346(3): Summary of General Theory: Lense-Thirring Precession of the Earth

The general expression for precession from previous notes is:

$$\Omega = \frac{G}{2c^2} \left| \underline{\nabla} \times \left(\frac{\underline{L} \times \underline{r}}{r^3} \right) \right| \quad -(1)$$

$$= \frac{G}{2c^2 r^3} \left| \left(3\frac{\underline{r}}{r} \left(\frac{\underline{r}}{r} \cdot \underline{L} \right) - \underline{L} \right) \right|$$

If principal axes are used:

$$\underline{L} = I \underline{\omega} \quad -(2)$$

where I is the moment of inertia and where $\underline{\omega}$ is the angular velocity vector. So the general expression for precession is:

$$\Omega = \frac{IG}{2c^2 r^3} \left| \left(3\frac{\underline{r}}{r} \left(\frac{\underline{r}}{r} \cdot \underline{\omega} \right) - \underline{\omega} \right) \right| \quad -(3)$$

Here I is in general the moment of inertia of a localized distribution of mass of any kind, for example the sun in the solar system. If the total mass of the sun is $\underline{M}$, the orbiting system is attracted by $\underline{M}$. The sun is inhomogeneous so its moment of inertia can be worked out following the method developed by Dr. Horst Eckardt in UFT 346. The sun rotates every 27 days around an axis which can be denoted $\underline{k}$. The orbit of the earth is not perpendicular to $\underline{k}$, it is inclined at an angle θ to $\underline{k}$:

$$\theta = 7.25^\circ \quad -(4)$$

The rotation of the sun can be measured with sunspot analysis, and is the rotation of a localized mass

2) distribution. If the earth were orbiting in a plane perpendicular to $\underline{k}$ then:

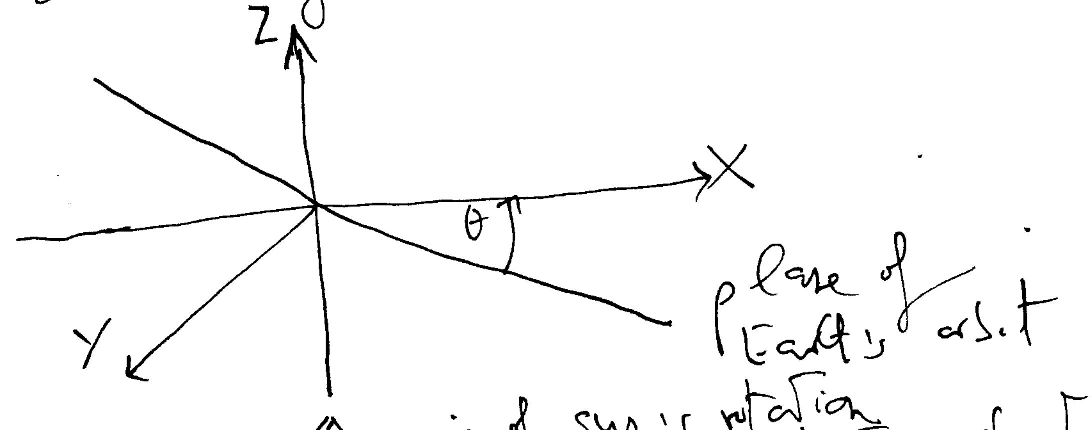
$$\underline{r} = X \underline{i} + Y \underline{j} \quad -(5)$$

In this case eq. (3) reduces to:

$$\Omega = \frac{I \sigma \omega}{2c^2} \quad -(6)$$

However, the plane of the earth's orbit is inclined at an angle θ as in Fig (1):

Fig. (1)



This inclination is described by a rotation about the Y axis:

$$X' = X\cos\theta + Z\sin\theta \quad -(7)$$
$$Z' = -X\sin\theta + Z\cos\theta \quad -(8)$$
$$Y' = Y \quad -(9)$$

