49(3): Lorentz Force Law and Ohm's Law from the ECE2 Field Equations.

Consider the ECE2 field equations of electrodynamics of (UFT317) in the notation of that paper:

$$\underline{\nabla} \cdot \underline{B} = \underline{\kappa} \cdot \underline{B} \quad -(1)$$

$$\underline{\nabla} \cdot \underline{E} = \underline{\kappa} \cdot \underline{E} \quad -(2)$$

$$\frac{\partial \underline{B}}{\partial t} + \underline{\nabla} \times \underline{E} = -\left(\kappa_0 c \underline{B} + \underline{\kappa} \times \underline{E} \right) \quad -(3)$$

$$\underline{\nabla} \times \underline{B} - \frac{1}{c^2} \frac{\partial \underline{E}}{\partial t} = \frac{\kappa_0}{c} \underline{E} + \underline{\kappa} \times \underline{B} \quad -(4)$$

where:

$$\kappa_0 = 2 \left(\frac{q_0}{r^{(0)}} - \omega_0 \right) \quad -(5)$$

$$\underline{\kappa} = 2 \left(\frac{1}{r^{(0)}} \underline{q} - \underline{\omega} \right) \quad -(6)$$

The charge current density is defined by:

$$J^\mu = \left(c\rho, \underline{J} \right) \quad -(7)$$

and the A^μ and W^μ four potentials by:

$$A^\mu = \left(\frac{\phi}{c}, \underline{A} \right) \quad -(8)$$

and

$$W^\mu = \left(\frac{\phi_W}{c}, \underline{W} \right) \quad -(9)$$

The field potential relations are:

$$\underline{E} = -\underline{\nabla} \phi_W - \frac{\partial \underline{W}}{\partial t}, \quad -(10)$$

$$\underline{B} = \underline{\nabla} \times \underline{W} \quad -(11)$$

with:

$$\phi_W = W^{(0)} \omega_0 = c W_0 \quad -(12)$$

$$\underline{W} = W^{(0)} \underline{\omega} \quad -(13)$$

2) From these equations:

$$\rho = \epsilon_0 \underline{\kappa} \cdot \underline{E} \quad -(14)$$

and

$$\underline{J} = \frac{1}{\mu_0}\left(\frac{\kappa_0}{c}\underline{E} + \underline{\kappa} \times \underline{B}\right) \quad -(15)$$

in general. From note 249(2), Ohm's law is:

$$\underline{J} = \sigma\left(\underline{E} + \underline{v} \times \underline{B}\right) \quad -(16)$$

where σ is the conductivity, and the Lorentz force density (force per unit volume) is:

$$\underline{F_0} = \rho\left(\underline{E} + \underline{v} \times \underline{B}\right). \quad -(17)$$

Here ρ is charge density (coulombs per unit volume), and the Lorentz force is:

$$\underline{F} = \int \underline{F}_0 \, dV \quad -(18)$$

Therefore the Lorentz force equation is contained in the inhomogeneous field equation of ECE2:

$$\boxed{\underline{F}_0 = \frac{\rho}{\mu_0 \sigma}\left(\frac{\kappa_0}{c}\underline{E} + \underline{\kappa} \times \underline{B}\right) = \frac{\rho}{\mu_0 \sigma}\left(\underline{\nabla} \times \underline{B} - \frac{1}{c^2}\frac{\partial \underline{E}}{\partial t}\right)} \quad -(19)$$

For one electron, the Lorentz force is:

$$\boxed{\begin{aligned}\underline{F} &= e\left(\underline{E} + \underline{v} \times \underline{B}\right) \\ &= \frac{e}{\mu_0 \sigma}\left(\frac{\kappa_0}{c}\underline{E} + \underline{\kappa} \times \underline{B}\right) = \frac{e}{\mu_0 \sigma}\left(\underline{\nabla} \times \underline{B} - \frac{1}{c^2}\frac{\partial \underline{E}}{\partial t}\right)\end{aligned}} \quad -(20)$$

3) Note carefully that the definitions (14) and (15) are fully relativistic, because they are derived from fully relativistic field equations of ECE2. However the usual definition:

$$\underline{F} = e(\underline{E} + \underline{v} \times \underline{B}) \quad -(21)$$

is a well defined limit.

