351(4): The Equation of a Sound Wave in Fluid Electrodynamics

In Section 4 of his paper, "Fluid Maxwell equations" Kambe derives the equation of a sound wave from:

$$\underline{\nabla} \cdot \underline{E}_F = q \quad -(1)$$

$$a_0^2 \underline{\nabla} \times \underline{H}_F - \frac{\partial \underline{E}_F}{\partial t} = \underline{J}_F \quad -(2)$$

so:

$$\frac{\partial^2 \underline{E}_F}{\partial t^2} + a_0^2 \underline{\nabla} \times (\underline{\nabla} \times \underline{E}_F) = -\frac{\partial \underline{J}_F}{\partial t}. \quad -(3)$$

He uses the identity:

$$\underline{\nabla}(\underline{\nabla} \cdot \underline{v}) = \underline{\nabla} \times (\underline{\nabla} \times \underline{v}) + \nabla^2 \underline{v} \quad -(4)$$

and integrates to give the equation of a sound wave:

$$\boxed{\left(\frac{1}{a_0^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) \bar{h} = S(x, t)} \quad -(5)$$

where:

$$\bar{h} = h - h_0 \quad -(6)$$

and the source term is:

$$S(x, t) = \underline{\nabla} \cdot \underline{E}_F + \frac{\partial \bar{Q}}{\partial t} \quad -(7)$$

where:

$$\bar{Q} = (1 - \hat{a}^2) \underline{\nabla} \cdot \underline{v} - \frac{1}{a_0^2} (\underline{v} \cdot \underline{\nabla}) h \quad -(8)$$

and

$$\hat{a} = \frac{a}{a_0} \quad -(9)$$

Now translate the Kambe equations into equations

) of fluid electrodynamics using the geometrical analogy between the Kambe equation of fluid dynamics and the ECE2 equations of electrodynamics. The conversion factors are:

$$q = \frac{1}{\epsilon_0} \frac{\rho^2}{\rho_m} \; ; \; \underline{E}_F = \left(\frac{\rho}{\rho_m} \right) \underline{E} \quad -(10)$$

$$\underline{J}_F = \frac{1}{\epsilon_0} \frac{\rho}{\rho_m} \underline{J} \; ; \; \underline{H}_F = \frac{\rho}{\rho_m} \underline{B} \quad -(11)$$

Here $\underline{B}$ is the magnetic flux density of electrodynamics, $\underline{E}$ the electric field strength of electrodynamics, ρ is charge density of electrodynamics, $\underline{J}$ is the current density of electrodynamics and ρ_m is the mass density in $kg\, m^{-3}$.

The S.I. units are:

$$q = s^{-2} \; ; \; E_F = m s^{-2} \; ; \; J_F = m s^{-3} \; ; \; H_F = s^{-1} \quad -(12)$$

and

$$\rho = C m^{-3} \; ; \; E = J C^{-1} m^{-1} \; ; \; B = J C^{-1} m^{-2} s^{-1} \; ;$$
$$J = C m^{-2} s^{-1} \; , \; \epsilon_0 = J^{-1} C^2 m^{-1} \quad -(13)$$

It follows that the Coulomb law of fluid electrodynamics is:

$$\boxed{\begin{aligned} \underline{\nabla} \cdot \left(\frac{\rho}{\rho_m} \underline{E} \right) &= \frac{1}{\epsilon_0} \frac{\rho^2}{\rho_m} \\ &= \frac{\rho}{\rho_m} \underline{\nabla} \cdot \underline{E} + \underline{E} \cdot \underline{\nabla} \left(\frac{\rho}{\rho_m} \right) \end{aligned}} \quad -(14)$$

This contains more information than the Coulomb law of electrodynamics. It reduces to the latter if:

$$\underline{\nabla}\left(\frac{\rho}{\rho_m}\right) = \underline{0} \quad -(15)$$

i.e. when the ratio of charge density to mass density is constant. Eq. (14) then becomes:

$$\underline{\nabla} \cdot \underline{E} = \rho / \epsilon_0 \quad -(16)$$

which is the usual Coulomb law, QED.

