330(3): Overview of New Schemes of Quantization

The basic spin orbit hamiltonian is:

$$\hat{H}_{SO}\phi = \frac{1}{4m^2c^2}\,\underline{\sigma}\cdot\underline{p}\,U\,\underline{\sigma}\cdot\underline{p}\,\phi \quad -(1)$$

where

$$\underline{p} = \gamma\,\underline{p}_0 \quad -(2)$$

is the relativistic momentum. Therefore eq. (1) may be written as:

$$\hat{H}_{SO}\phi = \frac{1}{4m^2c^2}\,\underline{\sigma}\cdot\gamma\underline{p}_0\,U\,\underline{\sigma}\cdot\gamma\underline{p}_0\,\phi \quad -(2)$$

giving rise to many new spectral effects. There are two quantization schemes possible:

$$\underline{p}\,\phi_r = -i\hbar\,\underline{\nabla}\,\phi_r \quad -(3)$$

where ϕ_r is the relativistic wavefunction, and:

$$\underline{p}_0\,\phi = -i\hbar\,\underline{\nabla}\,\phi \quad -(4)$$

where ϕ is the non relativistic wavefunction.

In the usual procedure based on the Dirac equation, the scheme (3) is used, although this is rarely if ever made clear.

If scheme (4) is used several new types of spectra appear, because:

2)

$$\gamma = \left(1 - \frac{p_0^2}{m^2c^2}\right)^{-1/2} \quad -(5)$$

where

$$H_0 = \frac{p_0^2}{2m} + U \quad -(6)$$

and

$$p_0^2 = 2m\left(H_0 - U\right) \quad -(7)$$

So

$$\gamma = \left(1 - \frac{2\left(H_0 - U\right)}{mc^2}\right)^{-1/2} \quad -(8)$$

Scheme 1

This is based on:

$$\hat{H}_{SO}\phi = \frac{1}{4m^2c^2}\,\underline{\sigma}\cdot\gamma\left(-i\hbar\underline{\nabla}\right)U\,\underline{\sigma}\cdot\gamma p_0\phi \quad -(9)$$

$$= \frac{-i\hbar}{4m^2c^2}\,\underline{\sigma}\cdot\gamma\underline{\nabla}\left(U\underline{\sigma}\cdot\gamma\underline{p_0}\,\phi\right)$$

Scheme 2

This is based on:

$$\hat{H}_{SO}\phi = \frac{1}{4m^2c^2}\,\underline{\sigma}\cdot\underline{p}_0\,\gamma U\,\underline{\sigma}\cdot\gamma\underline{p_0}\,\phi \quad -(10)$$

$$= \frac{-i\hbar}{4m^2c^2}\,\underline{\sigma}\cdot\underline{\nabla}\left(\gamma U\underline{\sigma}\cdot\gamma p_0\phi\right)$$

The spectra of eq. (9) are different from the spectra of eq. (10) because:

$$U = -\frac{e^2}{4\pi\epsilon_0 r} \quad -(11)$$

and $\underline{\nabla}$ acts on γ to give a non zero result.

Scheme 3

The vector potential $\underline{A}$ is introduced using

$$\underline{p_0} \rightarrow \underline{p_0} - e\underline{A} \quad -(12)$$

giving:

$$\hat{H}_{so}\phi = \frac{1}{4m^2c^2}\,\underline{\sigma}\cdot\gamma\left(\underline{p_0} - e\underline{A}\right)U\,\underline{\sigma}\cdot\gamma\left(\underline{p_0} - e\underline{A}\right)\phi$$

$$= -\frac{i\hbar}{4m^2c^2}\,\underline{\sigma}\cdot\gamma\left(\underline{\nabla} - e\underline{A}\right)U\,\underline{\sigma}\cdot\gamma\left(\underline{p_0} - e\underline{A}\right)\phi$$

$$-(13)$$

Scheme 4

This develops scheme 2 as follows:

$$\hat{H}_{so}\phi = -\frac{i\hbar}{4m^2c^2}\,\underline{\sigma}\cdot\left(\underline{\nabla} - e\underline{A}\right)\left(\gamma U\,\underline{\sigma}\cdot\gamma\left(\underline{p_0} - e\underline{A}\right)\phi\right)$$

$$-(14)$$

4) These four schemes all give different results and different spectroscopic patterns.

For example eq. (14) gives new types of Zeeman effect, ESR, NMR and MRI.

In ECE2 theory the scalar and vector potentials are due to spin curvature. The latter can therefore be observed in all types of spectroscopy. There are also new Evans / Morris effects along the lines of UFT 308.

There are no a priori laws or rules that forbid the use of the scheme (3) or (4), and there are no rules that can be used to determine which scheme is valid. In the usual textbook theory, the scheme that is used is:

$$\hat{H}_{SO}\,\phi = \frac{-i\hbar}{4m^2c^2}\,\underline{\sigma}\cdot\underline{\nabla}\left(U\,\underline{\sigma}\cdot\underline{p}_0\,\phi\right) \quad -(15)$$

in which the first $\underline{p}$ on the RHS of eq. (1) is defined by:

$$\underline{p}\,\phi_r = -i\hbar\,\underline{\nabla}\,\phi_r \quad -(16)$$

and in which the second $\underline{p}$ is erroneously replaced by $\underline{p}_0$.