440(2): Calculation of the Lamb Shift from Vacuum Particle Momentum.

Consider the four-momentum of the vacuum particle:

$$p^\mu = \left(\frac{U_1}{c}, \underline{p}_1 \right) \quad -(1)$$

$$= \hbar \left(\frac{\omega_1}{c}, \underline{\kappa}_1 \right)$$

In this notation ω_1 is the angular frequency and $\underline{\kappa}_1$ is the wave vector of the vacuum particle. The four momentum is a special case of the ECE2 W^μ potential:

$$W^\mu = \frac{\hbar}{e} \left(\Omega^0, \underline{\Omega} \right) \quad -(2)$$

where

$$\Omega^\mu = \left(\Omega^0, \underline{\Omega} \right) \quad -(3)$$

the spin connection four vector. So:

$$p^\mu = e W^\mu = \hbar \left(\Omega^0, \underline{\Omega} \right) \quad -(4)$$

which implies that:

$$\omega_1 = c\Omega^0, \quad -(5)$$

$$\underline{\kappa}_1 = \underline{\Omega}. \quad -(6)$$

In ECE2 theory the vacuum wavenumber and the vacuum frequency of a wave / particle are defined by the spin connection. In the Aharonov Bohm vacuum, the spin connection is non-zero, but the torsion and curvature are zero. So the Maurer Cartan structure equations of the AB vacuum are as follows:

and

$$T = d \wedge q + \omega \wedge q = 0 \quad -(7)$$

$$R = d \wedge \omega + \omega \wedge \omega = 0 \quad -(8)$$

in minimal notation (see UFT papers).

Consider now the interaction of an elementary particle of mass m with the AB vacuum defined by eqs. (7) and (8). This is referred to as the ECE2 vacuum.

For the free elementary particle, hypothetically free of the vacuum, its motion is governed by the Einstein energy equation:

$$E^2 = c^2 p^2 + m^2 c^4 \quad -(9)$$

where

$$E = \gamma m c^2 = \hbar \omega \quad -(10)$$

and

$$\underline{p} = \gamma m \underline{v} = \hbar \underline{\kappa} \quad -(11)$$

are the de Broglie / Einstein wave particle duality equations. The Lorentz factor is:

$$\gamma = \left(1 - \frac{v^2}{c^2} \right)^{-1/2} \quad -(12)$$

For an elementary particle of mass m interacting with the ECE2 vacuum, the hamiltonian is:

$$H = E + U_1 \quad -(13)$$

and is a constant of motion.

From eq. (13):

)

$$E = H - \bar{U}_1 \quad -(14)$$

d this equation is the minimal prescription. It is denoted:

$$E \rightarrow H - \bar{U}_1 \quad -(15)$$

Similarly:

$$\underline{p} \rightarrow \underline{p} - \underline{p}_1 \quad -(16)$$

Eq. (16) means that the total momentum of the elementary particle in contact with the ECE2 vacuum is $\underline{p}_{total}$, where:

$$\underline{p}_{total} = \underline{p} + \underline{p}_1 \quad -(17)$$

From eqs. (9), (14) and (16):

$$(H - \bar{U}_1)^2 = c^2 (\underline{p} - \underline{p}_1) \cdot (\underline{p} - \underline{p}_1) + m^2 c^4 \quad -(18)$$

In the SU(2) basis:

$$(H - \bar{U}_1)^2 = c^2 \underline{\sigma} \cdot (\underline{p} - \underline{p}_1) \underline{\sigma} \cdot (\underline{p} - \underline{p}_1) + m^2 c^4 \quad -(19)$$

i.e.:

$$H - \bar{U}_1 - mc^2 = \frac{\underline{\sigma} \cdot (\underline{p} - \underline{p}_1) \underline{\sigma} \cdot (\underline{p} - \underline{p}_1)}{m \left(1 + \frac{H}{mc^2}\right) \left(1 - \frac{\bar{U}_1}{mc^2 \left(1 + \frac{H}{mc^2}\right)}\right)} \quad -(20)$$

The anomalous g factor of the electron is defined by:

$$\boxed{g = 1 + \frac{H}{mc^2}} \quad -(21)$$

i.e.

$$\begin{aligned} g &= 1 + \gamma + \frac{U_1}{mc^2} \\ &= 1 + \gamma + \frac{\hbar\omega_1}{mc^2} \qquad -(22) \\ &= 1 + \frac{\hbar(\omega + \omega_1)}{mc^2} \end{aligned}$$

If the elementary particle is at rest, its frequency is defined by $\hbar\omega_0 = mc^2$ $-(23)$
which is the de Broglie rest frequency equation. Therefore for an elementary particle at rest and in contact with the vacuum:

$$\begin{aligned} g &= 1 + \frac{\hbar\omega_0}{mc^2} + \frac{\hbar\omega_1}{mc^2} \\ &= 2 + \frac{\hbar\omega_1}{mc^2} \qquad -(24) \end{aligned}$$

In general, g is given by eq. (21).

