351(7): The Spin Connection as Jacobean in Fluid Dynamics.

The following calculation by Dr Horst Eckardt shows that the spin connection is the Jacobian, a fundamental result. Consider the convective derivative of fluid dynamics:

$$\frac{D\underline{v}}{Dt} = \frac{\partial \underline{v}}{\partial t} + (\underline{v} \cdot \underline{\nabla})\underline{v} \quad -(1)$$

$$= \frac{\partial \underline{v}}{\partial t} + \left(v_X \frac{\partial}{\partial X} + v_Y \frac{\partial}{\partial Y} + v_Z \frac{\partial}{\partial Z} \right) \underline{v}$$

Considering the X component:

$$\frac{D v_X}{Dt} = \frac{\partial v_X}{\partial t} + v_X \frac{\partial v_X}{\partial X} + v_Y \frac{\partial v_X}{\partial Y} + v_Z \frac{\partial v_X}{\partial Z}$$

$$= \frac{\partial v_X}{\partial t} + \underline{v} \cdot \underline{\nabla} v_X \quad -(2)$$

and similarly for the Y and Z components.

The covariant derivative of Cartan geometry is:

$$\frac{D V^a}{D x^\mu} = \frac{\partial V^a}{\partial x^\mu} + \omega^a_{\ \mu b} V^b \quad -(3)$$

Considering $\mu = 0$, it follows that:

$$\omega^1_{\ 01} = \frac{\partial v_X}{\partial X}; \quad \omega^1_{\ 02} = \frac{\partial v_X}{\partial Y}; \quad \omega^1_{\ 03} = \frac{\partial v_X}{\partial Z} \quad -(4)$$

and in general:

$$\omega^a_{\ 0b} = \boxed{\frac{\partial v^a}{\partial r^b}} \quad -(5)$$

2) where:

$$\underline{v} = v_x \underline{i} + v_y \underline{j} + v_z \underline{k} \quad -(6)$$

and

$$\underline{r} = X \underline{i} + Y \underline{j} + Z \underline{k} \quad -(7)$$

Therefore the entire subject of fluid dynamics is an example of Cartan geometry, and the convective derivative is an example of the covariant derivative of Cartan geometry.

The convective derivative is fundamental to the Navier Stokes and vorticity equations, and to the equation of continuity.