320(1): Lorentz Covariance and Lorentz Force of ECE2

The ECE2 field equations are:

$$\partial_\mu \tilde{G}^{\mu\nu} = 0 \quad -(1)$$

and

$$\partial_\mu G^{\mu\nu} = J^\nu \quad -(2)$$

where:

$$\tilde{G}^{\mu\nu} = \begin{bmatrix} 0 & -c\Omega^1 & -c\Omega^2 & -c\Omega^3 \\ c\Omega^1 & 0 & g^3 & -g^2 \\ c\Omega^2 & -g^3 & 0 & g^1 \\ c\Omega^3 & g^2 & -g^1 & 0 \end{bmatrix} \quad -(3)$$

and

$$G^{\mu\nu} = \begin{bmatrix} 0 & -g^1 & -g^2 & -g^3 \\ g^1 & 0 & -c\Omega^3 & c\Omega^2 \\ g^2 & c\Omega^3 & 0 & -c\Omega^1 \\ g^3 & -c\Omega^2 & c\Omega^1 & 0 \end{bmatrix}. \quad -(4)$$

Here

$$\left.\begin{array}{l} g^1 = g_X,\ g^2 = g_Y,\ g^3 = g_Z \\ \Omega^1 = \Omega_X,\ \Omega^2 = \Omega_Y,\ \Omega^3 = \Omega_Z \end{array}\right\} \quad -(5)$$

Lorentz transformation gives the results:

$$\underline{g}' = \gamma\left(\underline{g} + \underline{v} \times \underline{\Omega}\right) - \frac{\gamma^2}{\gamma+1}\frac{\underline{v}}{c}\left(\frac{\underline{v}}{c}\cdot\underline{g}\right) \quad -(6)$$

$$\underline{\Omega}' = \gamma\left(\underline{\Omega} - \frac{\underline{v}}{c^2}\times\underline{g}\right) - \frac{\gamma^2}{\gamma+1}\frac{\underline{v}}{c}\left(\frac{\underline{v}}{c}\cdot\underline{\Omega}\right) \quad -(7)$$

2) These exactly parallel the well known results in electrodynamics (Jackson, third edition):

$$\underline{E}' = \gamma\left(\underline{E} + \underline{v} \times \underline{B}\right) - \frac{\gamma^2}{\gamma+1}\frac{\underline{v}}{c}\left(\frac{\underline{v}}{c} \cdot \underline{E}\right) \quad -(8)$$

and

$$\underline{B}' = \gamma\left(\underline{B} - \frac{\underline{v}}{c^2} \times \underline{E}\right) - \frac{\gamma^2}{\gamma+1}\frac{\underline{v}}{c}\left(\frac{\underline{v}}{c} \cdot \underline{B}\right) \quad -(9)$$

Here γ is the Lorentz factor:

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad -(10)$$

where $\underline{v}$ is the velocity of the ticked frame with respect to the static frame of the observer.

The gravitational Lorentz force is:

$$\underline{F} = m\left(\underline{g} + \underline{v} \times \underline{\Omega}\right) \quad -(11)$$

where $\underline{g}$ is the acceleration due to gravity and where $\underline{\Omega}$ is the gravitomagnetic field. Note that in magnitude:

$$g^{(0)} = c\,\Omega^{(0)} \quad -(12)$$

3) which exactly parallels:

$$E^{(0)} = cB^{(0)} \quad -(13)$$

If

$$v << c \quad -(14)$$

$$\underline{g}' = \gamma(\underline{g} + \underline{v} \times \underline{\Omega}) \quad -(15)$$

and

$$\underline{\Omega}' = \gamma\left(\underline{\Omega} - \frac{1}{c^2}\underline{v} \times \underline{g}\right) \quad -(16)$$

So the Lorentz force is:

$$\underline{F} = \frac{1}{\gamma} m\underline{g}' \quad -(17)$$

Define the force as:

$$\underline{F} = \frac{d\underline{p}}{dt} = \frac{1}{\gamma} m\underline{g}' \quad -(18)$$

and use:

$$\gamma = \frac{dt}{d\tau} \quad -(19)$$

to find that:

$$\underline{F} = \frac{d\underline{p}}{d\tau} = m\underline{g}' \quad -(20)$$

Here τ is the proper time, the time in the ticked frame. The time in the observer frame is t, and in the

4) observer frame:

$$\underline{F} = \frac{d\underline{p}}{dt} = m(\underline{g} + \underline{v} \times \underline{\Omega}) \quad -(21)$$

This generalizes the Newtonian result:

$$\underline{F} = m\underline{g} \quad -(22)$$

Note carefully that these are all equations of ECE2 generally covariant unified field theory, and no longer of unnunified special relativity. They are all derived from Cartan geometry with torsion and curvature both non zero.

