343(4): Calculation of the Thomas Phase Shift from the Newtonian Limit.

In previous notes it has been shown that the orbit in the Newtonian limit for a rotating frame (r, θ_1) is:

$$r = \frac{\alpha_1}{1+\epsilon_1 \cos\theta_1} \quad - (1)$$

This produces the non-relativistic orbital velocity:

$$v_{N1}^2 = \frac{L_1^2}{m^2 r^4}\left(r^2 + \left(\frac{dr}{d\theta_1}\right)^2\right) \quad - (2)$$

The relativistic angular velocity used in Thomas precession theory is:

$$\Omega = \frac{v}{r} \quad - (3)$$

where the relativistic velocity v is defined by:

$$v^2 = \gamma^2 v_{N1}^2 = \frac{v_{N1}^2}{1 - \frac{v_{N1}^2}{c^2}} \quad - (4)$$

Therefore the Thomas phase shift is:

$$\Delta\phi = \Omega d\tau - \cancel{\omega t} \quad - (5)$$

As in UFT110:

$$d\tau = \left(1 - \frac{v_\theta^2}{c^2}\right)^{1/2} dt \quad - (6)$$

and

2)

$$\Omega = \omega \left(1 - \frac{v_\theta^2}{c^2}\right)^{-1} \quad -(7)$$

if the infinitesimal line element of ECE2 relativity is rotated. This means that:

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - dr^2 - r^2 d\theta^2 \quad -(8)$$

is rotated using:

$$d\theta_1 = d\theta + \omega_\theta \, dt \quad -(9)$$

In the rotating frame, eq. (8) becomes:

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - dr^2 - r^2 d\theta_1^2 \quad -(10)$$

$$= dt^2 \left(c^2 - v_{N1}^2\right)$$

where v_{N1}^2 is given by eq. (2).

It follows that the Thomas angular velocity Ω for eqs (3) and (7) must be the same:

$$\Omega = \frac{v}{r} = \omega \left(1 - \frac{v_\theta^2}{c^2}\right)^{-1} \quad -(11)$$

The Thomas angular velocity is therefore:

$$\Omega = \frac{\gamma v_{N1}}{r} \quad -(12)$$

and the Thomas phase shift can be calculated from the orbit (1). The square of the Thomas angular velocity is therefore:

$$\Omega^2 = \frac{1}{r^2} \frac{V_{N1}^2}{1 - \frac{V_{N1}^2}{c^2}} \quad -(13)$$

and can be calculated from eq. (2).

Here:

$$V_{N1}^2 = \frac{L_1^2}{m^2 r^4} \left(r^2 + \left(\frac{dr}{d\theta_1} \right)^2 \right) \quad -(14)$$

where

$$\frac{dr}{d\theta_1} = \frac{\alpha_1 \epsilon_1 \sin\theta_1}{(1 + \epsilon_1 \cos\theta_1)^2} \quad -(15)$$

$$= \frac{\epsilon_1 r^2}{\alpha_1} \sin\theta_1 .$$

Here,

$$\theta_1 = \theta + \omega_\theta t \quad -(16)$$

Finally, in order to obtain an idea of the properties of Eq. (1) it can be developed as follows. It can be written as:

$$r = \frac{\alpha_1}{1 + \epsilon_1 \cos(\theta + \omega_\theta t)} \quad -(17)$$

in which:

$$\frac{dr}{dt} = \left(\frac{2}{m} (H - U) - \frac{L_1^2}{m^2 r^2} \right)^{1/2} \quad -(18)$$

Therefore:

$$t = \int \left(\frac{2}{m} (H - U) - \frac{L_1^2}{m^2 r^2} \right)^{-1/2} dr \quad -(19)$$

in which:

$$U = -\frac{mMG}{r} \quad -(20)$$

Eqs. (17) and (19) can be solved simultaneously using computer algebra, and a graph of r versus θ plotted. This gives the effect of rotating (r, θ) to give (r, θ_1) in the Newtonian orbit.

From eq. (17):

$$1 + \epsilon_1 \cos(\theta + \omega_\theta t) = \frac{\alpha_1}{r} \quad -(21)$$

and

$$\cos(\theta + \omega_\theta t) = \frac{1}{\epsilon_1}\left(\frac{\alpha_1}{r} - 1\right) \quad -(22)$$

so

$$\theta + \omega_\theta t = \cos^{-1}\left(\frac{1}{\epsilon_1}\left(\frac{\alpha_1}{r} - 1\right)\right) \quad -(23)$$

Therefore θ can be expressed in terms of r as follows:

$$\theta = \cos^{-1}\left(\frac{1}{\epsilon_1}\left(\frac{\alpha_1}{r} - 1\right)\right) - \omega_\theta \int \left(\frac{2}{m}(H - U) - \frac{L_1^2}{m^2 r^2}\right)^{-1/2} dr \quad -(24)$$

and θ can be plotted against r. This plot can be inverted numerically to give r versus θ. The Newtonian result is:

$$\theta = \cos^{-1}\left(\frac{1}{\epsilon}\left(\frac{\alpha}{r} - 1\right)\right) \quad -(25)$$