320(4): General Considerations of the Gravitational Lorentz Force for an Orbit.

In general the velocity of the orbit in the static frame of the observer is:

$$\underline{v} = \dot{r}\,\underline{e}_r + \omega r\,\underline{e}_\theta \quad -(1)$$

and the acceleration in the static frame of the observer is:

$$\underline{a} = (\ddot{r} - r\dot{\theta}^2)\,\underline{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\underline{e}_\theta \quad -(2)$$

It is seen that $\underline{v}$ is parallel to $\underline{a}$. In general the force in the static observer frame is:

$$\underline{F} = m\ddot{r}\,\underline{e}_r + m\left(-r\dot{\theta}^2\,\underline{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\underline{e}_\theta\right) \quad -(3)$$

In the Newtonian theory in a non rotating frame:

$$\underline{v}_N = \dot{r}\,\underline{e}_r \quad -(4)$$

and

$$\underline{a}_N = \ddot{r}\,\underline{e}_r \quad -(5)$$

The other terms are generated by frame rotation. In a planar orbit it was shown in UFT 235 that:

$$r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 \quad -(6)$$

so

$$\underline{v} = \dot{r}\,\underline{e}_r + \omega r\,\underline{e}_\theta \quad -(7)$$

and

$$\underline{a} = (\ddot{r} - r\dot{\theta}^2)\,\underline{e}_r \quad -(8)$$

2) The non Newtonian terms are due to frame rotation and are:

$$\underline{v}_{rot} = \omega r \underline{e}_\theta = \underline{\omega} \times \underline{r} \quad -(9)$$

and

$$\underline{a}_{rot} = - r \dot{\theta}^2 \underline{e}_r \quad -(10)$$

$$= - r \omega^2 \underline{e}_r = - \underline{\omega} \times (\underline{\omega} \times \underline{r})$$

Here $\underline{v}_{rot}$ is the orbital velocity of a particle m in orbit around a particle M in a plane. The orbital velocity is the velocity of the rotating frame w.r.t respect to the static observer frame.

Therefore $\underline{v}_{rot}$ is the velocity of the Lorentz force equation:

$$\underline{F} = \underline{F}_N + \underline{v}_{rot} \times \underline{\Omega} \quad -(11)$$

where $\underline{\Omega}$ is the gravitomagnetic field.

By experiment in non-relativistic orbital dynamics:

$$\underline{F} = - \frac{mMG}{r^2} \underline{e}_r \quad -(12)$$

So

$$\underline{F}_N = \ddot{r} \underline{e}_r = \left(- \frac{mMG}{r^2} + \omega^2 r \right) \underline{e}_r \quad -(13)$$

This is the 1689 Leibnitz equation of orbits.

3) Similarly, the Newtonian velocity of the static frame of the observer is:

$$\underline{V}_N = \dot{r}\,\underline{e}_r = \underline{V} - \omega r\,\underline{e}_\theta \quad -(14)$$

$$= \underline{V} - \underline{\omega} \times \underline{r}$$

$$= \underline{V} - \underline{V}_{rot}$$

The Newtonian force from eq. (11) is:

$$\underline{F}_N = \underline{F} - \underline{V}_{rot} \times \underline{\Omega} \quad -(15)$$

$$= \underline{F} + \underline{\omega} \times (\underline{\omega} \times \underline{r})$$

So:

$$-\underline{V}_{rot} \times \underline{\Omega} = \underline{\omega} \times (\underline{\omega} \times \underline{r}) \quad -(16)$$

i.e.

$$-\omega r\,\underline{e}_\theta \times \underline{\Omega} = \omega^2 r\,\underline{e}_r \quad -(17)$$

i.e.

$$\boxed{\underline{\Omega} = -\omega\,\underline{k}} \quad -(18)$$