327(7): <u>Precession Calculated with the Apsidal Angle Method in the Near Circular Approximation</u>

Consider the Leibnitz orbit equation of 1689:

$$F(r) = m\frac{d^2 r}{dt^2} - \frac{L_0^2}{mr^3} \quad -(1)$$

where L_0 is the non relativistic angular momentum. For a nearly circular orbit:

$$\frac{d^2 r}{dt^2} \sim 0 \quad -(2)$$

This the first approximation used. If r_c is the radius of the circular orbit then:

$$F(r_c) = -\frac{L_0^2}{m r_c^3} \quad -(3)$$

Define a small perturbation:

$$x = r - r_c \quad -(4)$$

then

$$r = r_c + x. \quad -(5)$$

and:

$$m\frac{d^2 x}{dt^2} - \frac{L^2}{m(r_c + x)^3} = F(r_c + x) \quad -(6)$$

because

$$\frac{d^2}{dt^2}(r_c + x) = \frac{d^2 x}{dt^2} \quad -(7)$$

Using a Maclaurin expansion:

$$F(r_c + x) = F(r_c) + x\frac{dF}{dr}(r_c) \quad -(8)$$

$$:= F(r_c) + xF'(r_c)$$

2) So eq. (6) is:

$$m\ddot{x} - \frac{L_0^2}{m r_c^3\left(1+\frac{x}{r_c}\right)^3} \sim F(r_c) + xF'(r_c) \quad -(9)$$

Using a binomial approximation:

$$\left(1+\frac{x}{r_c}\right)^{-3} \doteqdot 1 - 3\frac{x}{r_c} \quad -(10)$$

for:

$$x \ll r_c \quad -(11)$$

so

$$\ddot{x} + \left(-\frac{3F(r_c)}{r_c} - F'(r_c)\right)x = 0 \quad -(11)$$

This equation is arrived at with ~~three~~ two further approximations, so there have been three approximations so far. From eq. (11), stable orbits mean that:

$$\left(F(r_c) + \frac{r_c}{3}F'(r_c)\right) < 0 \quad -(12)$$

The apsis is a point in an orbit when r is a maximum or minimum. The perihelia and aphelia are apsides. The apsidal angle ϕ is the angle through which the radius vector rotates between consecutive apsides.

It is defined by:

$$\boxed{\phi = \frac{T}{2}\dot{\theta}} \quad -(13)$$

3) where T is the period of oscillation. From eq. (11):

$$T = \frac{2\pi}{\left(-\frac{3F(r_c)}{r_c} - F'(r_c)\right)^{1/2}} \quad -(14)$$

From Lagrangian theory:

$$\dot{\theta} = \frac{L_0}{mr^2} \quad -(15)$$

so

$$\boxed{\phi = \pi\left(3 + r_c \frac{F'(r_c)}{F(r_c)}\right)^{-1/2}} \quad -(16)$$

In Newtonian dynamics:

$$F(r_c) = -\frac{mMG}{r_c^2} \quad -(17)$$

and

$$F'(r_c) = \frac{2mMG}{r_c^3} \quad -(18)$$

so

$$\boxed{\phi = \pi} \quad -(19)$$

in Newtonian dynamics. The orbit is closed and there is no precession.

In Einsteinian dynamics the effective force is claimed to be a sum:

$$F = -\frac{mMG}{r^2} - \frac{3MGL_0^2}{mc^2 r^4} \quad - (20)$$

$$:= -\frac{A}{r_c^2} - \frac{B}{r_c^5}$$

where

$$A := mMG, \quad B = \frac{3MGL_0^2}{mc^2} \quad - (21)$$

so

$$F' = \frac{2A}{r_c^3} + \frac{4B}{r_c^4} \quad - (22)$$

Therefore:

$$\phi = \pi\left(3 - 2\left(\frac{A/r_c^2 + 2B/r_c^4}{A/r_c^2 + B/r_c^4}\right)\right)^{-1/2}$$

$$= \pi\left(3 - 2\left(\frac{1 + (2B/(A\, r_c^2))}{1 + (B/(A\, r_c^2))}\right)\right)^{-1/2} \quad - (22)$$

Introducing a fourth approximation it is assumed that in the denominator:

$$1 + \left(\frac{B}{A\, r_c^2}\right) \sim 1 \quad - (23)$$

However, it is also assumed that the approximation (23) does not apply in the numerator. This is logical nonsense but accepting it for the sake of argument only:

5)

$$\phi \doteq \pi \left(3 - 2 \left(1 + \left(\frac{2B}{Ar_c^2} \right) \right) \right)^{-1/2} \quad -(24)$$

$$= \pi \left(1 - \left(\frac{2B}{Ar_c^2} \right) \right)^{-1/2}$$

Introducing a fifth approximation:

$$\boxed{\phi \sim \pi \left(1 + \frac{B}{Ar_c^2} \right)} \quad -(25)$$

Using the definitions of A and B, the change in apsidal angle is

$$\Delta \phi \sim \frac{3\pi L_0^2}{m^2 c^2 r_c^2} \quad -(26)$$

in a rotation of π. For a rotation of 2π:

$$\Delta \phi \doteq \frac{6\pi L_0^2}{m^2 c^2 r_c^2} \quad -(27)$$

Finally use:

$$L_0^2 = m^2 MG \alpha \quad -(28)$$

and

$$\alpha = a(1 - \epsilon^2) \quad -(29)$$

to arrive at

$$\Delta \phi = \frac{6\pi MG\alpha}{c^2 r_c^2} \quad -(30)$$

For a circular orbit:

$$\alpha = r_c. \quad -(31)$$

So

$$\boxed{\Delta\phi = \frac{6\pi MG}{ac^2(1-\epsilon^2)}} \quad -(32)$$

This is claimed to be a highly accurate result fitted by the experimental data, but in reality it is a crude approximation. In fact five approximations have been used.

Eq. (32) has been arrived at using:

$$\frac{B}{Ar_c^2} = \frac{3L_0^2}{m^2c^2r_c^2} = \frac{3MG\alpha}{c^2r_c^2} \quad -(33)$$

$$= \frac{3MG}{c^2\alpha} = \frac{3MG}{c^2a(1-\epsilon^2)}$$

So from eqns. (22) and (33) the true apsidal angle is

$$\phi = \pi\left(3 - 2\left(\frac{1 + \frac{6MG}{ac^2(1-\epsilon^2)}}{1 + \frac{3MG}{ac^2(1-\epsilon^2)}}\right)\right)^{-1/2} \quad -(34)$$

and this is not the experimental result.

7) It is claimed in the standard literature that eq. (32) is a precise result, but this experimental claim is very controversial. It seems clear that all the methods used to calculate precession are flawed in some way, and the method has not verified Einsteinian general relativity at all.