327(8): Precession from the Minkowski Metric

As in previous notes the orbit from the Minkowski metric of special relativity is:

$$r = \frac{\alpha}{1+\epsilon\cos\theta} \quad -(1)$$

where α and ϵ are the respectively the relativistic half right latitude and the relativistic ellipticity. Therefore:

$$\cos\theta = \frac{1}{\epsilon}\left(\frac{\alpha}{r} - 1\right) \quad -(2)$$

Define θ by:

$$\theta = \theta_0 + \Delta\theta \quad -(3)$$

where $\Delta\theta$ is the precession angle of the Minkowski metric.

At the perihelion:

$$\theta_0 = 2\pi \quad -(3)$$

and

$$r = r_{min} = \frac{\alpha_0}{1+\epsilon_0} \quad -(4)$$

of the classical or Newtonian orbit with:

$$r = \frac{\alpha_0}{1+\epsilon_0\cos\theta_0} \quad -(5)$$

Therefore at the perihelion:

$$\cos(2\pi + \Delta\theta) = \cos\Delta\theta \quad -(6)$$

and

$$\sin\Delta\theta \sim \Delta\theta = \left(1-\cos^2\Delta\theta\right)^{1/2} \quad -(7)$$

It follows from eqs. (2) and (4) that:

2)

$$\cos \Delta\theta = \frac{1}{\epsilon}\left(\frac{\alpha}{\alpha_0}\left(1+\epsilon_0\right) - 1\right) \quad -(8)$$

At the perihelion. As:

$$\alpha \rightarrow \alpha_0 \text{ and } \epsilon \rightarrow \epsilon_0 \quad -(9)$$

eq. (8) gives:

$$\Delta\theta \rightarrow 0 \quad -(10)$$

self consistently.

Therefore the precession is given by:

$$\boxed{\Delta\theta = \left(1 - \frac{1}{\epsilon^2}\left(\frac{\alpha}{\alpha_0}\left(1+\epsilon_0\right) - 1\right)^2\right)^{1/2}} \quad -(11)$$

where

$$\frac{\alpha}{\alpha_0} = \gamma^2 = \left(1 - \left(\frac{v_0}{c}\right)^2\right)^{-1} \quad -(12)$$

The relativistic ϵ can be worked out as in previous notes, and to an excellent approximation

$$\epsilon \sim \epsilon_0 \quad -(13)$$

Therefore:

$$\Delta\theta = \left(1 - \frac{1}{\epsilon_0^2}\left(\frac{\alpha}{\alpha_0}\left(1+\epsilon_0\right) - 1\right)^2\right)^{1/2} \quad -(14)$$