravity is defined in note 358(1):

$$\underline{g}\,(\text{matter}) = \left(\left(\underline{v}_F \cdot \underline{\nabla}\right)\underline{v}_F\right)(\text{spacetime}) \qquad -(1)$$

$$= \left(-\frac{\partial \underline{v}_F}{\partial t} - \underline{\nabla}\,\underline{\Phi}\right)(\text{spacetime})$$

where:

$$\underline{\Phi}\,(\text{spacetime}) = h_F(\text{spacetime}). \qquad -(2)$$

Consider the ECE2 gravitational field equation:

$$\underline{\nabla} \cdot \underline{g}\,(\text{matter}) = 4\pi G \rho_m (\text{matter})$$

$$= \underline{\kappa}\,(\text{matter}) \cdot \underline{g}\,(\text{matter}) \qquad -(3)$$

in the notation of the ECE2 papers. It follows that:

$$\underline{\nabla} \cdot \underline{g}\,(\text{matter}) = \left(\underline{\nabla} \cdot \left(\left(\underline{v}_F \cdot \underline{\nabla}\right)\underline{v}_F\right)\right)(\text{spacetime})$$

$$= q_F(\text{spacetime}) \qquad -(4)$$

$$= 4\pi G \rho_m (\text{matter})$$

Therefore:

$$\boxed{\rho_m(\text{matter}) = \frac{q_F(\text{spacetime})}{4\pi G}} \qquad -(5)$$

Therefore mass density is a spacetime property,

d) it is the charge q_F of fluid spacetime within a proportionality constant $4\pi G$. Here:

$$q_F(\text{spacetime}) = (\underline{\nabla} \cdot \underline{E}_F)(\text{spacetime}) \quad -(6)$$

In the particular case of the Newtonian gravitational field:

$$\underline{g}(\text{matter}) = -\frac{MG}{r^2}\underline{e}_r \quad -(7)$$

then

$$\underline{\nabla} \cdot \left(-\frac{MG}{r^2}\underline{e}_r\right) = \underline{\kappa} \cdot \left(-\frac{MG}{r^2}\underline{e}_r\right), \quad -(8)$$

i.e.

$$\frac{\partial}{\partial r}\left(\frac{1}{r^2}\right) = \frac{\kappa_r}{r^2} \quad -(9)$$

and

$$\kappa_r = -\frac{2}{r} \quad -(10)$$

In general:

$$\boxed{\left(\underline{\nabla} \cdot ((\underline{v}_F \cdot \underline{\nabla})\underline{v}_F)\right)(\text{spacetime}) = 4\pi G \rho_m(\text{matter})} \quad -(11)$$

So in fluid gravitation, mass density is defined in general by the velocity field of spacetime.

Conversely, mass density induces a velocity field $\underline{v}_F$ in spacetime.

3) In previous work the wave equation of spacetime:

$$\Box \Phi_F = \rho_F \quad -(12)$$

was inferred, given the Lorenz condition of spacetime:

$$\frac{\partial \Phi_F}{\partial t} + a_0^2 \, \underline{\nabla} \cdot \underline{v}_F . \quad -(13)$$

Eq. (13) was deduced in previous work from the spacetime continuity equation:

$$\frac{\partial \rho_F}{\partial t} + \underline{\nabla} \cdot \underline{J}_F = 0 \quad -(14)$$

where the spacetime current is:

$$\underline{J}_F = a_0^2 \, \underline{\nabla} \times \left(\underline{\nabla} \times \underline{v}_F \right) - \frac{\partial}{\partial t} \left(\left(\underline{v}_F \cdot \underline{\nabla} \right) \underline{v}_F \right) \quad -(15)$$

It follows that:

$$\boxed{\Box \Phi_F (\text{spacetime}) = 4\pi G \rho_m (\text{matter})} \quad -(16)$$

which infers a link between the spacetime potential:

$$\Phi_F = h_F \, (\text{spacetime}) \quad -(17)$$

and the mass density of matter. The origin of mass density in matter is spacetime enthalpy, a form of energy