325(9): Definitive Refutation of the Method of Marion and Thornton.

The equation to be solved is:

$$\frac{d^2u}{d\theta^2} + u = \frac{1}{\alpha} + \delta u^2 \quad -(1)$$

where

$$u = \frac{1}{r} \quad -(2)$$

Here α is the half right latitude defined by:

$$L^2 = m^2 MG\alpha \quad -(3)$$

and

$$\delta = \frac{3GM}{c^2}. \quad -(4)$$

This equation comes from the Einsteinian force law defined by:

$$-\frac{m}{L^2} r^2 F(r) = \frac{m^2 MG}{L^2} + \frac{3MG}{c^2 r^2} \quad -(5)$$

i.e.

$$F(r) = -\frac{mMG}{r^2} - \frac{3MGL^2}{mc^2 r^4} \quad -(6)$$

It is known that eq. (6) does not produce:

$$r = \frac{\alpha}{1 + \epsilon \cos(x\theta)} \quad -(7)$$

The method used by Marion and Thornton is to claim that to a rough first approximation, a solution of eq. (1) is:

$$u_1 = \frac{1}{\alpha}\left(1+\epsilon\cos\theta\right) \quad -(8)$$

From eqs. (1) and (8):

$$\frac{d^2u}{d\theta^2} + u = \frac{1}{\alpha} + \frac{\delta}{\alpha^2}\left(1+2\epsilon\cos\theta + \epsilon^2\cos^2\theta\right) \quad -(9)$$

They then assume:

$$u_2 = u_1 + u_p \quad -(10)$$

where

$$u_p = \frac{\delta}{\alpha^2}\left[\left(1+\frac{\epsilon^2}{2}\right) + \epsilon\theta\sin\theta - \frac{\epsilon^2}{6}\cos 2\theta\right] \quad -(11)$$

and

$$\frac{d^2u_p}{d\theta^2} + u_p = \frac{\delta}{\alpha^2}\left(1+2\epsilon\cos\theta + \frac{\epsilon^2}{2}\left(1+\cos 2\theta\right)\right) \quad -(12)$$

so the solution is claimed to be:

$$u = \frac{1}{\alpha}\left(1+\epsilon\cos\theta\right) + \frac{\delta\epsilon}{\alpha^2}\theta\sin\theta + \frac{\delta}{\alpha^2}\left(1+\frac{\epsilon^2}{2}\right) - \frac{\delta\epsilon^2}{6\alpha^2}\cos 2\theta \quad -(13)$$

Therefore this is the claimed Einsteinian orbit.

Computer algebra can be used to plot the orbit (13) to find out what happens when δ is

3) increased.

Secondly the function (13) can be used in the Binet equation

$$F(r) = -\frac{L^2}{mr^2}\left(\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r}\right) \quad -(14)$$

to produce a force law in order to check whether it gives eq. (6).

In order to do this eq. (13) has to be solved for θ in terms of r, using:

$$\cos 2\theta = 2\cos^2\theta - 1 \quad -(15)$$

Eq. (13) is:

$$\frac{1}{r} = \frac{1}{\alpha}\left(1 + \epsilon\cos\theta\right) + \frac{\delta}{\alpha^2}\left[1 + \frac{\epsilon^2}{2} + \epsilon\theta\sin\theta - \frac{\epsilon^2}{6}\cos 2\theta\right] \quad -(16)$$

$-(17)$

Therefore:

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) = -\frac{\epsilon}{\alpha}\cos\theta + \frac{\delta}{\alpha^2}\left[2\cos\theta - \theta\sin\theta + \frac{2\epsilon^2}{3}\cos 2\theta\right]$$

and:

$$F(r) = -\frac{L^2}{mr^2}\left(\left[-\frac{\epsilon}{\alpha}\cos\theta + \frac{1}{r}\right] + \frac{\delta}{\alpha^2}\left[2\cos\theta - \theta\sin\theta + \frac{2\epsilon^2}{3}\cos 2\theta\right]\right) \quad -(18)$$

$$= -\frac{mMG}{r^2} - \frac{3MGL^2}{mc^2 r^4}$$

The Newtonian part is:

$$-\frac{L^2}{mr^2}\left(-\frac{\epsilon}{\alpha}\cos\theta + \frac{1}{r}\right) = -\frac{mMG}{r^2} \quad -(19)$$

and the Non-Newtonian part is: $\quad -(20)$

$$-\frac{L^2}{mr^2}\frac{\delta}{\alpha^2}\left[2\cos\theta - \theta\sin\theta + \frac{2\epsilon^2}{3}\cos 2\theta\right\} = -\frac{3MGL^2}{mc^2 r^4}$$

From eq. (19):

$$r = \frac{\alpha}{1+\epsilon\cos\theta} \quad -(21)$$

So

$$\cos\theta = \frac{1}{\epsilon}\left(1 - \frac{\alpha}{r}\right) \quad -(22)$$

However, eq. (22) is incompatible with eq. (13), and using eq. (22) in eq. (18) does not give eq. (6).