349(1): The Field Equations of Hydrodynamics, Gravitation and Electrodynamics in ECE2 Relativity.

(Reference: Kambe "On Fluid Maxwell Equations")

Kambe has reduced the equations of hydrodynamics to the format of the Maxwell Heaviside equations, which have been developed in UFT317 into the ECE2 field equations of electrodynamics. The equations of hydrodynamics are:

$$\frac{D\underline{v}}{\partial t} = \frac{\partial \underline{v}}{\partial t} + (\underline{v}\cdot\underline{\nabla})\underline{v} = -\frac{1}{\rho}\underline{\nabla}p, \quad -(1)$$

which is Euler's equation; the continuity equation:

$$\frac{\partial \rho}{\partial t} + \underline{v}\cdot\underline{\nabla}\rho + \rho\,\underline{\nabla}\cdot\underline{v} = 0, \quad -(2)$$

the vorticity equation:

$$\frac{\partial \underline{\omega}}{\partial t} + \underline{\nabla}\times(\underline{\omega}\times\underline{v}) = \underline{0}, \quad -(3)$$

and the entropy equation:

$$\frac{\partial S}{\partial t} + \underline{v}\cdot\underline{\nabla}S = 0 \quad -(4)$$

Here ρ is the fluid density, S is the entropy per unit mass, and p is the pressure. Eq. (4) is the adiabatic equation, which means that each fluid particle keeps its initial entropy.

Kambe shows that under well defined conditions, eqs (1) to (4) reduce to:

$$\frac{\partial \underline{v}}{\partial t} + (\underline{v}\cdot\underline{\nabla})\underline{v} + \underline{\nabla}h = 0 \quad -(5)$$

$$\frac{\partial h}{\partial t} + \underline{v}\cdot\underline{\nabla}h + a^2\,\underline{\nabla}\cdot\underline{v} = 0 \quad -(6)$$

2) $$\frac{\partial \underline{\omega}}{\partial t} + \underline{\nabla} \times (\underline{\omega} \times \underline{v}) = \underline{0} \quad -(7)$$

In the above equations, the vorticity is defined by:

$$\underline{\omega} = \underline{\nabla} \times \underline{v}, \quad -(8)$$

and h is the enthalpy per unit mass. The S.I. unit of enthalpy is joules per kilogram.

Units Check

In eq. (5), the units of $d\underline{v}/dt$ are $m\,s^{-2}$, the units of $\underline{\nabla} h$ are $J\,kg^{-1}\,m^{-1} = kg\,m^2 s^{-2} kg^{-1} m^{-1} = m\,s^{-2}$ ✓ Kambe means, therefore, by "enthalpy per unit mass", the usual enthalpy. The units of h are $J\,kgm^{-1} = m^2 s^{-2}$ and the units of $\underline{v}$ are $m\,s^{-1}$.

Kambe derives the wave equations:

$$\Box h = 0 \quad -(9)$$

and

$$\Box \underline{v} = \underline{0} \quad -(10)$$

where the d'Alembertian is:

$$\Box = \frac{\partial^2}{\partial t^2} - a_0^2 \nabla^2 \quad -(11)$$

where a_0 is the speed of sound. So it is possible to derive the four potential of fluid dynamics:

$$\boxed{\phi^{\mu}_{FD} = (h, a_0 \underline{v})} \quad -(12)$$

This is directly analogous to the four potential of ECE2 electrodynamics:

$$\overline{W}^{\mu} = \overline{W}^{(0)} \omega^{\mu} = \left(\phi_W, c \underline{W} \right) - (13)$$

defined in UFT317. In eq. (13) ω^{μ} is the spin connection four vector, not to be confused with the vorticity $\underline{\omega}$.

