549(2): Transition to Turbulence via the Lorentz Force Equation.
(Google "Turbulent flow Lorentz force" first site from Stanford / NASA, A. Thess et al. (2006))

Consider Ohm's law in the presence of a magnetic field:

$$\underline{J} = \sigma \left(\underline{E} + \underline{v} \times \underline{B} \right) \quad -(1)$$

where $\underline{J}$ is the current density, σ the conductivity, $\underline{E}$ the electric field strength, $\underline{v}$ the velocity and $\underline{B}$ the magnetic flux density. Eq. (1) is related to the Lorentz force equation for a distribution of charge. The Lorentz force equation for the elementary charge e is:

$$\underline{F} = e \left(\underline{E} + \underline{v} \times \underline{B} \right) \quad -(2)$$

so

$$d\underline{F} = de \left(\underline{E} + \underline{v} \times \underline{B} \right) \quad -(3)$$

or

$$\underline{F}_0 = \rho \left(\underline{E} + \underline{v} \times \underline{B} \right) \quad -(4)$$

where $\underline{F}_0$ denotes force per unit volume and where ρ is the charge density or charge per unit volume. When considering the motion of a charge continuum:

$$\underline{J} = \rho \underline{v} \quad -(5)$$

where $\underline{J}$ is the current density. So:

$$\boxed{\underline{F}_0 = \rho \underline{E} + \underline{J} \times \underline{B}} \quad -(6)$$

and the Lorentz force of continuum charge is:

$$\underline{F} = \int \underline{F}_0 \, dV \quad -(7)$$

2) From eqs. (1) and (4):

$$\frac{F_0}{J} = \frac{\rho}{\sigma} \quad -(8)$$

so

$$\boxed{F_0 = \frac{\rho}{\sigma} J} \quad -(9)$$

and

$$F = \int \rho \frac{J}{\sigma} \, dV. \quad -(10)$$

The four current density is:

$$J^{\mu} = (c\rho, \underline{J}) \quad -(11)$$

<u>Units Check</u>

The units of ρ are $C m^{-3}$, so those of $\underline{J}$ are $C s^{-1} m^{-2}$. The units of ρ are C, and the units of conductivity are

$$\sigma = C^2 J^{-1} s^{-1} m^{-1} \quad -(12)$$

so

$$F_0 = \cancel{C} m^{-3} \cancel{C} \cancel{s^{-1}} m^{-2} \cancel{C^{-2}} J \cancel{s} m = J m^{-2}$$

$$= kg m\, m s^{-2} \quad -(13)$$

Therefore the <u>Lorentz force density</u> F_0 <u>can be expressed</u> in <u>terms of</u> current density $\underline{J}$.

In the paper by Thess et al. a magnetic flux density:

$$\underline{B} = B_r \underline{e}_r + B_z \underline{k} \quad -(14)$$

is applied to a conducting fluid. It is assumed that:

$$\underline{\nabla} \cdot \underline{B} = 0 \quad -(15)$$

) For low Reynolds number flows:

$$\underline{E} = -\underline{\nabla}\phi \quad -(16)$$

is assumed. In ECE2:

$$\underline{E} = -\underline{\nabla}\phi_W \quad -(17)$$

So low Reynold's number flows correspond to

$$E \gg B \quad -(18)$$

A steady flow is considered to be:

$$\underline{v} = v(r)\,\underline{k} \quad -(19)$$

The authors take the divergence of Ohm's Law, Eq. (1):

$$\begin{aligned}\underline{\nabla}\cdot\underline{J} &= \sigma\left(\underline{\nabla}\cdot\underline{E} + \underline{\nabla}\cdot\underline{v}\times\underline{B}\right)\\ &= \sigma\left(-\nabla^2\phi + \underline{B}\cdot\underline{\nabla}\times\underline{v} - \underline{v}\cdot\underline{\nabla}\times\underline{B}\right)\end{aligned} \quad -(20)$$

where

$$\underline{w} = \underline{\nabla}\times\underline{v} \quad -(21)$$

is the vorticity. From eqs. (5) and (19):

$$\underline{J} = \rho v(r)\,\underline{k} \quad -(22)$$

and for steady flow $v(r)$ is a constant, so:

$$\underline{\nabla}\cdot\underline{J} = 0 \quad -(23)$$

The authors assume that the curl of the magnetic field (4) is zero,

$$\underline{\nabla}\times\underline{B} = \underline{0} \quad -(24)$$

so

$$\boxed{\nabla^2\phi = \underline{B}\cdot\underline{w}} \quad -(25)$$

For flows of type (14) the vorticity is $\parallel \underline{k}$

4) so

$$\nabla^2 \phi = 0 \quad -(26)$$

The scalar potential ϕ has to satisfy homogeneous boundary conditions:

$$\phi = 0 \text{ at } r = 0 \quad -(27)$$

and

$$\frac{\partial \phi}{\partial r} = 0 \text{ at } r = R. \quad -(28)$$

Under these conditions the Lorentz term in eq. (1) gives the eddy currents:

$$\boxed{\underline{J} = \sigma \underline{v} \times \underline{B} = \sigma v(r) B_r \underline{e}_\theta} \quad -(29)$$

These are azimuthal and flow parallel to the wall of the pipe.

The authors consider the Lorentz force density:

$$\underline{F}_0 = \underline{J} \times \underline{B} \quad -(30)$$

and integrate over the volume of the pipe to give the Lorentz force:

$$F = -2\pi\sigma \int_{-\infty}^{\infty} \int_0^R v(r) B_r^2 \, r \, dr \, dZ \quad -(30)$$

The magnetic field is produced by a single coil wrapped around the pipe, with radius L,

$$R << L \quad -(31)$$

Under these conditions:

$$B_r = \frac{3B_0}{2L^2} \frac{rZ}{\left(1 + \frac{Z^2}{L^2}\right)^{5/2}} ; \quad B_Z = \frac{B_0}{\left(1 + \frac{Z^2}{L^2}\right)^{3/2}}$$

This field is a maximum at the axis of the cylinder, at -(32)

$$Z = 0 \quad -(33).$$

From eqs. (30) to (33):

$$F = -\frac{45\pi^2}{256} \frac{\sigma B_0^2}{L} \int_0^R v(r) r^3 dr \quad -(34)$$

Finally the authors consider models of transition to turbulence.

In precise analogy, the Lorentz force from the ECE2 vacuum can produce turbulent motion in the electrons of a circuit, and this will be considered in the next note.

Finally in this note, Eq. (13) of UFT317 defines the current density:

$$\boxed{\underline{J} = \frac{1}{\mu_0} \underline{\kappa} \times \underline{B}} \quad -(35)$$

where

$$\underline{\kappa} = 2\left(\frac{1}{r^{(0)}} \underline{q} - \underline{\omega}\right). \quad -(36)$$

Here $\underline{q}$ is a tetrad vector, $\underline{\omega}$ is a spin connection vector, and $r^{(0)}$ has the units of metres. By comparison of eqs. (1) and (35):

$$\underline{J} = \sigma\, \underline{v} \times \underline{B} = \frac{1}{\mu_0}\, \underline{\kappa} \times \underline{B} \quad -(37)$$

so:

$$\boxed{\underline{\kappa} = \mu_0\, \sigma\, \underline{v}} \quad -(38)$$

in ECE2 relativity. Therefore the ECE2 field equations contain the Lorentz force equation as the result of geometry, and define hydrodynamic flow velocity in terms of the tetrad and spin connection vectors.