Eq. (2) gives the Ampère Maxwell law of fluid electrodynamics

$$\boxed{a_0^2 \underline{\nabla} \times \left(\frac{\rho}{\rho_m}\underline{B}\right) - \frac{\partial}{\partial t}\left(\frac{\rho}{\rho_m}\underline{E}\right) = \frac{1}{\epsilon_0}\frac{\rho}{\rho_m}\underline{J}} \quad -(17)$$

in which:

$$\underline{\nabla} \times \left(\frac{\rho}{\rho_m}\underline{B}\right) = \frac{\rho}{\rho_m}\underline{\nabla} \times \underline{B} + \left(\underline{\nabla}\frac{\rho}{\rho_m}\right) \times \underline{B} \quad -(18)$$

and

$$\frac{\partial}{\partial t}\left(\frac{\rho}{\rho_m}\underline{E}\right) = \frac{\rho}{\rho_m}\frac{\partial \underline{E}}{\partial t} + \underline{E}\frac{\partial}{\partial t}\left(\frac{\rho}{\rho_m}\right) \quad -(19)$$

Eq. (17) becomes the Ampère Maxwell law of electrodynamics if:

$$\frac{\partial}{\partial t}\left(\frac{\rho}{\rho_m}\right) = 0, \quad \underline{\nabla}\left(\frac{\rho}{\rho_m}\right) = \underline{0}, \quad a_0 \rightarrow c \quad -(20)$$

in which case:

$$\underline{\nabla} \times \underline{B} - \frac{1}{c^2}\frac{\partial \underline{E}}{\partial t} = \mu_0 \underline{J} \quad -(21)$$

Q.E.D., where

$$\mu_0 = \frac{1}{\epsilon_0 c^2} . \quad -(22)$$

In fluid electrodynamics the assumed constant speed of sound a_0 replaces the speed of light c of electrodynamics. Under the conditions:

$$\frac{\partial}{\partial t}\left(\frac{\rho}{\rho_m}\right) = 0 \quad -(23)$$

and

$$\underline{\nabla}\left(\frac{\rho}{\rho_m}\right) = \underline{0} \quad -(24)$$

Eq. (17) can be written as:

$$\underline{\nabla} \times \underline{B} - \frac{1}{a_0^2}\frac{\partial \underline{E}}{\partial t} = \mu \underline{J} \quad -(25)$$

where

$$\mu = \frac{1}{\epsilon_0 a_0^2} . \quad -(26)$$

Using eqs. (3), (10) and (11), the sound equation of fluid electrodynamics is:

$$\frac{\partial^2}{\partial t^2}\left(\frac{\rho}{\rho_m}\underline{E}\right) + a_0^2 \underline{\nabla} \times \left(\underline{\nabla} \times \left(\frac{\rho}{\rho_m}\underline{E}\right)\right) = -\frac{1}{\epsilon_0}\frac{\partial}{\partial t}\left(\frac{\rho}{\rho_m}\underline{J}\right) \quad -(27)$$

Under the conditions (23) and (24):

$$\boxed{\frac{\partial^2 \underline{E}}{\partial t^2} + a_0^2 \underline{\nabla} \times (\underline{\nabla} \times \underline{E}) = -\frac{1}{\epsilon_0}\frac{\partial \underline{J}}{\partial t}} \quad -(28)$$

5) which using the identity (4) can be expressed as eq. (5) with the source term for the sound wave:

$$S(x,t) = \underline{\nabla} \cdot \underline{E}_F + \frac{\partial \overline{Q}}{\partial t} \quad -(29)$$

where

$$\underline{E}_F = \frac{\rho}{\rho_m} \underline{E} \quad -(30)$$

In these equations:

$$q = \underline{\nabla} \cdot ((\underline{v} \cdot \underline{\nabla})\underline{v}) \quad -(31)$$

and

$$\underline{J}_F = \frac{\partial^2 \underline{v}}{\partial t^2} + \underline{\nabla}\left(\frac{\partial h}{\partial t}\right) + a_0^2\, \underline{\nabla} \times (\underline{\nabla} \times \underline{v}) \quad -(32)$$

So the charge density and current density of fluid electrodynamics are defined by:

$$\boxed{\rho = \epsilon_0 \rho_m \underline{\nabla} \cdot ((\underline{v} \cdot \underline{\nabla})\underline{v})} \quad -(33)$$

and

$$\boxed{\underline{J} = \frac{\epsilon_0 \rho_m}{\rho}\left(\frac{\partial^2 \underline{v}}{\partial t^2} + \underline{\nabla}\left(\frac{\partial h}{\partial t}\right) + a_0^2\, \underline{\nabla} \times (\underline{\nabla} \times \underline{v})\right)} \quad -(34)$$

where the velocity field is:

$$\underline{v} = \underline{v}(x,t) \quad -(35)$$

6) In conventional electrodynamics the vacuum is thought of as a region where ρ and $\underline{J}$ are zero, but in fluid electrodynamics the vacuum is really structured, a fluid with velocity field (35). In eq. (33) ϵ_0 is the vacuum permittivity and ρ_m is the mass density of the vacuum.

It follows that $\underline{E}$ and $\underline{B}$ can be generated in a circuit through its interaction with the ubiquitous vacuum or aether or spacetime via eqs. (14) and (17).
