51(1): Derivation of the Kambe Equations with Reynolds Number

(Ref. google "Kambe, fluid Maxwell equations")

The Kambe equations are:

$$\underline{\nabla} \cdot \underline{H} = 0 \quad -(1)$$

$$\underline{\nabla} \cdot \underline{E} = q \quad -(2)$$

$$\underline{\nabla} \times \underline{E} + \frac{\partial \underline{H}}{\partial t} = \underline{0} \quad -(3)$$

$$a_0^2 \, \underline{\nabla} \times \underline{H} - \frac{\partial \underline{E}}{\partial t} = \underline{J} \quad -(4)$$

where:

$$\underline{H} = \underline{\nabla} \times \underline{v}, \quad -(5)$$

$$\underline{E} = -\frac{\partial \underline{v}}{\partial t} - \underline{\nabla} h, \quad -(6)$$

$$q = \underline{\nabla} \cdot \left((\underline{v} \cdot \underline{\nabla}) \underline{v} \right) \quad -(7)$$

$$\underline{J} = \frac{\partial^2 \underline{v}}{\partial t^2} + \underline{\nabla} \left(\frac{\partial h}{\partial t} \right) + a_0^2 \, \underline{\nabla} \times (\underline{\nabla} \times \underline{v}) \quad -(8)$$

They are equations of a fluid of velocity field $\underline{v}(t, x)$. The equations use the convective derivative:

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \underline{v} \cdot \underline{\nabla} \quad -(9)$$

which is the Lagrange derivative of fluid mechanics. It can be derived from the covariant derivative of Cartan geometry. The fluid velocity is:

$$\underline{v} = \frac{D \underline{r}}{Dt} = \frac{\partial \underline{r}}{\partial t} + (\underline{v} \cdot \underline{\nabla}) \underline{r} \quad -(10)$$

and the fluid acceleration is

$$\underline{a} = \frac{D \underline{v}}{Dt} = \frac{\partial \underline{v}}{\partial t} + (\underline{v} \cdot \underline{\nabla}) \underline{v} \quad -(11)$$

2) From vector analysis:

$$(\underline{v}\cdot\underline{\nabla})\underline{v} = -\underline{v}\times(\underline{\nabla}\times\underline{v}) + \underline{\nabla}\left(\frac{1}{2}|\underline{v}|^2\right) \quad -(12)$$

The vorticity is defined as:

$$\underline{w} = \underline{\nabla}\times\underline{v} \quad -(13)$$

and

$$\underline{a} = \frac{\partial \underline{v}}{\partial t} + \underline{w}\times\underline{v} + \underline{\nabla}\left(\frac{1}{2}\left(|\underline{v}|^2\right)\right) \quad -(14)$$

The Kambe equations (1) to (8) have the same overall structure as the ECE2 equations of electrodynamics and gravitation.

Kambe derives these equations from well known equations of fluid dynamics.

1) The Euler equation:

$$\frac{\partial \underline{v}}{\partial t} + (\underline{v}\cdot\underline{\nabla})\underline{v} = \frac{1}{\rho}\underline{\nabla}p \quad -(15)$$

where ρ is mass density and p is pressure. The units of ρ are kilogram per cubic metre ($kg\, m^{-3}$), the units of pressure are $kg\, m\, m^{-2}$, so the units of $\underline{\nabla}p/\rho$ are $m\, s^{-2}$. The mechanic pressure p is defined as:

$$p = w\rho \quad -(16)$$

where w is the specific thermodynamic work.

2) The continuity equation:

$$\frac{\partial \rho}{\partial t} + \underline{v}\cdot\underline{\nabla}\rho + \rho\,\underline{\nabla}\cdot\underline{v} = 0. \quad -(17)$$

3) The entropy equation:

$$\frac{\partial S}{\partial t} + \underline{v} \cdot \underline{\nabla} S = \frac{DS}{Dt} = 0 \quad -(18)$$

4) The vorticity equation:

$$\frac{\partial \underline{w}}{\partial t} + \underline{\nabla} \times (\underline{w} \times \underline{v}) = 0 \quad -(19)$$

Note carefully that in general, eq. (19) must be:

$$\frac{\partial \underline{w}}{\partial t} + \underline{\nabla} \times (\underline{w} \times \underline{v}) = \frac{1}{R} \nabla^2 \underline{w} \quad -(20)$$

where R is the Reynolds number. Therefore Kambe develops his theory using:

$$\frac{1}{R} \nabla^2 \underline{w} = 0. \quad -(21)$$

Eq. (20) is given in "Vector Analysis Problem Solver", Problem 11-22. The units of pressure in eq. (15) are those used in the subject of fluid dynamics. Finally, Kambe considers isentropic flow:

$$S = S_0 \quad -(22)$$

where S is the entropy per unit mass.

The Kambe equation (1) follows from the definition (5). Eq. (3) follows from the definitions (5) and (6), where h is enthalpy per unit mass ($J \, kgm^{-1}$), i.e.

$$-\underline{\nabla} \times \frac{\partial \underline{v}}{\partial t} - \underline{\nabla} \times \underline{\nabla} h + \frac{\partial}{\partial t} \underline{\nabla} \times \underline{v} = \underline{0} \quad -(23)$$

Eq. (2) is the definition of the thermodynamic charge q.

