325(3): The Einsteinian Orbit

The starting equation is:

$$\frac{1}{r} = \left(\frac{1}{\alpha} + \frac{3}{2}\frac{r_0}{r^2}\right)\left(1 + \epsilon\cos\theta\right) \quad -(1)$$

So

$$1 + \epsilon\cos\theta = \frac{1}{r\left(\frac{1}{\alpha} + \frac{3}{2}\frac{r_0}{r^2}\right)} \quad -(2)$$

and

$$\cos\theta = \frac{1}{\epsilon}\left(\frac{1}{r\left(\frac{1}{\alpha} + \frac{3}{2}\frac{r_0}{r^2}\right)} - 1\right) \quad -(3)$$

so

$$\boxed{\theta = \cos^{-1}\left[\frac{1}{\epsilon}\left(\frac{1}{\left(\frac{r}{\alpha} + \frac{3}{2}\frac{r_0}{r}\right)} - 1\right]\right.} \quad -(4)$$

The Newtonian result is

$$\theta_0 = \cos^{-1}\left(\frac{1}{\epsilon}\left(\frac{\alpha}{r} - 1\right)\right) \quad -(5)$$

Therefore $\theta - \theta_0$ can be plotted against r.

Here

$$r_0 = \frac{2MG}{c^2} \quad -(6)$$

and at the perihelion:

$$r = r_{min} = a(1 - \epsilon) = \frac{\alpha}{1+\epsilon} \quad -(7)$$

Therefore at the perihelion:

2)

$$\theta_0 = \cos^{-1} 1 = 0, 2\pi, \ldots \quad -(8)$$

We choose

$$\theta_0 = 2\pi \quad -(9)$$

At the perihelia:

$$\theta = \cos^{-1}\left[\frac{1}{\epsilon}\left(\frac{1}{\frac{1}{1+\epsilon} + \frac{3}{2}\frac{r_0}{\alpha}(1+\epsilon)} - 1\right)\right]$$

$$= \cos^{-1}\left[\frac{1}{\epsilon}\left(\frac{1+\epsilon}{1 + \frac{3}{2}\left(\frac{(1+\epsilon)^2 r_0}{\alpha}\right)} - 1\right)\right]$$

So at the perihelia: $-(9)$

$$\theta - \theta_0 = \cos^{-1}\left[\frac{1}{\epsilon}\left(\frac{1+\epsilon}{1+\frac{3}{2}\frac{r_0}{\alpha}(1+\epsilon)^2} - 1\right)\right] - 2\pi \quad -(10)$$

This is the true precession of the Einstein theory for eq. (1).