346(1): Precession Due to a Localized Current Distribution.

First consider the ECE2 theory of magnetostatics based on:

$$\underline{\nabla} \cdot \underline{B} = 0 \quad -(1)$$

$$\underline{\nabla} \times \underline{B} = \mu_0 \underline{J} \quad -(2)$$

and assume that:

$$\underline{B} = \underline{\nabla} \times \underline{A}(\underline{r}) \quad -(3)$$

In the usual theory described in Jackson, third edition, chapter 5 it is assumed that:

$$\underline{\nabla} \cdot \underline{A} = 0 \quad -(4)$$

to arrive at

$$\nabla^2 \underline{A} + \mu_0 \underline{J} = \underline{0} \quad -(5)$$

However, eq. (5) can be derived within the structure of ECE2 by using the ECE2 wave equation derived from the tetrad postulate.

The above theory can be developed within the structure of ECE2 following UFT318. However it is developed here in the manner of Jackson, chapter five, giving the solution:

$$\underline{A}(\underline{r}) = \frac{\mu_0}{4\pi} \int \frac{\underline{J}(\underline{r}')}{|\underline{r} - \underline{r}'|} d^3 r' \quad -(6)$$

Consider the general current distribution localized in a small region of space:

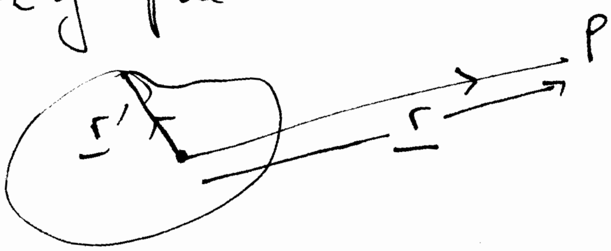

Fig (1)

Now expand as:

$$\frac{1}{|\underline{r} - \underline{r}'|} = \frac{1}{|\underline{r}|} + \frac{\underline{r} \cdot \underline{r}'}{|\underline{r}|^3} + \cdots \quad -(7)$$

So:

$$A_i(\underline{r}) = \frac{\mu_0}{4\pi}\left(\frac{1}{|\underline{r}|}\int J_i(\underline{r}')\,d^3r' + \frac{\underline{r}\cdot}{|\underline{r}|^3}\int J_i(\underline{r}')\,\underline{r}'\,d^3r' + \ldots\right) \quad -(8)$$

It is assumed that $\underline{J}$ is a localized, divergenceless, current distribution. In general, if $J(r')$ is localized but not divergenceless it can be shown that there exists the identity:

$$\int\left(f\,\underline{J}\cdot\underline{\nabla}'g + g\,\underline{J}\cdot\underline{\nabla}'f + fg\,\underline{\nabla}'\cdot\underline{J}\right)d^3r' = 0 \quad -(9)$$

and that this identity implies:

$$\int \underline{J}(\underline{r}')\,d^3r' = \underline{0} \quad -(10)$$

Therefore the monopole term in the expansion (8) vanishes. The dipole term is the second term on the right hand side of Eq. (8) and can be developed as follows:

$$\begin{aligned}\underline{r}\cdot\int\underline{r}'J_i\,d^3r' &= \sum_j r_j\int r_j'J_i\,d^3r' \\ &= -\frac{1}{2}\sum_j r_j\int\left(r_i'J_j - r_j'J_i\right)d^3r' \\ &= -\frac{1}{2}\sum_{j,k}\epsilon_{ijk}r_j\int\left(\underline{r}'\times\underline{J}\right)_k d^3r' \quad -(11)\\ &= -\frac{1}{2}\left(\underline{r}\times\int\left(\underline{r}'\times\underline{J}\right)d^3r'\right)_i\end{aligned}$$

The magnetization is the magnetic dipole moment density:

3)

$$\underline{M}(\underline{r}) = \frac{1}{2} \underline{r} \times \underline{J}(\underline{r}) \quad -(12)$$

The magnetic dipole moment is:

$$\underline{m} = \frac{1}{2} \int \underline{r}' \times \underline{J}(\underline{r}') \, d^3 r' \quad -(13)$$

Therefore:

$$\underline{A}(\underline{r}) = \frac{\mu_0}{4\pi r^3} \underline{m} \times \underline{r}. \quad -(14)$$

The gravitomagnetic equivalent is:

$$\boxed{\underline{A}_g(\underline{r}) = \frac{G}{c^2 r^3} \underline{m}_g \times \underline{r}} \quad -(15)$$

where the gravitomagnetic dipole moment is:

$$\underline{m}_g = \frac{1}{2} \underline{L} \quad -(16)$$

where $\underline{L}$ is the orbital angular momentum of a localized distribution of mass.

Therefore the gravitomagnetic vector potential is

$$\boxed{\underline{A}_g(\underline{r}) = \frac{G}{2c^2 r^3} \underline{L} \times \underline{r}} \quad -(17)$$

The gravitomagnetic field is:

$$\underline{\Omega} = \underline{\nabla} \times \underline{A}_g(\underline{r}) \quad -(18)$$

and the angular momentum is:

$$\underline{L} = \int \underline{r}' \times \underline{J}(\underline{r}') \, d^3 r' \quad -(19)$$

) For one particle this reduces to:

$$\underline{L} = \underline{r} \times \underline{p} \quad - (20)$$

Therefore eq. (19) is a generalization of eq. (20) for a localized distribution of mass.