Therefore:

4) $$\rho = \underline{\nabla} \cdot \underline{E} = \underline{\nabla} \cdot \left((\underline{v} \cdot \underline{\nabla}) \underline{v} \right) \quad -(24)$$

so
$$\underline{E} = (\underline{v} \cdot \underline{\nabla}) \underline{v} \quad -(25)$$

and
$$\underline{a} = \frac{\partial \underline{v}}{\partial t} + \underline{E} \quad -(26)$$

Using eq.(12):
$$\underline{E} = \underline{w} \times \underline{v} + \underline{\nabla} \left(\frac{1}{2} v^2 \right) \quad -(27)$$

Eq.(4) is derived using:
$$\underline{E} = -\frac{\partial \underline{v}}{\partial t} - \underline{\nabla} h \quad -(28)$$

and
$$\frac{\partial \underline{E}}{\partial t} = -\frac{\partial^2 \underline{v}}{\partial t^2} - \frac{\partial}{\partial t} (\underline{\nabla} h) \quad -(29)$$

Add:
$$a_0^2 \underline{\nabla} \times \underline{H} = a_0^2 \underline{\nabla} \times (\underline{\nabla} \times \underline{v}) \quad -(30)$$

to left side of eq.(29) and use:
$$-\frac{\partial \underline{E}}{\partial t} = \frac{\partial^2 \underline{v}}{\partial t^2} + \underline{\nabla} \frac{\partial h}{\partial t} \quad -(31)$$

to find eq.(4), QED

Kambe develops the Euler equation with:
$$\frac{1}{\rho} \underline{\nabla} p = \underline{\nabla} h \quad -(32)$$

For isentropic flows:
$$\Delta p = a^2 \Delta \rho \quad -(33)$$

where:

5)

$$a = \left(\left(\frac{\partial p}{\partial \rho}\right)_S\right)^{1/2} \quad -(34)$$

is the speed of sound. The subscript S denotes differentiation with constant S. Therefore:

$$\frac{\partial \rho}{\partial t} = \left(\frac{\rho}{a^2}\right)\frac{\partial h}{\partial t}, \quad \underline{\nabla}\rho = \left(\frac{\rho}{a^2}\right)\underline{\nabla}h \quad -(35)$$

so the continuity equation transforms into:

$$\frac{\rho}{a^2}\left(\frac{\partial h}{\partial t} + \underline{v}\cdot\underline{\nabla}h + a^2\,\underline{\nabla}\cdot\underline{v}\right) = 0 \quad -(36)$$

The set of fluid equations used by Kambe is therefore:

$$\frac{\partial \underline{v}}{\partial t} + (\underline{v}\cdot\underline{\nabla})\underline{v} + \underline{\nabla}h = 0 \quad -(37)$$

$$\frac{\partial h}{\partial t} + \underline{v}\cdot\underline{\nabla}h + a^2\,\underline{\nabla}\cdot\underline{v} = 0 \quad -(38)$$

and

$$\frac{\partial \underline{w}}{\partial t} + \underline{\nabla}\times(\underline{w}\times\underline{v}) = \underline{0} \quad -(39)$$

These are equivalent to eqns. (1) to (8).

It follows that electrodynamic equations with the structure (1) to (8) can be transformed into the structure (37) to (38).

More generally, eq. (39) is:

$$\frac{\partial \underline{w}}{\partial t} + \underline{\nabla}\times(\underline{w}\times\underline{v}) = \frac{1}{R}\nabla^2\underline{w} \quad -(40)$$

6) In ECE2 electrodynamics:

$$\mathbf{B} = \nabla \times \mathbf{W} \quad -(41)$$

The units of $\mathbf{W}$ are the same as those of $\mathbf{A}$, and from the minimal prescription:

$$\mathbf{p} = m\mathbf{v} = e\mathbf{A} = e\mathbf{W} \quad -(42)$$

so

$$\boxed{\mathbf{v} = \frac{e}{m}\mathbf{W}} \quad -(43)$$

In hydrodynamics: $\mathbf{w} = \nabla \times \mathbf{v} \quad -(44)$

and from eqs. (43) and (44):

$$\boxed{\mathbf{w} = \frac{e}{m}\nabla \times \mathbf{W} = \frac{e}{m}\mathbf{B}} \quad -(45)$$

Eq. (40) is therefore:

$$\boxed{\frac{\partial \mathbf{B}}{\partial t} + \nabla \times (\mathbf{B} \times \mathbf{v}) = \frac{1}{R}\nabla^2 \mathbf{B}} \quad -(46)$$

Therefore the hydrodynamic equation (40) has been transformed into the electromagnetic equation (46).

It can be envisaged that the velocity field $\mathbf{v}$ in eq. (46) is that of spacetime or aether. The velocity flow $\mathbf{v}$ is the fluid velocity of the aether. The magnetic Reynolds number is that of the aether, and the magnetic flux density $\mathbf{B}$ is induced in a circuit. The fluid velocity or aether velocity becomes turbulent at a given Reynolds number.