The Maxwell Heaviside (MH) field equations are:

$$\underline{\nabla} \cdot \underline{B} = 0 \quad -(22)$$

$$\frac{\partial \underline{B}}{\partial t} + \underline{\nabla} \times \underline{E} = \underline{0} \quad -(23)$$

$$\underline{\nabla} \cdot \underline{E} = \rho / \epsilon_0 \quad -(24)$$

$$\underline{\nabla} \times \underline{B} - \frac{1}{c^2} \frac{\partial \underline{E}}{\partial t} = \mu_0 \underline{J} \quad -(25)$$

and do not contain the Lorentz force equation as is well known. Also, the MH equations do not contain Ohm's Law. Therefore ECE2 is preferred to MH because the former is generally covariant and its field equations contain both the Lorentz force equation and Ohm's law.

In general, the ECE2 field equations reduce to the mathematical format of MH if:

$$\underline{\kappa} \cdot \underline{B} = 0 \quad -(26)$$

$$\frac{\kappa_0}{c} \underline{E} + \underline{\kappa} \times \underline{B} = \mu_0 \underline{J} \quad -(27)$$

$$\kappa_0 c \underline{B} + \underline{\kappa} \times \underline{E} = \underline{0} \quad -(28)$$

$$\underline{\kappa} \cdot \underline{E} = \frac{\rho}{\epsilon_0} \quad -(29)$$

4) In the presence of polarization and magnetization:

$$\underline{D} = \epsilon_0 \underline{E} + \underline{P} \quad -(30)$$

$$\underline{B} = \mu_0 \underline{H} + \mu_0 \underline{M} \quad -(31)$$

Here:

$\underline{E}$ = electric field strength (volt m^{-1}) -(32)

$\underline{D}$ = electric displacement (C m^{-2}) -(33)

ρ = charge density (C m^{-3}) -(34)

$\underline{H}$ = magnetic field strength (A m^{-1}) -(35)

$\underline{B}$ = magnetic flux density (tesla) -(36)

$\underline{J}$ = current density (A m^{-2}) -(37)

$\underline{P}$ = polarization -(38)

$\underline{M}$ = magnetization -(39)

ϵ_0 = vacuum permittivity -(40)

μ_0 = vacuum permeability -(41)

So in general:

$$\underline{\nabla} \cdot \underline{D} = \rho \quad -(42)$$

$$\underline{\nabla} \times \underline{H} - \frac{\partial \underline{D}}{\partial t} = \underline{J} \quad -(43)$$

$$\underline{\nabla} \cdot \underline{B} = 0 \quad -(44)$$

$$\underline{\nabla} \times \underline{E} + \frac{\partial \underline{B}}{\partial t} = \underline{0} \quad -(45)$$

The equivalent of the inhomogeneous field equations (42) and (43) in ECE2 theory are found from Cartan geometry with the appropriate field tensor definitions, and are:

5)

$$\underline{\nabla} \cdot \underline{D} = \underline{\kappa} \cdot \underline{D} \quad -(46)$$

$$\underline{\nabla} \times \underline{H} - \frac{\partial \underline{D}}{\partial t} = \underline{J} = \frac{\kappa_0}{c} \underline{D} + \underline{\kappa} \times \underline{H} \quad -(47)$$

Therefore in the presence of polarization and magnetization:

$$\underline{F_0} = \frac{f}{\sigma} \left(\frac{\kappa_0}{c} \underline{D} + \underline{\kappa} \times \underline{H} \right) = \frac{f}{\sigma} \left(\underline{\nabla} \times \underline{H} - \frac{\partial \underline{D}}{\partial t} \right) \quad -(48)$$

$$\underline{F} = \int \underline{F_0} \, dV \quad -(49)$$

If it is assumed that there is no magnetic charge current density:

$$\underline{\kappa} \cdot \underline{B} = 0 \quad -(50)$$

and

$$\kappa_0 c \underline{B} + \underline{\kappa} \times \underline{E} = \underline{0} \quad -(51)$$

Note that eq. (51) implies eq. (50) because from eq. (51):

$$\underline{B} = -\frac{1}{\kappa_0 c} \underline{\kappa} \times \underline{E} \quad -(52)$$

so

$$\underline{\kappa} \cdot \underline{\kappa} \times \underline{E} = 0 \quad -(53)$$

