346(4): Gravitomagnetic Theory of the Perihelion Precession of the Earth

· In the general theory this is explained with:

$$\Omega = \frac{G}{2c^3}\left|\nabla \times \left(\frac{L \times r}{r^3}\right)\right| \quad -(1)$$

$$= \frac{G}{2c^2 r^3}\left|3\frac{r}{r}\left(\frac{r}{r}\cdot L\right) - L\right|$$

where r is the distance from the sun to the earth, and where L is an angular momentum. From note 346(3):

$$r = i \times \cos\theta - k \times \sin\theta + Y j \quad -(2)$$

where θ is the angle at which the plane of the orbit of the earth is inclined to the plane perpendicular to the axis of spin of the sun, denoted k. By observation:

$$\theta = 7.25° \quad -(3)$$

The observed precession of the perihelion of the earth is (Meria and Thomas):

$$\Omega = (5.0 \pm 1.2)''\ a\ century \quad -(4)$$
$$= 7.681 \times 10^{-15}\ rad\ s^{-1}$$

This is much larger than the Lense Thirring precession of the earth, which is

$$\Omega_{LT} = 5.741 \times 10^{-21}\ rad\ s^{-1} \quad -(5)$$

from ECE2 theory, assuming in a rough approximation that

$$\theta \sim 0 \quad -(6)$$

2) The perihelia precession does not depend on the spin of the sun. Once every 27 days about its own axis. From the earth the sun appears to be rotating w.r.t an angular momentum of:

$$L = \underline{M} r^2 \underline{\omega} \quad -(6)$$

is the first approximation of a circular orbit. The magnitude of the angular momentum is:

$$L = M r^2 \omega \quad -(7)$$

For the sun:
$$\frac{mG}{c^2} = 1.475 \times 10^3 \text{ metres.} \quad -(8)$$

The earth to sun distance is:
$$r = 1.49598 \times 10^{11} \text{ metres.} \quad -(9)$$

The earth rotates around the sun once every year, i.e once every 3.156×10^7 secs. So:

$$\omega = \frac{2\pi}{3.157 \times 10^7} \text{ radians } s^{-1} \quad -(10)$$

The perihelion precession for these data and equation is:

$$\Omega = 0.981 \times 10^{-15} \text{ rad } s^{-1} \quad -(11)$$

An exact result can be obtained by evaluating the effective L from eqns (1) to (3). An exact result for the geodesic

) precession observed in Gravity Probe B can also
be obtained with an effective $\underline{L}$.

The fundamental assumption is that a gravito-
magnetic vector potential is generated as follows:

$$\underline{A}_g(\underline{r}) = \frac{G}{c^2 r^3} \, \underline{m}_g \times \underline{r} \quad - (12)$$

between the gravitomagnetic dipole moment $\underline{m}_g$ of a
localized mass distribution and the vector $\underline{r}$. Here:

$$\underline{m}_g = \frac{1}{2} \underline{L}, \quad - (13)$$

So

$$\underline{A}_g(\underline{r}) = \frac{G}{2c^2 r^3} \, \underline{L} \times \underline{r}. \quad - (14)$$

The gravitomagnetic field is defined by:

$$\underline{\Omega}_g = \underline{\nabla} \times \underline{A}_g. \quad - (15)$$

The following torque is generated:

$$\underline{T_g} = \underline{m}_g \times \underline{\Omega}_g \quad - (16)$$

between the earth sun system. The torque (16) gives
a precession frequency of:

$$\Omega = \frac{1}{2} \Omega_g \quad - (17)$$

$$= \frac{G}{c^2} \left| \underline{\nabla} \times \left(\frac{\underline{m}_g \times \underline{r}}{r^3} \right) \right|$$

Therefore the angular momentum $\underline{L}$ is the

4) **orbital angular momentum** of the sun as seen from the earth.

The Lense Thirring precession of the earth with respect to the sun is due to the latter's spin angular momentum about its own axis.

Similarly the Lense Thirring precession seen by Gravity Probe B is due to the spin of the earth about its own axis, and the geodesic precession is due to the orbital angular momentum of the earth as seen from Gravity Probe B.

The exact perihelion precession of the Earth can be found by using an effective $\underline{L}$, and similarly for the geodesic precession of the earth seen from Gravity Probe B.

Finally, note that these theories are derived in the ECE2 gravitational field equations which are Lorentz covariant in a space with finite torsion and curvature.