320(2): The Gravitomagnetic Field of the Earth.

Consider the gravitomagnetic Lorentz force:

$$\underline{F} = \frac{d\underline{p}}{dt} = m(\underline{g} + \underline{v} \times \underline{\Omega}) \quad -(1)$$

in the non-relativistic limit, and apply this to the orbit of a mass m around the earth, or any orbit in a plane. As shown in UFT236, the velocity is:

$$\underline{v} = \frac{d\underline{r}}{dt} = \frac{dr}{dt}\underline{e}_r + r\frac{d\underline{e}_r}{dt} \quad -(2)$$

where

$$\underline{e}_r = \cos\theta\,\underline{i} + \sin\theta\,\underline{j} \quad -(3)$$

and

$$\underline{e}_\theta = -\sin\theta\,\underline{i} + \cos\theta\,\underline{j} \quad -(4)$$

so

$$\frac{d\underline{e}_r}{dt} = \frac{d\theta}{dt}\underline{e}_\theta = \omega\,\underline{e}_\theta \quad -(5)$$

So

$$\underline{v} = \frac{dr}{dt}\underline{e}_r + \omega r\,\underline{e}_\theta \quad -(6)$$

$$= \frac{dr}{dt}\underline{e}_r + \underline{\omega} \times \underline{r}$$

The acceleration is:

$$\underline{a} = \frac{d\underline{v}}{dt} = (\ddot{r} - r\dot{\theta}^2)\underline{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\underline{e}_\theta \quad -(7)$$

2) where:

$$(\ddot{r}-r\dot{\theta}^2)\underline{e}_r = \frac{d^2r}{dt^2}\underline{e}_r + \underline{\omega}\times(\underline{\omega}\times\underline{r}) \quad -(8)$$

and:

$$(r\ddot{\theta}+2\dot{r}\dot{\theta})\underline{e}_\theta = \frac{d\underline{\omega}}{dt}\times\underline{r} + 2\underline{\omega}\times\dot{\underline{r}} \quad -(9)$$

As shown in UFT 235, for all planar orbits:

$$\underline{a} = \frac{d\underline{v}}{dt} = \frac{d^2\underline{r}}{dt^2} = \frac{d^2r}{dt^2}\underline{e}_r + \underline{\omega}\times(\underline{\omega}\times\underline{r}) \quad -(10)$$

where:

$$\underline{\omega}\times(\underline{\omega}\times\underline{r}) = -\omega^2 r\,\underline{e}_r = -\frac{L^2}{m^2r^3}\underline{e}_r \quad -(11)$$

Therefore:

$$\underline{a} = \left(\frac{d^2r}{dt^2} - \frac{L^2}{m^2r^3}\right)\underline{e}_r \quad -(12)$$

$$= \left(\frac{d^2r}{dt^2} - \omega^2 r\right)\underline{e}_r$$

The force on the orbiting mass m is:

$$\underline{F} = m\underline{a} = m\left(\frac{d^2r}{dt^2} - \omega^2 r\right)\underline{e}_r \quad -(13)$$

). However, the force between the orbiting m and the earth's mass M is given by the Hooke inverse square law:

$$\underline{F} = -\frac{mMG}{r^2}\,\underline{e}_r \quad -(14)$$

From eqs. (13) and (14):

$$\frac{d^2 r}{dt^2} = -\frac{MG}{r^2} + \omega^2 r \quad -(15)$$

Here $\omega^2 r$ is the centrifugal acceleration, which is positive valued and outward from the earth and

$$g = -\frac{MG}{r^2} \quad -(16)$$

is the acceleration due to gravity, which is inwards and negative valued.

The force is:

$$\underline{F} = m\left(\underline{g} + \omega^2 r\,\underline{e}_r\right). \quad -(17)$$

By comparison of eqns. (1) and (17):

$$\underline{v} \times \underline{\Omega} = \omega^2 r\,\underline{e}_r \quad -(18)$$

where

$$\underline{v} = \frac{dr}{dt}\,\underline{e}_r + \omega r\,\underline{e}_\theta \quad -(19)$$

Now use:

$$\underline{e}_r = \underline{e}_\theta \times \underline{k} \quad -(20)$$

$$\underline{e}_\theta = \underline{k} \times \underline{e}_r \quad -(21)$$

$$\underline{k} = \underline{e}_r \times \underline{e}_\theta \quad -(22)$$

to find that:

$$\boxed{\underline{\Omega} = \omega \underline{k}} \quad -(23)$$

where ω is the angular velocity of the earth:

$$\omega = \frac{d\theta}{dt} = 7.292115 9 \times 10^{-5} \text{ rad s}^{-1} \quad -(24)$$

The acceleration due gravity $\underline{g}$ at the earth's surface is

$$\underline{g} = 9.80665 \text{ m s}^{-2} \quad -(25)$$

Therefore $\underline{g}$ is about ~~six~~ five orders of magnitude greater than $\underline{\Omega}$ for the earth.

This note opens up a new era of cosmology in which the existence is recognized of the gravito-magnetic field.