The units of the gravitomagnetic field $\underline{\Omega}$ are the same as those of angular velocity, and the motion of a mass m in a static gravitomagnetic field is a precession:

$$\frac{d\underline{p}}{dt} = m\underline{v} \times \underline{\Omega} \quad -(23)$$

and

$$\frac{d\underline{g}}{dt} = \underline{0} \quad -(24)$$

In previous papers the equinoctial precession was investigated with the gravitomagnetic field.

As in previous UFT papers, when there is frame rotation in a plane, the velocity is defined by:

$$\underline{v} = \frac{d\underline{r}}{dt} = \frac{d}{dt}\left(r\underline{e}_r\right) - (25)$$

$$= \dot{r}\underline{e}_r + r\dot{\underline{e}}_r$$

and the acceleration by:

$$\underline{a} = \frac{d\underline{v}}{dt} = \frac{d}{dt}\left(\dot{r}\underline{e}_r + r\dot{\underline{e}}_r\right) - (26)$$

The Newtonian part of eq. (26) is:

$$\underline{a} = \frac{d}{dt}(\dot{r})\,\underline{e}_r = \frac{d^2 r}{dt^2}\,\underline{e}_r - (27)$$

In addition, there are other terms from application of the Leibnitz Theorem, so:

$$\underline{a} = \left(\frac{d}{dt}(\dot{r})\right)\underline{e}_r + \dot{r}\frac{d\underline{e}_r}{dt} + \frac{d}{dt}\left(r\dot{\underline{e}}_r\right)$$

$$- (28)$$

In the context of gravitation:

$$\underline{g} = \frac{d^2 r}{dt^2}\,\underline{e}_r - (29)$$

is the Newtonian or inertial acceleration due to gravity.

6)

Eq. (29) is true if and only if there is no rotational motion, i.e. no rotation of the frame of reference. More generally:

$$\underline{F} = m\underline{g} + m\left(\dot{r}\frac{d\underline{e}_r}{dt} + \frac{d}{dt}\left(r\,\dot{\underline{e}}_r\right)\right)$$

$$= m\underline{g} + m\left(\frac{d\underline{\omega}}{dt}\times\underline{r} + 2\underline{\omega}\times\frac{dr}{dt}\underline{e}_r + \underline{\omega}\times(\underline{\omega}\times\underline{r})\right) \quad -(30)$$

Here

$$\underline{\omega} = \frac{d\underline{\theta}}{dt} \quad -(31)$$

is the angular velocity of the rotating axes. Eq. (30) was derived by G. G. Coriolis in 1835.

It is therefore concluded that in the presence of a rotating frame:

$$\boxed{\underline{v}\times\underline{\Omega} = \frac{d\underline{\omega}}{dt}\times\underline{r} + 2\underline{\omega}\times\frac{dr}{dt}\underline{e}_r + \underline{\omega}\times(\underline{\omega}\times\underline{r}) \quad -(32)}$$

In Eq. (32) $\underline{v}$ is the velocity of one frame

7) with respect to another in the Lorentz transform.

From eq. (25) the velocity is defined by:

$$\underline{v} = \frac{d\underline{r}}{dt} = \frac{dr}{dt}\underline{e}_r + r\dot{\underline{e}}_r$$

$$= \frac{dr}{dt}\underline{e}_r + \underline{\omega} \times \underline{r}. \quad -(33)$$

The Newtonian part of the velocity is:

$$\underline{v}_N = \frac{dr}{dt}\underline{e}_r, \quad -(34)$$

and the velocity due to frame rotation is:

$$\underline{v}_r = \underline{\omega} \times \underline{r}. \quad -(35)$$

In a non-rotating coordinate system:

$$\underline{F} = m\underline{g} \quad -(36)$$

so

$$\underline{v} \times \underline{\Omega} = \underline{0} \quad -(37)$$

is the traditional Newtonian view. In general, however:

$$\underline{v} = \underline{v}_N + \underline{\omega} \times \underline{r} \quad -(38)$$

so

$$(\underline{v}_N + \underline{\omega} \times \underline{r}) \times \underline{\Omega} \quad -(39)$$

$$= \frac{d\underline{\omega}}{dt} \times \underline{r} + 2\underline{\omega} \times \frac{dr}{dt}\underline{e}_r + \underline{\omega} \times (\underline{\omega} \times \underline{r})$$
