331(6): Splittings of the n → n+1 Series

As shown in note 306(6) the Zeeman lines of each transition are $n^2 + 1$ degenerate in general. However for absorptions there is the further restriction of:

$$\Delta l = 1 \quad — (1)$$

in addition to:

$$\Delta m_l = 0, \pm 1 \quad — (2)$$

Examples of such lines for atomic H are:

1) $n = 2$ to $n = 3$, the $H\alpha$ line at 15,241.4 cm^{-1} (visible)
2) $n = 3$ to $n = 4$, Infra-red, 5,334.4 cm^{-1}.
3) $n = 4$ to $n = 5$, Infra-red, 2,469.1 cm^{-1}
4) $n = 13$ to $n = 14$, Far infra-red, 81.52 cm^{-1}.
5) $n = 50$ to $n = 51$, Microwave 1.704 cm^{-1}.

It is also possible that $n = 2$ goes to $n = 4, 5, 6 \ldots$ etc. This builds up the well known Lyman, Balmer, Paschen, Pfund series of atomic absorptions

For $n = 2$ to $n = 3$

The possible transitions for left cp polarization:

$$\Delta m_l = 1 \quad — (3)$$

are the five degenerate transitions:

a) $2s \rightarrow 2p$ ($n = 2, l = 0, m = 0$ to $n = 3, l = 1, m = 1$)
b) $2p \rightarrow 3s$ ($n = 2, l = 1, m = -1$ to $n = 3, l = 0, m = 0$)
*c) $2p \rightarrow 3d$ ($n = 2, l = 1, m = -1$ to $n = 3, l = 2, m = 0$)
*d) $2p \rightarrow 3d$ ($n = 2, l = 1, m = 0$ to $n = 3, l = 2, m = 1$)
*e) $2p \rightarrow 3d$ ($n = 2, l = 1, m = 1$ to $n = 3, l = 2, m = 2$)

) For absorption:

$$\Delta l = 1 \quad -(4)$$

so there are three degenerate lines marked with an asterisk. These are split into three lines by the new relativistic effect, as discussed in Note 331(5). Similar results are obtained for:

$$\Delta m = 0 \text{ (linear polarization)} \quad -(5)$$

and right CP polarization:

$$\Delta m = -1 \quad -(6)$$

The new energy levels are defined by:

$$E = -\frac{m e^4}{32\pi^2\epsilon_0^2\hbar^2 n^2} - \frac{e\hbar}{2m} B_Z \left(1 + \frac{2.66567 \times 10^{-5}}{n^2} \right) \quad -(7)$$

Now consider the $n=4$ to $n=5$ infra-red line at 2,469.1 cm^{-1}. We have transitions as in Note 306(6) from:

4s	$n=4$,	$l=0$,	$m=0$
4p	$n=4$,	$l=1$,	$m=-1, 0, 1$
4d	$n=4$,	$l=2$,	$m=-2, -1, 0, 1, 2$
4f	$n=4$,	$l=3$,	$m=-3, -2, -1, 0, 1, 2$

to

5s	$n=5$,	$l=0$,	$m=0$
5p	$n=5$,	$l=1$,	$m=-1, 0, 1$
5d	$n=5$,	$l=2$,	$m=-2, -1, 0, 1, 2$
5f	$n=5$,	$l=3$	$m=-3, -2, -1, 0, 1, 2, 3$
5g	$n=5$,	$l=4$,	$m=-4, -3, -2, -1, 0, 1, 2, 3, 4$

3) For

$$\Delta m = 1 \quad -(8)$$

there are seventeen degenerate transitions:

1) $4s \to 5p$ $(n=4, \ell=0, m=0 \to n=5, \ell=1, m=1)$
2) $4p \to 5s$ $(n=4, \ell=1, m=-1 \to n=5, \ell=0, m=0)$
*3) $4p \to 5d$ $(n=4, \ell=1, m=-1 \to n=5, \ell=2, m=0)$
*4) $4p \to 5d$ $($ " 0 " $1)$
*5) $4p \to 5d$ $($ " 1 " $2)$
*6) $4d \to 5f$ $(n=4, \ell=2, m=-2$ to $n=5, \ell=3, m=-1)$
*7) " $($ " -1 " $0)$
*8) " $($ " 0 " $1)$
*9) " $($ " 1 " $2)$
*10) " $($ " 2 " $3)$
*11) $4f \to 5g$ $(n=4, \ell=3, m=-3$ to $n=5, \ell=4, m=-2)$
*12) " " -2 " -1
*13) " " -1 " 0
*14) " " 0 " 1
*15) " " 1 " 2
*16) " " 2 " 3
*17) " " 3 " 4

For

$$\Delta \ell = 1 \quad -(9)$$

there are fifteen degenerate transitions marked with an asterisk.

So eq. (7) splits these into fifteen different lines.

4) Similar results are obtained for:

$$\Delta n = 0, \quad \Delta n = -1 \qquad - (10)$$

so there are ~~forty five~~ lines in the relativistic spectrum.

In general there $3(n^2 - 1)$ lines in the relativistic spectrum, occurring in three groups, one symmetric ($\Delta n = 0$) and two asymmetric mirror image groupings ($\Delta n = \pm 1$). The table gives the number of lines:

Transition	Wavenumber of Main Line	Number of Lines
$n = 2$ to $n = 3$	15,241.4	9
$n = 3$ to $n = 4$	5,334.1	24
$n = 4$ to $n = 5$	2,469.1	45
$n = 13$ to $n = 14$	81.52	804
$n = 50$ to $n = 51$	1.704	7497