Kambe also derives the Lorenz condition:

$$\partial_{\mu} \phi^{\mu}_{FD} = 0 - (14)$$

where

$$\partial_{\mu} = \left(\frac{1}{a_0} \frac{\partial}{\partial t}, \underline{\nabla} \right) - (15)$$

From eqs. (12), (14) and (15):

$$\frac{1}{a_0} \frac{\partial h}{\partial t} + a_0 \underline{\nabla} \cdot \underline{v} = 0 - (16)$$

i.e.

$$\boxed{\frac{\partial h}{\partial t} + a_0^2 \underline{\nabla} \cdot \underline{v} = 0} - (17)$$

In ECE2 electrodynamics the Lorenz condition

$$\partial_{\mu} \overline{W}^{\mu} = 0 - (18)$$

is

where

$$\partial_{\mu} = \left(\frac{1}{c} \frac{\partial}{\partial t}, \underline{\nabla} \right) - (19)$$

i.e.

$$\boxed{\frac{\partial \phi_W}{\partial t} + c^2 \underline{\nabla} \cdot \underline{W} = 0} - (20)$$

Kambe defines the <u>fluid electric field strength</u>

$$\underline{E}_{FD} = -\frac{\partial \underline{v}}{\partial t} - \underline{\nabla} h \quad -(21)$$

and the fluid magnetic flux density:

$$\underline{B}_{FD} = \underline{\nabla} \times \underline{v} \quad -(22)$$

Note that the fluid magnetic flux density is the vorticity.

The analogies in ECE2 electrodynamics are:

$$\underline{E} = -\frac{\partial \underline{W}}{\partial t} - \underline{\nabla} \phi_W \quad -(23)$$

and

$$\underline{B} = \underline{\nabla} \times \underline{W} \quad -(24)$$

The analogies in ECE2 gravitational physics are

$$\underline{g} = -\frac{\partial \underline{W}_g}{\partial t} - \underline{\nabla} \phi_{Wg} \quad -(25)$$

and

$$\underline{\Omega} = \underline{\nabla} \times \underline{v}_g \quad -(26)$$

where $\underline{g}$ is the acceleration due to gravity and where $\underline{\Omega}$ is the gravitomagnetic field.

With these definitions, it is possible to rewrite Eqs. (5) to (7) as:

$$\underline{\nabla} \cdot \underline{B}_{FD} = 0 \quad -(27)$$

$$\underline{\nabla} \cdot \underline{E}_{FD} = q_{FD} \quad -(28)$$

$$\underline{\nabla} \times \underline{E}_{FD} + \frac{\partial \underline{B}_{FD}}{\partial t} = \underline{0} \quad -(29)$$

$$\underline{\nabla} \times \underline{B}_{FD} - \frac{1}{a_0^2} \frac{\partial \underline{E}_{FD}}{\partial t} = \frac{1}{a_0^2} \underline{J}_{FD} \quad -(30)$$

) which have the same structure as the ECE2 field equations of electromagnetism:

$$\nabla \cdot \mathbf{B} = 0 \quad -(31)$$

$$\nabla \cdot \mathbf{E} = \boldsymbol{\kappa} \cdot \mathbf{E} = \rho / \epsilon_0 \quad -(32)$$

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = \mathbf{0} \quad -(33)$$

$$\nabla \times \mathbf{B} - \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} = \mu_0 \mathbf{J} = \boldsymbol{\kappa} \times \mathbf{B} \quad -(34)$$

where

$$\boldsymbol{\kappa} = 2 \left(\frac{\mathbf{q}}{r^{(0)}} - \boldsymbol{\omega}_1 \right) \quad -(35)$$

where the spin connection vector is $\boldsymbol{\omega}_1$, the tetrad vector is $\mathbf{q}$ and $r^{(0)}$ has the units of metres. In eqs. (31) to (35), the charge density four vector is:

$$J^\mu = (c\rho, \mathbf{J}) \quad -(36)$$

and the continuity equation is:

$$\partial_\mu J^\mu = 0 \quad -(37)$$

i.e.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{J} = 0. \quad -(38)$$

In direct analogy, Kambe defines the continuity equation

$$\frac{\partial q_{FD}}{\partial t} + \nabla \cdot \mathbf{J}_{FD} = 0 \quad -(39)$$

i.e.