For some applications in magnetostatics it is possible to express $\underline{B}$ as:

$$\underline{B} = - \underline{\nabla} \phi_m \quad - (21)$$

where ϕ_m is the magnetic scalar potential. Eq. (21) can also be developed in ECE2 theory.

Therefore any precession can be expressed as:

$$\boxed{\Omega = \frac{G}{2c^2 r^3} \left| \underline{\nabla} \times (\underline{L} \times \underline{r}) \right|} \quad - (22).$$

We refer to this as the ECE2 law of precession.

Therefore any precession is defined as being due to the angular momentum of a localized mass distribution. The perihelion precession for example is due to $\underline{L}$ of the sun. Here r is the distance from the sun to the planet. The Lense-Thirring precession is due to the $\underline{L}$ of the spinning earth, where r is the distance from the earth to the Gravity Probe B satellite. The geodetic precession is due to the $\underline{L}$ of the earth spinning with respect to a frame of reference fixed on Gravity Probe B.

Any precession can be described by Eq. (22).

5) In general:

$$\underline{\nabla} \times (\underline{L} \times \underline{r}) = \underline{L}(\underline{\nabla} \cdot \underline{r}) - (\underline{\nabla} \cdot \underline{L})\underline{r} + (\underline{r} \cdot \underline{\nabla})\underline{L} - (\underline{L} \cdot \underline{\nabla})\underline{r} \quad -(23)$$

and computer algebra can be used to work out the magnitude $|\underline{\nabla} \times (\underline{L} \times \underline{r})|$ in order to avoid human error.

If it is assumed that $\underline{L}$ is defined in one axis, the $\underline{k}$ axis, then:

$$\underline{L} = MR^2 \omega \underline{k} \quad -(24)$$

for one particle rotating on the rim of a circle of radius R. More generally however, the relation between angular momentum $\underline{L}$ and angular velocity $\underline{\omega}$ for a localized distribution of mass must be worked out from the integral (19). For a spinning sphere:

$$\underline{L} = \frac{2}{5} MR^2 \omega \underline{k} \quad -(25)$$

$$= I \underline{\omega}$$

where I is the moment of inertia of the sphere:

$$I = \frac{2}{5} MR^2 \quad -(26)$$

and where

$$\underline{\omega} = \omega \underline{k}. \quad -(27)$$

In general therefore $\underline{L}$ can be related to $\underline{\omega}$ by:

$$\underline{L} = I \underline{\omega} \quad -(28)$$

and generalizations of this equation.

6)

Examples

i) Perihelion Precession of the Earth

Experimentally:

$$\Omega = 11.45'' \text{ per year} = 5.551 \times 10^{-5} \text{ radians per year}$$

$$= \frac{5.551 \times 10^{-5}}{3.156 \times 10^{7}} \text{ rad s}^{-1} \quad -(29)$$

$$= 1.759 \times 10^{-12} \text{ rad s}^{-1}$$

So

$$\Omega = \frac{G}{2c^2 r^3} \left| \underline{\nabla} \times (\underline{L} \times \underline{r}) \right| \quad -(30)$$

$$= 1.759 \times 10^{-12}$$

where

$$G = 6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2} \quad -(31)$$

$$c = 2.998 \times 10^{8} \text{ m s}^{-1} \quad -(32)$$

$$r = 1.496 \times 10^{9} \text{ m} \quad -(33)$$

So

$$\frac{G}{2c^2 r^3} = \frac{6.67 \times 10^{-11}}{2 \times 2.998^2 \times 1.496^3 \times 10^{39}} \quad -(34)$$

$$= 1.108 \times 10^{-51}$$

So

$$\left| \underline{\nabla} \times (\underline{L} \times \underline{r}) \right| = \frac{1.759 \times 10^{-12}}{1.108 \times 10^{-51}}$$

$$= 1.588 \times 10^{39} \text{ Js} \quad -(35)$$

Finally we denote:

$$\alpha = \left| \underline{\nabla} \times (\underline{L} \times \underline{r}) \right| = 1.588 \times 10^{39} \text{ Js}$$

$$-(36)$$

1) We denote α as the precession factor of the earth - sun system.

Units Check

MG/c^2 = metres, $G/c^2 = m\,kg^{-1}$, so

$$\boxed{\Omega = \frac{G}{2c^2 r^3}\alpha} \quad -(37)$$

$= m\,kg^{-1}\,J\,s\,m^{-3} = m\,kg^{-1}\,kg\,m^2 s^{-2}\,s\,m^{-3} = s^{-1}$ ✓✓

Note that α has the same units as the Planck constant $\hbar$.

2) Lense Thirring Precession from Gravity Probe B

In this case:

$$\Omega_{LT} = 6.284 \times 10^{-15}\ \text{rad}\ s^{-1} \quad -(38)$$

$$= \frac{G}{2c^2 r^3}\alpha$$

where:

$$r = 7.02 \times 10^{6}\ m \quad -(39)$$

so:

$$6.284 \times 10^{-15} = \frac{6.67 \times 10^{-11}}{2 \times 2.998^2 \times 10^{16} \times 7.02^3 \times 10^{18}}\alpha \quad -(40)$$

so

$$\alpha = \frac{6.284 \times 10^{-15} \times 2 \times 2.998^2 \times 10^{16} \times 7.02^3 \times 10^{18}}{6.67 \times 10^{-11}}\ Js$$

$$= 5858.9 \times 10^{8}\ Js$$

$$\alpha = 5.859 \times 10^{11}\ Js$$