Eq. (53) is always true because:

$$\underline{\kappa} \cdot \underline{\kappa} \times \underline{E} = \underline{E} \cdot \underline{\kappa} \times \underline{\kappa} = 0 \quad -(54)$$

Q.E.D. By definition:

$$\underline{B} = \underline{\nabla} \times \underline{W} \quad -(55)$$

so:

$$\underline{\kappa} \cdot \underline{\nabla} \times \underline{W} = 0 \quad -(56)$$

Using the vector identity:

$$\underline{\nabla} \cdot \underline{F} \times \underline{G} = \underline{G} \cdot \underline{\nabla} \times \underline{F} - \underline{F} \cdot \underline{\nabla} \times \underline{G} \quad -(57)$$

it follows that: -(58)

$$\underline{\nabla} \cdot \left(\underline{\kappa} \times \underline{\overline{W}} \right) = \underline{\overline{W}} \cdot \left(\underline{\nabla} \times \underline{\kappa} \right) - \underline{\kappa} \cdot \left(\underline{\nabla} \times \underline{\overline{W}} \right)$$

so

$$\underline{\kappa} \cdot \underline{\nabla} \times \underline{\overline{W}} = \underline{\overline{W}} \cdot \underline{\nabla} \times \underline{\kappa} - \underline{\nabla} \cdot \underline{\kappa} \times \underline{\overline{W}} \quad -(59)$$

It follows that in the absence of a magnetic monopole:

$$\boxed{\underline{\overline{W}} \cdot \underline{\nabla} \times \underline{\kappa} = \underline{\nabla} \cdot \underline{\kappa} \times \underline{\overline{W}}} \quad -(60)$$

where:

$$\underline{\overline{W}} = \overline{W}^{(0)} \underline{\omega} , \quad -(61)$$

and

$$\underline{\kappa} = 2 \left(\frac{1}{r^{(0)}} \underline{v} - \underline{\omega} \right) \quad -(62)$$

The geometrical condition for a vanishing magnetic charge density is, from Eqs. (60) – (62):

$$\boxed{\underline{\omega} \cdot \underline{\nabla} \times \underline{v} = \underline{\nabla} \cdot \underline{v} \times \underline{\omega}} \quad -(63)$$

Therefore in ECE2 there can be a zero or non-zero magnetic charge-current density, depending on geometry. Only experiment can determine whether there is a finite magnetic charge-current density. Under the purely geometrical condition (63) the ECE2 field equations reduce to:

$$\underline{\nabla} \cdot \underline{B} = 0 \quad -(64)$$

$$\frac{\partial \underline{B}}{\partial t} + \underline{\nabla} \times \underline{E} = \underline{0} \quad -(65)$$

$$\underline{\nabla} \cdot \underline{D} = \underline{\kappa} \cdot \underline{D} \quad -(66)$$

$$\underline{\nabla} \times \underline{H} - \frac{\partial \underline{D}}{\partial t} = \frac{\kappa_0}{c} \underline{D} + \underline{\kappa} \times \underline{H} \quad -(67)$$

In the absence of polarization and magnetization the inhomogeneous field equations become:

$$\underline{\nabla} \cdot \underline{E} = \underline{\kappa} \cdot \underline{E} \quad -(68)$$

and

$$= \rho / \epsilon_0$$

$$\underline{\nabla} \times \underline{B} - \frac{1}{c^2} \frac{\partial \underline{E}}{\partial t} = \mu_0 \underline{J} = \frac{\kappa_0}{c} \underline{E} + \underline{\kappa} \times \underline{B} \quad -(69)$$

in which the Lorentz force density is:

$$\underline{F_0} = \frac{\rho}{\sigma} \underline{J} \quad -(70)$$

and in which Ohm's Law is:

$$\underline{J} = \frac{1}{\mu_0} \left(\frac{\kappa_0}{c} \underline{E} + \underline{\kappa} \times \underline{B} \right) = \sigma \left(\underline{E} + \underline{v} \times \underline{B} \right) \quad -(71)$$

So the ECE2 field equations contain the Lorentz force and Ohm's Law, QED. It follows that conductivity can be derived from Cartan geometry.