$$\boxed{\partial_\mu J^\mu_{FD} = 0} \quad -(40)$$

) where:

$$\rho_{FD} = \underline{\nabla} \cdot \left((\underline{v} \cdot \underline{\nabla}) \underline{v} \right) \quad -(41)$$

is the fluid charge density, and:

$$\underline{J}_{FD} = \frac{\partial^2 \underline{v}}{\partial t^2} + \underline{\nabla} \frac{\partial h}{\partial t} + a_0^2 \, \underline{\nabla} \times (\underline{\nabla} \times \underline{v}) \quad -(42)$$

is the fluid current density.

Units Check

In Eq. (28), $\underline{\nabla} \cdot \underline{E}_{FD} = \rho_{FD}$. The units of both sides are s^{-2} ✓. In Eq. (30) the units of $\partial^2 \underline{v} / \partial t^2$ (current density) are $m s^{-3}$ and the units of $\partial \underline{E}_{FD} / \partial t$ are $m s^{-3}$ ✓

Eq. (41) means that the fluid charge (density) is defined in terms of the velocity $\underline{v}$ of the fluid. The charge density ρ of electrodynamics is defined by:

$$\underline{\nabla} \cdot \underline{E} = \frac{\rho}{\epsilon_0} \quad -(43)$$

where ϵ_0 is the vacuum permittivity. The units of ρ are $C m^{-3}$, so

$$\boxed{\rho = e A \rho_{FD}} \quad -(44)$$

where A has the units of s^2 / m^3. Therefore the electric charge density originates in the flow of a fluid. This can be thought of as the flow of spacetime itself.

Kambe also derives the Lorentz force equation of fluid dynamics:

7) $$\frac{d\underline{P}_{FD}}{dt} = m\left(\underline{E}_{FD} + \underline{u} \times \underline{B}_{FD}\right) - m\underline{\nabla}\phi_g \quad -(45)$$

where $\underline{P}$ is the canonical momentum of fluid dynamics.
The analogy of eq. (45) in ECE2 electrodynamics is:

$$\underline{F} = \ddot{\underline{r}} = -e\left(\underline{E} + \dot{\underline{r}} \times \underline{B}\right) - m\underline{\nabla}\phi_W \quad -(46)$$

where the canonical momentum of ECE2 is:

$$\underline{p} = m\dot{\underline{r}} - e\underline{W} \quad -(47)$$

Finally the field equations of hydrodynamics are directly analogous to the field equations of ECE2 gravitation:

$$\underline{\nabla} \cdot \underline{\Omega} = 0 \quad -(48)$$

$$\underline{\nabla} \times \underline{g} + \frac{\partial \underline{\Omega}}{\partial t} = \underline{0} \quad -(49)$$

$$\underline{\nabla} \cdot \underline{g} = \underline{\kappa} \cdot \underline{g} = 4\pi G \rho_m \quad -(50)$$

and

$$\underline{\nabla} \times \underline{\Omega} - \frac{1}{c^2}\frac{\partial \underline{g}}{\partial t} = \underline{\kappa} \times \underline{\Omega} = \frac{4\pi G}{c^2}\underline{J}_m \quad -(51)$$

where

$$\underline{g} = -\underline{\nabla}\phi_g - \frac{\partial \underline{v}_g}{\partial t} \quad -(52)$$

and

$$\underline{\Omega} = \underline{\nabla} \times \underline{W}_g = \underline{\nabla} \times \underline{v}_g \quad -(53)$$

All these equations emerge from Cartan geometry

8) is a unified field theory. Many new conceptual analogies can now be made. For example the fluid magnetic flux density from Eq (22) is directly analogous to the gravitomagnetic field. They are both vortices of spacetime.

The equations governing the transition to turbulence in hydrodynamics and aerodynamics can be transferred to electrodynamics and gravitational theory. Therefore spacetime turbulence can be picked up from its effects in circuits.