355(5): Simplification of the Spacetime Navier Stokes Equation with the Lorenz Condition

From previous work the electric field strength induced in the circuit is:

$$\underline{E}(\text{circuit}) = \left(\frac{\rho_m}{\rho}\right)_{\text{circuit}} \left((\underline{v} \cdot \underline{\nabla})\underline{v}\right)_{\text{spacetime}} \quad -(1)$$

$$= \left(\frac{\rho_m}{\rho}\right)_{\text{circuit}} \left(-\underline{\nabla}\,\underline{\Phi} - \frac{\partial \underline{v}}{\partial t}\right)_{\text{spacetime}}$$

where $\underline{\Phi}$ is defined in UFT353.

The Spacetime Lorenz condition is:

$$\left(\frac{1}{a_0^2}\frac{\partial \underline{\Phi}}{\partial t} + \underline{\nabla}\cdot\underline{v}\right)_{\text{spacetime}} = 0 \quad -(2)$$

So

$$\underline{\Phi} = -a_0^2 \int \underline{\nabla}\cdot\underline{v}\, dt \quad -(3)$$

Therefore:

$$a_0^2\, \underline{\nabla}\left(\int \underline{\nabla}\cdot\underline{v}\, dt\right) - \frac{\partial \underline{v}}{\partial t} = (\underline{v}\cdot\underline{\nabla})\underline{v} \quad -(4)$$

and:

$$\boxed{\frac{D\underline{v}}{Dt} = \frac{\partial \underline{v}}{\partial t} + (\underline{v}\cdot\underline{\nabla})\underline{v} = a_0^2\, \underline{\nabla}\left(\int \underline{\nabla}\cdot\underline{v}\, dt\right)} \quad -(5)$$

This is a linear differential equation

2) that can be solved for $\underline{v}$. Having solved for $\underline{v}$, the electric field in the circuit can be found from:

$$\underline{E}(\text{circuit}) = \left(\frac{m}{\rho}\right)_{\text{circuit}} \left((\underline{v}\cdot\underline{\nabla})\underline{v}\right)(\text{spacetime}) \quad -(6)$$

Eq. (5) has advantages over the simplified vorticity equation:

$$\frac{\partial \underline{v}}{\partial t} = \underline{v}\times(\underline{\nabla}\times\underline{v}) - \frac{1}{R}\,\underline{\nabla}\times(\underline{\nabla}\times\underline{v}) \quad -(7)$$

The magnetic flux density in the circuit is:

$$\underline{B}(\text{circuit}) = \left(\frac{m}{\rho}\right)_{\text{circuit}} \underline{\nabla}\times\underline{v}\,(\text{spacetime}) \quad -(8)$$

As in Note 355(2) the Lorenz condition (2) corresponds to the wave equation:

$$\boxed{\square W^{\mu}(\text{circuit}) = \mu_0 J^{\mu}(\text{spacetime})} \quad -(9)$$

where

$$W^{\mu}(\text{circuit}) = \left(\frac{\phi_W}{c}, \underline{W}\right)(\text{circuit}) \quad -(10)$$

is the four potential of ECE2 electrodynamics.

The spacetime four current $J^{\mu}(\text{spacetime})$ is defined as:

$$J^\mu(\text{spacetime}) = (a_0 q, \underline{J})(\text{spacetime}) \quad -(11)$$

w.r.t.:

$$V^\mu = \left(\frac{\Phi}{a_0}, \underline{V}\right). \quad -(12)$$

So:

$$\Box V^\mu(\text{circuit}) = \frac{1}{a_0^2} J^\mu_F(\text{spacetime}) \quad -(13)$$

and

$$\Box \Phi(\text{matter}) = q_F(\text{spacetime}) \quad -(14)$$

$$\Box \underline{V}(\text{matter}) = \frac{1}{a_0^2} \underline{J}(\text{spacetime}) \quad -(15)$$

From previous work:

$$\boxed{\begin{aligned} \Box W^\mu(\text{circuit}) &= \left(\frac{a_0}{c}\right)^2 \left(\frac{\rho_m}{\rho}\right)(\text{circuit})\, \Box V^\mu(\text{circuit}) \quad -(16) \\ &= \frac{1}{c^2}\left(\frac{\rho_m}{\rho}\right)(\text{circuit})\, J^\mu_F(\text{spacetime}) \end{aligned}}$$

and:

$$\underline{E}(\text{circuit}) = \left(\frac{\rho_m}{\rho}\right)(\text{circuit})\left((\underline{v}\cdot\underline{\nabla})\underline{v}\right)(\text{spacetime}) \quad -(17)$$

$$\underline{B}(\text{circuit}) = \left(\frac{\rho_m}{\rho}\right)(\text{circuit})\left(\underline{\nabla}\times\underline{v}\right)(\text{spacetime}) \quad -(18)$$