6(1): Combined Theory of the g Factor and Lamb Shift.

Consider the total energy of a free elementary particle:

$$H = E = (c^2 p_f^2 + m^2 c^4)^{1/2} \quad -(1)$$

Here H is the hamiltonian and E is the total relativistic energy:

$$E = \gamma m c^2, \quad -(2)$$

γ is the Lorentz factor and $\underline{p}$ the relativistic momentum:

$$\underline{p}_f = \gamma m \underline{v} \quad -(3)$$

where $\underline{v}$ is defined by:

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad -(4)$$

Finally, m is the mass of the elementary particle and c is the vacuum speed of light.

In the presence of the vacuum:

$$H \rightarrow E + U_1 \quad -(5)$$

where U_1 is the ECE2 vacuum potential:

$$U_1 = \hbar \omega_1 \quad -(6)$$

where

$$\hbar \omega_1 = \gamma_1 m_1 c^2. \quad -(7)$$

where m_1 is the mass of the vacuum particle and γ_1 the Lorentz factor of the vacuum wave / particle.

Similarly the total momentum of the elementary particle in the presence of the vacuum is:

2)

$$\underline{p} = \underline{p}_f + \underline{p}_1 \quad -(8)$$

where $\underline{p}_1$ is the momentum of the vacuum particle. So:

$$\underline{p}_f \rightarrow \underline{p} - \underline{p}_1 \quad -(9)$$

Eqs. (5) and (9) are what is usually referred to as the minimal prescription, i.e.

$$\boxed{\begin{aligned} E &\rightarrow H - U_1 \\ \underline{p}_f &\rightarrow \underline{p} - \underline{p}_1 \end{aligned}} \quad -(10)$$

In ECE2 theory:

$$\underline{p}_1 = \hbar \underline{\kappa}_1 \quad -(11)$$

Therefore eq. (1) becomes:

$$(H - U_1)^2 = c^2 (\underline{p} - \underline{p}_1)\cdot(\underline{p} - \underline{p}_1) + m^2 c^4 \quad -(12)$$

i.e. $$(H - U_1 - mc^2)(H - U_1 + mc^2) = c^2 (\underline{p} - \underline{p}_1)\cdot(\underline{p} - \underline{p}_1) \quad -(13)$$

In the SU(2) basis:

$$H - U_1 - mc^2 = \frac{c^2 \underline{\sigma}\cdot(\underline{p} - \underline{p}_1)\underline{\sigma}\cdot(\underline{p} - \underline{p}_1)}{H - U_1 + mc^2} \quad -(14)$$

In the Dirac approximation:

$$H = ? \; mc^2 \quad -(15)$$

) and $H + mc^2 = ?\ 2mc^2$ — (16)

This incorrect approximation is the origin of the g factor of the electron in the Dirac theory:

$$g(\text{Dirac}) = 2 \quad -(17)$$

The correct g factor is however:

$$\boxed{g = 1 + \frac{H}{mc^2}} \quad -(18)$$

In a hydrogen atom the vacuum potential U_1 is augmented by the Coulombic potential U_c between the electron and proton:

$$U_c = -\frac{e^2}{4\pi\epsilon_0 r} \quad -(19)$$

where ϵ_0 is the vacuum permittivity and e the proton charge. The distance between electron and proton is $\underline{r}$. In ECE2 the magnetic flux density is

$$\underline{B} = \underline{\nabla} \times \underline{W} \quad -(20)$$

and the momentum of the electron becomes:

$$\underline{p} \rightarrow \underline{p} - \underline{p}_1 - e\underline{W} \quad -(21)$$

So in the H atom, eq. (14) becomes the basis of both the g factor of the electron and the Lamb shift:

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$$H' - U_1 - U_c - mc^2 = \frac{c^2 \underline{\sigma} \cdot (\underline{p} - \underline{p}_1 - e\underline{W}) \underline{\sigma} \cdot (\underline{p} - \underline{p}_1 - e\underline{W})}{H - U_1 - U_c + mc^2} \quad -(22)$$

in which the relativistic hamiltonian is:

$$H = E + U_1 + U_c \quad -(23)$$

The experimentally measured g factor of the electron is

$$g = 1 + \frac{H}{mc^2} = 2.002319314, \quad -(24)$$

so the hamiltonian is:

$$\boxed{H = 1.002319314\, mc^2} \quad -(25)$$

and is a constant of motion.

The Lamb shift is explained by the fact that the orbital angular momentum is changed to:

$$\underline{L} = \underline{r} \times (\underline{p} - \underline{p}_1) \quad -(26)$$

$$= \underline{r} \times (\underline{p} - \hbar \underline{\kappa})$$

In orbitals with finite $\underline{L}$, the energy level is affected by the vacuum's $\underline{p}_1$. Orbitals with no angular momentum such as S orbitals are not affected. This theory will be developed in the next note.