The anomalous g factor of the electron is

5) measured experimentally to be:

$$g = 2.00231931 4. \quad -(25)$$

Accepting this result for the sake of argument, the hamiltonian is:

$$H = 1.00231931 4\, mc^2 \quad -(26)$$

Therefore eq. (20) is:

$$H - U_1 - mc^2 = \frac{\underline{\sigma} \cdot (\underline{p} - \underline{p}_1)\, \underline{\sigma} \cdot (\underline{p} - \underline{p}_1)}{mg\left(1 - \frac{U_1}{gmc^2}\right)} \quad -(27)$$

Assume that: $U_1 << gmc^2 \quad -(28)$

then:

$$H - U_1 - mc^2 \doteq \frac{1}{mg} \underline{\sigma} \cdot (\underline{p} - \underline{p}_1)\left(1 + \frac{U_1}{gmc^2}\right) \underline{\sigma} \cdot (\underline{p} - \underline{p}_1) \quad -(29)$$

This equation is quantized as follows:

$$\left(H - U_1 - mc^2\right)\phi = \frac{1}{mg} \underline{\sigma} \cdot \left(-i\hbar \underline{\nabla} - \underline{p}_1\right)\left(1 + \frac{U_1}{gmc^2}\right) \underline{\sigma} \cdot (\underline{p} - \underline{p}_1)\phi \quad -(30)$$

$$= \frac{i\hbar}{g^2 m^2 c^2} \underline{\sigma} \cdot \underline{\nabla} U\, \underline{\sigma} \cdot (\underline{p} - \underline{p}_1)\phi + \cdots$$

Consider the hydrogen atom, in which the Coulomb

) potential is:
$$U_c = -\frac{e^2}{4\pi\epsilon_0 r} \quad -(31)$$

In this case:
$$(H - U_1 - U_c - mc^2)\phi = -\frac{i\hbar}{g^2 m^2 c^2}\,\underline{\sigma}\cdot\underline{\nabla}(U_1 + U_c)\,\underline{\sigma}\cdot(\underline{p} - \underline{p}_1)\phi \quad -(32)$$

It is assumed that U_1 has no dependence on $\underline{r}$, so
it follows that:
$$\underline{\nabla}(U_1 + U_c) = \frac{e^2}{4\pi\epsilon_0 r^2} = \frac{e^2}{4\pi\epsilon_0 r^3}\,\underline{r} \quad -(33)$$

and:
$$(H - U_1 - U_c - mc^2)\phi = \frac{-i\hbar e^2}{4\pi\epsilon_0 g^2 m^2 c^2 r^3}\,\underline{\sigma}\cdot\underline{r}\,\underline{\sigma}\cdot(\underline{p} - \underline{p}_1)\phi \quad -(34)$$

The real and physical part of eq. (34) is:
$$\mathrm{Re}(H - U_1 - U_c - mc^2)\phi = \frac{\hbar e^2}{4\pi\epsilon_0 g^2 m^2 c^2 r^3}\,\underline{\sigma}\cdot(\underline{L} - \underline{L}_1)\phi \quad (35)$$

where
$$\underline{L} = \underline{r} \times \underline{p} \quad -(36)$$

and
$$\underline{L}_1 = \underline{r} \times \underline{p}_1 \quad -(37)$$

The usual theory gives the result:
$$\mathrm{Re}(H - U_c - mc^2)\phi = \frac{\hbar e^2}{16\pi\epsilon_0 m^2 c^2 r^3}\,\underline{\sigma}\cdot\underline{L} \quad -(36)$$

7) i.e. the usual theory ignores the presence of the vacuum and uses:

$$g = 2 \quad -(37)$$

The usual theory uses:

$$\underline{S} = \frac{\hbar}{2} \underline{\sigma} \quad -(38)$$

to define the spin angular momentum $\underline{S}$ of the electron.
However, in the presence of the vacuum, eq. (38) is:

$$\underline{S} = \frac{\hbar}{g} \underline{\sigma} \quad -(39)$$

From eqs. (35) and (39):

$$\mathrm{Re}\left(H - U_1 - U_c - mc^2\right)\psi = \frac{e^2}{4\pi\epsilon_0 g m^2 c^2 r^3} \underline{S} \cdot \left(\underline{L} - \underline{L}_1\right)\psi \quad -(40)$$

The energy levels for this equation are given from the expectation value

$$\left\langle \frac{\underline{S} \cdot \underline{L}}{r^3} \right\rangle = \frac{\hbar^2}{2a_0^3}\left(\frac{J(J+1) - L(L+1) - S(S+1)}{n^3 L(L+1/2)(L+1)}\right) -(41)$$

where a_0 is the Bohr radius and n the principal quantum number. The Clebsch Gordan series is

$$J = L+S,\ L+S-1,\ \ldots,\ |L-S| \quad -(42)$$

<u>The Lamb shift</u> is given by:

$$E_{LS} = -\frac{e^2}{4\pi\epsilon_0 g m^2 c^2} \left\langle \frac{\underline{S} \cdot \underline{L}_1}{r^3} \right\rangle \quad -(43)$$