324(7): Velocity Curve of a Whirlpool Galaxy

The orbit of a star in a whirlpool galaxy is the hyperbolic spiral:

$$\frac{1}{r} = \frac{\theta}{r_0} \quad -(1)$$

Its velocity from eq. (2) of Note 324(6) is:

$$v^2 = \frac{\frac{L^2}{m^2}\left(\left(\frac{d}{d\theta}\left(\frac{1}{r}\right)\right)^2 + \frac{1}{r^2}\right)}{1 + \frac{L^2}{m^2c^2}\left(\left(\frac{d}{d\theta}\left(\frac{1}{r}\right)\right)^2 + \frac{1}{r^2}\right)} \quad -(2)$$

$$= \frac{\frac{L^2}{m^2}\left(\frac{1}{r_0^2} + \frac{1}{r^2}\right)}{1 + \frac{L^2}{m^2c^2}\left(\frac{1}{r_0^2} + \frac{1}{r^2}\right)} \quad -(3)$$

Therefore:

$$v \xrightarrow[r \to \infty]{} \frac{L}{m r_0}\left(1 + \frac{L^2}{m^2c^2r_0^2}\right)^{-1/2} \quad -(4)$$

$$= \underline{\text{constant}}.$$

The theory gives the correct $r \to \infty$ of the velocity curve of a whirlpool galaxy.

By definition:

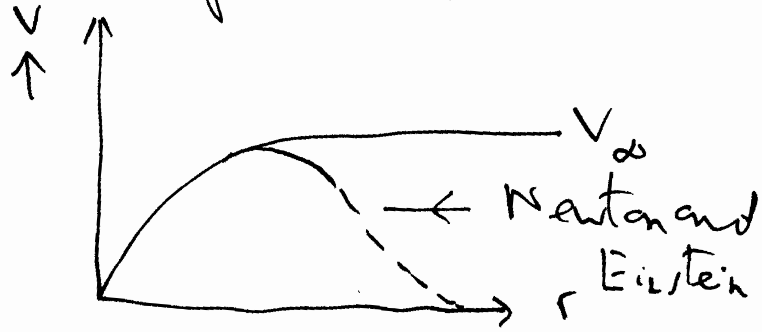
$$v^2 = \dot{r}^2 + \dot{\theta}^2 r^2 \quad -(5)$$

So

$$v \xrightarrow[r \to 0]{} 0 \quad -(6)$$

The velocity curve is therefore as sketched in the following:

Fig.(1)



where

$$v_\infty = \frac{L}{m r_0}\left(1 + \frac{L^2}{m^2 c^2 r_0^2}\right)^{-1/2} \quad -(7)$$

The Newtonian velocity is well known to be:

$$v^2(\text{Newton}) = mG\left(\frac{2}{r} - \frac{1}{a}\right)$$

$$= mG\left(\frac{2}{r} - \frac{1}{\alpha}(1-\epsilon^2)\right)$$

$$= \frac{mG}{r}\left(2 + \frac{(\epsilon^2-1)}{1+\epsilon\cos\theta}\right) \quad -(8)$$

So

$$v(\text{Newton}) \xrightarrow[r \to \infty]{} 0 \quad -(9)$$

as sketched in Fig (1).

The Einsteinian orbit is:

$$r = \frac{\alpha}{1+\epsilon\cos(x\theta)} \quad -(10)$$

which is a precessing ellipse or conic section, so in eq. (8):

$$v(\text{Einstein}) = \frac{mG}{r}\left(2 + \frac{(\epsilon^2 - 1)}{1+\epsilon\cos(x\theta)}\right) \quad -(11)$$

$$\xrightarrow[r \to \infty]{} 0$$

So both Newton and Einstein fail completely in whirlpool galaxies while ECE 2 describes the velocity curve straightforwardly from eqs. (2) and (5), given the observed orbit (1).

In the non-relativistic limit the integral form of Binet's equation is eq. (5) of note 324(4):

$$U = H - \frac{L^2}{2m}\left(\left(\frac{d}{d\theta}\left(\frac{1}{r}\right)\right)^2 + \frac{1}{r^2}\right) \quad -(12)$$

So

$$U = H - \frac{L^2}{2m}\left(\frac{1}{r_0^2} + \frac{1}{r^2}\right) \quad -(13)$$

for a hyperbolic spiral orbit. The force is

$$F = -\frac{\partial U}{\partial r} = -\frac{L^2}{mr^3} \quad -(14)$$

This result is also given by the Binet equation:

4)

$$F = -\frac{L^2}{mr^2}\left(\left(\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right)\right) + \frac{1}{r}\right) \quad -(15)$$

because from eq. (1):

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) = 0 \quad -(16)$$

Therefore the potential is:

$$U = \int \frac{L^2}{mr^3}\,dr \quad -(17)$$

$$= -\frac{L^2}{2mr^2}$$

From eqs. (13) and (17):

$$-\frac{L^2}{2mr^2} = H - \frac{L^2}{2mr_0^2} - \frac{L^2}{2mr^2} \quad -(18)$$

So

$$\boxed{H = \frac{L^2}{2mr_0^2}} \quad -(19)$$

This is the classical hamiltonian of the hyperbolic spiral orbit of a star in a whirlpool galaxy.

5) Conclusions

For the orbit of a star in a whirlpool galaxy:

$$\frac{1}{r} = \frac{\theta}{r_0} \quad -(20)$$

$$F = -\frac{L^2}{mr^3} \quad -(21)$$

$$U = -\frac{L^2}{2mr^2} \quad -(22)$$

$$H = \frac{L^2}{2mr_0^2} \quad -(23)$$

$$v_\infty = \frac{L}{mr_0}\left(1 + \frac{L^2}{m^2c^2r_0^2}\right)^{-1/2} \quad -(24)$$

These are Cotes spiral orbits. The star is attracted inward towards the centre of the galaxy and falls in to the centre in a spiral defined by eq. (20). There is no black hole at the centre of the galaxy, and there is no dark matter. The velocity of the star an infinite distance from the centre is given by eq. (24) and is constant.