325(7): Direct Comparison of ECE2 and Einstein.

1) The ECE2 Hamiltonian and lagrangian are:

$$H(\text{ECE2}) = (\gamma-1)mc^2 + U \quad -(1)$$

and

$$\mathcal{L}(\text{ECE2}) = (\gamma-1)mc^2 - U \quad -(2)$$

where

$$\gamma = \left(1 - \frac{v_0^2}{c^2}\right)^{-1/2} \quad -(3)$$

and

$$U = -\frac{mMG}{r} \quad -(4)$$

The velocity v_0 is defined by the metric:

$$c^2 d\tau^2 = (c^2 - v_0^2)\,dt^2 \quad -(5)$$

so

$$v_0^2 = \dot{r}^2 + r^2\dot{\theta}^2 \quad -(6)$$

This is the Newtonian orbital velocity.

As in UFT324, the Euler Lagrange equation gives:

$$\frac{\partial\mathcal{L}}{\partial\theta} = \frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{\theta}} = 0 \quad -(7)$$

and the conserved angular momentum:

$$L = \gamma m r^2\dot{\theta}, \quad -(8)$$

which is a constant of motion, so:

2) so

$$\frac{dL}{dt} = 0 \quad -(9)$$

Therefore:

$$\boxed{\dot{\theta} = \frac{L}{\gamma m r^2}} \quad -(10)$$

Now use the change of variable:

$$\frac{d}{d\theta}\left(\frac{1}{r}\right) = -\frac{1}{r^2}\frac{dr}{dt}\frac{dt}{d\theta} \quad -(11)$$

i.e

$$\dot{r} = \frac{dr}{dt} = -r^2 \frac{d\theta}{dt}\frac{d}{d\theta}\left(\frac{1}{r}\right) \quad -(12)$$

so

$$\boxed{\dot{r} = -\frac{L}{\gamma m}\frac{d}{d\theta}\left(\frac{1}{r}\right)} \quad -(13)$$

Eqs. (10) and (13) define the relativistic orbital velocity:

$$v^2 = \dot{r}^2 + \dot{\theta}^2 r^2 = \frac{L^2}{\gamma^2 m^2}\left(\left(\frac{d}{d\theta}\left(\frac{1}{r}\right)\right)^2 + \frac{1}{r^2}\right) \quad -(14)$$

$$= \frac{L^2}{m^2}\left(1 - \frac{v^2}{c^2}\right)\left(\left(\frac{d}{d\theta}\left(\frac{1}{r}\right)\right)^2 + \frac{1}{r^2}\right)$$

i.e:

3)

$$v^2 = \frac{\frac{L^2}{m^2}\left(\left(\frac{d}{d\theta}\left(\frac{1}{r}\right)\right)^2 + \frac{1}{r^2}\right)}{1 + \frac{L^2}{m^2c^2}\left(\left(\frac{d}{d\theta}\left(\frac{1}{r}\right)\right)^2 + \frac{1}{r^2}\right)} \quad -(15)$$

$$= \frac{v_N^2}{1 + \left(\frac{v_N}{c}\right)^2}$$

where the non-relativistic or Newtonian velocity is

$$v_N^2 = \frac{L^2}{m^2}\left(\left(\frac{d}{d\theta}\left(\frac{1}{r}\right)\right)^2 + \frac{1}{r^2}\right) \quad -(16)$$

So in ECE2 the relativistic orbital velocity is:

$$\boxed{v^2 = \frac{v_N^2}{1 + \left(\frac{v_N}{c}\right)^2}} \quad -(17)$$

Eq. (17) gives the precisely correct result for light deflection due to gravitation as shown in UFT 324, i.e. as:

$$v_N \rightarrow c \quad -(18)$$

4) the relativistic orbital velocity becomes:

$$v^2 = \frac{c^2}{2} \quad -(19)$$

and the Newtonian deflection of the orbit:

$$\Delta\phi = \frac{2MG}{R_0 v_N^2} \quad -(20)$$

becomes the experimentally observed:

$$\Delta\phi = \frac{4MG}{R_0 c^2} \quad -(21)$$

<u>Q.E.D.</u>, <u>to very high precision</u>.

