355(1): The Poynting Theorem and Conservation of Energy

The conventional Poynting theorem is conservation of energy for a system consisting of electric and magnetic fields acting on charges in a volume V of surface S. The time rate of change of electromagnetic energy within V plus the net energy flowing out of V through S per unit time is equal to the negative of the total work done on the charges within V. In this analysis, $\underline{E}$ and $\underline{B}$ are external to the particle and do not include the electric and magnetic fields generated by the moving charged particle.

The rate at which work is done on an electric charge by an electromagnetic field is calculated from the Lorentz force:

$$\underline{F} = e(\underline{E} + \underline{v} \times \underline{B}) \quad -(1)$$

where v is the velocity of the particle, $\underline{E}$ is the electric field strength and $\underline{B}$ is the magnetic flux density. The rate at which work is done is:

$$\underline{P} = \underline{F} \cdot \underline{v} \quad -(2)$$

$$= e\underline{E} \cdot \underline{v}$$

because $\underline{B}$ is "perpendicular to $\underline{v}$", as usually argued. More accurately:

$$\underline{v} \cdot \underline{v} \times \underline{B} = \underline{0} \quad -(3)$$

$$= \underline{B} \cdot \underline{v} \times \underline{v}.$$

If the current is defined as

$$\underline{J}_0 = e\underline{v} \quad -(4)$$

then

$$\underline{P} = \underline{J}_0 \cdot \underline{E} \quad -(5)$$

2) The units of $\underline{P}$ are energy / time, so:

$$\frac{dW}{dt} = \frac{dE_n}{dt} = \underline{J_0} \cdot \underline{E} \quad - (6)$$

where W is the work done.

In terms of current density $\underline{J}$:

$$\boxed{\frac{dW}{dt} = \int \underline{J} \cdot \underline{E} \, dV} \quad - (7)$$

is the total rate of doing work.

In conventional electrodynamics:

$$\underline{\nabla} \cdot \underline{B} = 0 \quad - (8)$$

$$\underline{\nabla} \times \underline{E} + \frac{\partial \underline{B}}{\partial t} = \underline{0} \quad - (9)$$

$$\underline{\nabla} \times \underline{H} - \frac{\partial \underline{D}}{\partial t} = \underline{J} \quad - (10)$$

$$\underline{\nabla} \cdot \underline{D} = \rho \quad - (11)$$

Under well defined conditions, ECE2 electrodynamics reduces to eqs. (8) to (11).

The energy per unit volume of the electromagnetic field is:

$$U = \frac{1}{2} \left(\underline{E} \cdot \underline{D} + \underline{B} \cdot \underline{H} \right) \quad - (12)$$

In general:

$$\underline{D} = \epsilon_0 \underline{E} + \underline{P} \quad - (13)$$

$$\underline{H} = \frac{1}{\mu_0} \left(\underline{B} - \underline{M} \right) \quad - (14)$$

Here $\underline{D}$ is the electric displacement, $\underline{H}$ is the magnetic field strength, $\underline{P}$ is the polarization and $\underline{M}$ is

) the magnetization.

From eqs (7) and (10) ...

$$\underline{J} \cdot \underline{E} = \left(\underline{\nabla} \times \underline{H} - \frac{\partial \underline{D}}{\partial t} \right) \cdot \underline{E} \quad -(15)$$

Now use:

$$\underline{\nabla} \cdot \left(\underline{E} \times \underline{H} \right) = \underline{H} \cdot \underline{\nabla} \times \underline{E} - \underline{E} \cdot \underline{\nabla} \times \underline{H} \quad -(16)$$

and

$$\underline{\nabla} \times \underline{E} + \frac{\partial \underline{B}}{\partial t} = \underline{0} \quad -(17)$$

to find that:

$$\underline{J} \cdot \underline{E} = -\left(\underline{\nabla} \cdot \left(\underline{E} \times \underline{H} \right) + \underline{E} \cdot \frac{\partial \underline{D}}{\partial t} + \underline{H} \cdot \frac{\partial \underline{B}}{\partial t} \right) \quad -(18)$$

From eq. (12):

$$\frac{\partial U}{\partial t} = \frac{1}{2} \left(\frac{\partial}{\partial t} \left(\underline{E} \cdot \underline{D} \right) + \frac{\partial}{\partial t} \left(\underline{B} \cdot \underline{H} \right) \right) \quad -(19)$$

In the absence of polarization and magnetization:

$$\frac{\partial U}{\partial t} = \epsilon_0 \underline{E} \cdot \frac{\partial \underline{E}}{\partial t} + \frac{1}{\mu_0} \underline{B} \cdot \frac{\partial \underline{B}}{\partial t} \quad -(20)$$

The Poynting vector is defined as:

$$\underline{S} = \underline{E} \times \underline{H} \quad -(21)$$

$$= \frac{1}{\mu_0} \underline{E} \times \underline{B}$$

so

$$\boxed{\underline{\nabla} \cdot \underline{S} + \frac{\partial U}{\partial t} + \underline{J} \cdot \underline{E} = 0} \quad -(22)$$

4) This is the equation of conservation of energy of conventional electromagnetic theory, in which:

1) density of reactive power $= \epsilon_0 \underline{E} \cdot \frac{\partial \underline{E}}{\partial t}$ – (23)
driving the build up of the electric field strength;

2) density of reactive power $= \frac{1}{\mu_0} \underline{B} \cdot \frac{\partial \underline{B}}{\partial t}$ – (24)
driving the build up of the magnetic flux density;

3) $\underline{J} \cdot \underline{E}$ is the density of electric power dissipated by the Lorentz force acting on e.

Eq. (22) can now be applied to Kambe fluid dynamics and to fluid electrodynamics, in which the vacuum current density is not zero.

This shows that total energy is conserved when energy from spacetime is transferred to a circuit.