So:

$$\underline{r} = (X\cos\theta + Z\sin\theta)\,\underline{i} + (-X\sin\theta + Z\cos\theta)\,\underline{k} + Y\underline{j} \quad -(10)$$

In the untilted XY plane:

$$Z = 0 \quad -(11)$$

So in the tilted plane:

$$\underline{r} = \underline{i} X \cos\theta - \underline{k} X \sin\theta + Y \underline{j} \quad -(12)$$

Note that:

$$X'^2 + Y'^2 + Z'^2 = X^2 + Y^2 + Z^2 \quad -(13)$$

Therefore the precession of the Earth's perihelion from eqns (1) and (2) is:

$$\Omega = \frac{GI}{2c^2} \left| \underline{\nabla} \times \left(\frac{(\underline{\omega} \times \underline{r})}{r^3} \right) \right| \quad -(14)$$

where

$$\underline{\omega} = \omega \underline{k} \quad -(15)$$

and

$$\underline{r} = \underline{i} X \cos\theta + Y \underline{j} - \underline{k} X \sin\theta \quad -(16)$$

also

$$\theta = 7.25^\circ \quad -(17)$$

The observed precession of the perihelion of the earth is:

$$\Omega = 5.0 \pm 1.2'' \text{ a century}$$

$$= 7.681 \times 10^{-15} \text{ radians / second}$$

$$-(18)$$

If the sun were a perfect sphere and if the orbital plane were XY, perpendicular to Z, then:

$$I = \frac{2}{5} MR^2 \quad -(19)$$

where M is the mass of the sun and where R is its radius. In this case, as in note 344(4):

$$\Omega = \frac{G\omega M R^2}{5c^2r^3} \quad -(20)$$

where

$$\frac{MG}{c^2} = 1.475 \times 10^3 \text{ m} \quad -(21)$$

$$R = 6.95700 \times 10^8 \text{ m} \quad -(22)$$

and the radius of the sun. The earth - sun distance is:

$$r = 1.49598 \times 10^{11} \text{ m} \quad -(23)$$

We have:

$$\frac{MG}{c^2} = 1.475 \times 10^3 \text{ m}. \quad -(24)$$

Eq. (20) is the Lense - Thirring precession of the earth about the sun:

$$\Omega = \frac{1.475 \times 10^3 \times 6.957^2 \times 10^{16}}{1.49598^3 \times 10^{33}} \omega \quad -(25)$$

$$= \frac{1.475 \times 48.40}{3.348} \times 10^{-14} \omega$$

$$= 2.132 \times 10^{-13} \omega$$

For the Lense Thirring precession the angular velocity ω is that of the sun's own rotation, once every 27

5) earth days, so:

$$\omega = \frac{2\pi}{27 \times 3600 \times 24} \text{ rad } s^{-1} \quad -(26)$$

$$= \frac{2\pi}{2.7 \times 2.4 \times 3.6} \times 10^{-7}$$

$$= 2.693 \times 10^{-8} \text{ rad } s^{-1}$$

So the Lense Thirring precession of the Earth is:

$$\Omega = 2.132 \times 2.693 \times 10^{-21} \text{ rads per second} \quad -(27)$$

$$= 5.741 \times 10^{-21} \text{ rads. per second}$$

As in Note 345(5):

$$1 \text{ year} = 3.156 \times 10^{7} \text{ seconds} \quad -(28)$$

$$1 \text{ radian} = 2.06265 \times 10^{5} \text{ arc seconds} \quad -(29)$$

So

$$\Omega = 3.156 \times 2.06265 \times 10^{12} \times 5.741 \times 10^{-21} \text{ arc seconds a year} \quad -(30)$$

$$= 3.741 \times 10^{-8} \text{ arc seconds a year}$$

The perihelion precession is: $\quad -(31)$

$$\Omega = 5.0 \pm 1.2'' \text{ a century}$$

$$= 7.681 \times 10^{-15} \text{ radians per second.}$$

Therefore the perihelion precession is due to a different source of angular velocity.