For the hyperbolic spiral orbit of a star in a whirlpool galaxy:

$$\frac{1}{r} = \frac{\theta}{r_0} \quad -(21)$$

the ECE2 theory gives:

$$v^2 = \frac{\frac{L^2}{m^2}\left(\frac{1}{r_0^2} + \frac{1}{r^2}\right)}{1 + \frac{L^2}{m^2 c^2}\left(\frac{1}{r_0^2} + \frac{1}{r^2}\right)} \quad -(22)$$

So

$$v \xrightarrow[r \to \infty]{} \frac{L}{m r_0}\left(1 + \frac{L^2}{m^2 c^2 r_0^2}\right)^{-1/2}$$

$$= \underline{\text{constant plateau}} \quad -(23)$$

5)

as observed experimentally; Q.E.D.

2) The Einsteinian Hamiltonian and Lagrangian are:

$$H(\text{Einstein}) = \frac{1}{2} m\left(\dot{r}^2 + r^2\dot{\theta}^2\left(1+\frac{r_0}{r}\right)\right) + U \quad -(24)$$

and
$$\mathcal{L}(\text{Einstein}) = \frac{1}{2} m\left(\dot{r}^2 + r^2\dot{\theta}^2\left(1+\frac{r_0}{r}\right)\right) - U \quad -(25)$$

where
$$U = -\frac{mMG}{r} \quad -(26)$$

and
$$r_0 = \frac{2MG}{c^2}. \quad -(27)$$

From eqs. (7) and (25):

$$L = \frac{\partial \mathcal{L}}{\partial \dot{\theta}} = \left(1+\frac{r_0}{r}\right) m r^2 \dot{\theta} \quad -(28)$$

so
$$\boxed{\dot{\theta} = \frac{L}{\left(1+\frac{r_0}{r}\right) m r^2}} \quad -(29)$$

From eq (12):

$$\dot{r} = -r^2 \dot{\theta} \frac{d}{d\theta}\left(\frac{1}{r}\right), \quad -(30)$$

$$\boxed{\dot{r} = \frac{-L}{\left(1+\frac{r_0}{r}\right) m}} \quad -(31)$$

6) So the Einsteinian orbital velocity is:

$$v^2(\text{Einstein}) = \frac{L^2}{m^2\left(1+\frac{r_0}{r}\right)^2}\left(\left(\frac{d}{d\theta}\left(\frac{1}{r}\right)\right)^2 + \frac{1}{r^2}\right), \quad -(32)$$

i.e.:

$$\boxed{v^2(\text{Einstein}) = \frac{v_N^2}{\left(1+\frac{r_0}{r}\right)^2}} \quad -(33)$$

As:

$$v_N \rightarrow c \quad -(34)$$

and at

$$r = R_0 \quad -(35)$$

the Newtonian light deflection due to gravitation is:

$$\Delta\phi = \frac{2mG}{R_0 v_N^2} \rightarrow \frac{2mG}{R_0 c^2}\left(1+\frac{r_0}{R_0}\right)^2 \quad -(36)$$

$$= \frac{2mG}{R_0 c^2}\left(1 + \frac{4mG}{R_0 c^2} + \left(\frac{2mG}{R_0 c^2}\right)^2\right)$$

and this is not the experimental result, QED.

The Newtonian orbital velocity is:

$$v_N^2 = mG\left(\frac{2}{r} - \frac{1}{a}\right) \quad -(37)$$

$$= MG\left(\frac{2}{r} - \frac{(1-\epsilon^2)}{\alpha}\right)$$

where

$$r = \frac{\alpha}{1+\epsilon\cos\theta} \quad -(38)$$

So:

$$v_N^2 = MG\left(\frac{2}{r} - \frac{(1-\epsilon^2)}{r}(1+\epsilon\cos\theta)\right)$$

$$\xrightarrow[r\to\infty]{} 0 \quad -(39)$$

Therefore the Einsteinian orbital velocity is:

$$v^2(\text{Einstein}) \doteq \frac{MG}{r\left(1+\frac{r_0}{r}\right)^2}\left(2-(1-\epsilon^2)(1+\epsilon\cos\theta)\right)$$

$$\xrightarrow[r\to\infty]{} 0 \quad -(40)$$

The Einsteinian theory fails completely in a whirlpool galaxy.

Finally the effective Einsteinian potential:

$$U = -\frac{mMG}{r}\left(1+\frac{\alpha}{r^2}\right) \quad -(41)$$

does not give a precessing ellipse:

$$r = \frac{\alpha}{1+\epsilon\cos(x\theta)} \quad -(42)$$

While ECE2 gives a precessing ellipse by numerical methods